NON-RIGID POINT SET REGISTRATION: COHERENT POINT DRIFT (CPD)

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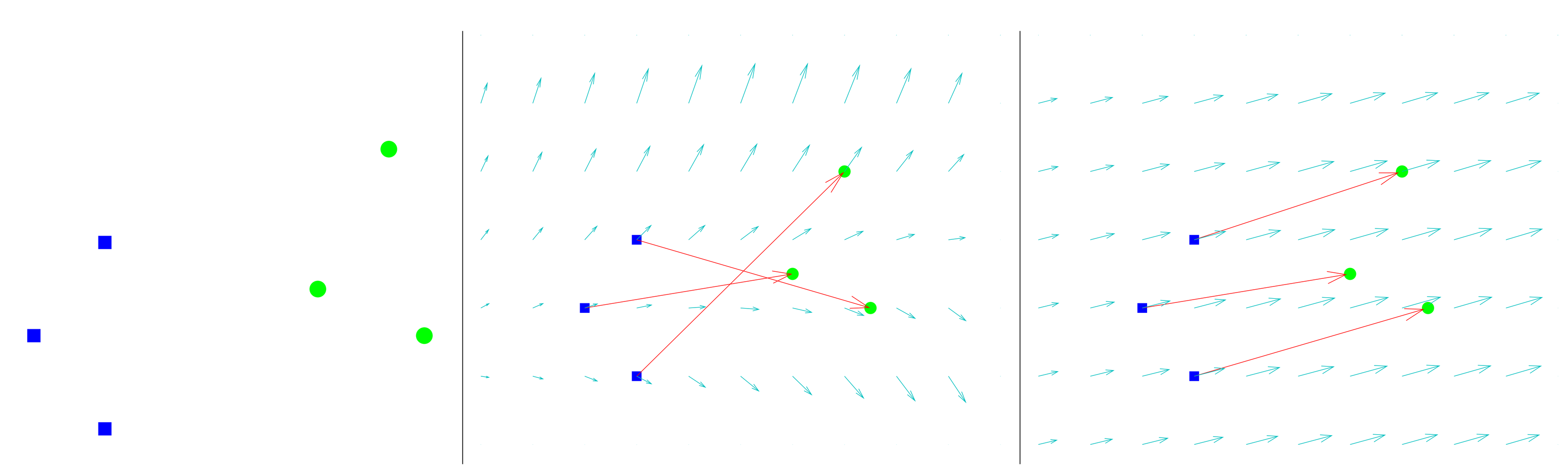
1 ABSTRACT

We introduce Coherent Point Drift (CPD), a novel probabilistic method for non-rigid registration of point sets. The registration is treated as a Maximum Likelihood (ML) estimation problem with motion coherence constraint over the velocity field such that one point set moves coherently to align with the second set. We formulate the motion coherence constraint and derive a solution of regularized ML estimation through the variational approach, which leads to an elegant kernel form. We also derive the EM algorithm for the penalized ML optimization with deterministic annealing. The CPD method simultaneously finds both the non-rigid transformation and the correspondence between two point sets without making any prior assumption of the transformation model except that of motion coherence. This method can estimate complex non-linear non-rigid transformations, and is shown to be accurate on 2D and 3D examples and robust in the presence of outliers and missing points.

2 PROBLEM STATEMENT

- The registration problem is to find meaningful correspondence between two point sets and to recover the underlying transformation that maps one point set to the second.
- Non-rigid registration assumes that the underlying transformation, required to align point sets, is complex, locally non-linear .
- GOAL - develop a non-rigid registration method to align two N-dimensional sets of points with complex non-linear underlying transformation in presence of noise, outliers and missing points.
- Intuition - points close to one another tend to move coherently.

Different correspondences give rise to less and more coherent velocity fields.



3 METHOD

- Given two point sets.
  - ⋆ $\mathbf{X}_{N\times D}$ - reference point set (data points);
  - ⋆ $\mathbf{Y}_{M\times D}$ - template point set (GMM centroids);
- Consider the points in $\mathbf{Y}$ as the centroids of a Gaussian Mixture Model, and fit it to the data points $\mathbf{X}$ by maximizing the likelihood function.
- Denote $\mathbf{Y}_0$ as the initial centroid positions and define a continuous velocity function v for the template point set such that the current position of centroids is defined as $\mathbf{Y} = v(\mathbf{Y}_0) + \mathbf{Y}_0$.
- Find $\mathbf{Y}$ by MAP. Minimize: $E(\mathbf{Y}) = -\sum_{n=1}^{N} \log \sum_{m=1}^{M} e^{-\frac{1}{2}\left\|\frac{\mathbf{x}_n-\mathbf{y}_m}{\sigma}\right\|^2} + \frac{\lambda}{2}\phi(v)$ (1)
- $\phi(v)$ is the regularization to ensure the velocity field v (displacement) to be smooth. One choise is to measure the high frequency content: $\phi(v) = \int \frac{|\tilde{v}(\mathbf{s})|^2}{\tilde{G}(\mathbf{s})} d\mathbf{s}$, where $\tilde{v}$ indicates the Fourier transform of the velocity. $\tilde{G}$ represents a symmetric low-pass filter.
- **It can be shown using a variational approach that the function which minimizes E has the form:**

$$v(\mathbf{z}) = \sum_{m=1}^{M} \mathbf{w}_m G(\mathbf{z} - \mathbf{y}_{0m}) \implies \mathbf{Y} = \mathbf{Y}_0 + \mathbf{G}\mathbf{W}, \quad \text{where } \mathbf{G}_{M\times M} : \text{Gaussian affinity matrix}$$

- The motivations to choose a Gaussian kernel form for G:
  - ⋆ It satisfies the required properties (symmetric, positive definite, and $\tilde{G}$ approaches zero as $\|\mathbf{s}\| \to \infty$).
  - ⋆ Gaussian form in both frequency and time domain without oscillations.
  - ⋆ The flexibility to control the range of filtered frequencies and thus the amount of spatial smoothness.
  - ⋆ It is equivalent to Motion Coherence Theory (MCT) prior: sum of weighted squares of all order derivatives $\int \sum_{m=1}^{\infty} \frac{\beta^{2m}}{m!2^m}(D^m v)^2$.

Motion Coherence
- penalizes all order derivatives
- easily generalizes to N-dimensions
- the extra parameter provides the flexibility to control the locality of spatial smoothness

Thin Plate Spline (TPS)
- penalizes second order derivatives
- problem with generalization to more that 3D
- doesn't have the control of spatial smoothness locality

- EM optimization:

Minimization of E in Eq. 1 is equivalent to minimization of its upper bound Q:

$$Q(\mathbf{W}) = \sum_{n=1}^{N}\sum_{m=1}^{M} P^{old}(m|\mathbf{x}_n)\frac{\|\mathbf{x}_n - \mathbf{y}_{0m} - \mathbf{G}(m,\cdot)\mathbf{W}\|^2}{2\sigma^2} + \frac{\lambda}{2}\mathrm{tr}(\mathbf{W}^T\mathbf{G}\mathbf{W})$$

The minimum of Q function is a solution of a linear system of equation:

$$[\mathrm{diag}(\mathbf{P}\cdot\mathbf{1})\mathbf{G} + \lambda\sigma^2\mathbf{I}]\cdot\mathbf{W} = [\mathbf{P}\mathbf{X} - \mathrm{diag}(\mathbf{P}\cdot\mathbf{1})\mathbf{Y}_0] \;\; (\mathrm{M-step}) \qquad (2)$$

where $\mathbf{P}$ is the matrix of posterior probabilities with elements $p_{mn} = \frac{e^{-\frac{1}{2}\|\frac{\mathbf{y}_m - \mathbf{x}_n}{\sigma}\|^2}}{\sum_{m=1}^{M} e^{-\frac{1}{2}\|\frac{\mathbf{y}_m - \mathbf{x}_n}{\sigma}\|^2} + (2\pi\sigma^2)^{\frac{D}{2}}/a}$.

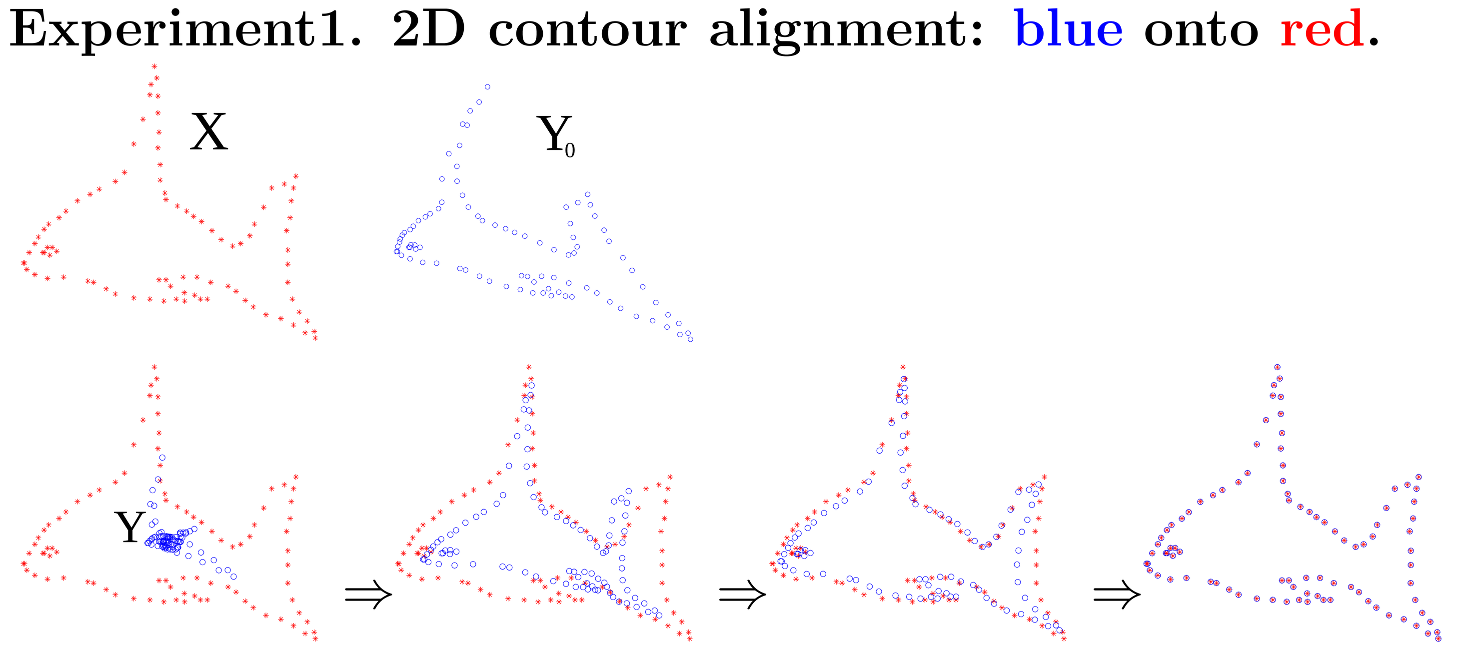
- Additional uniform pdf component is added to the mixture model in oder to account for outliers.

CPD algorithm:

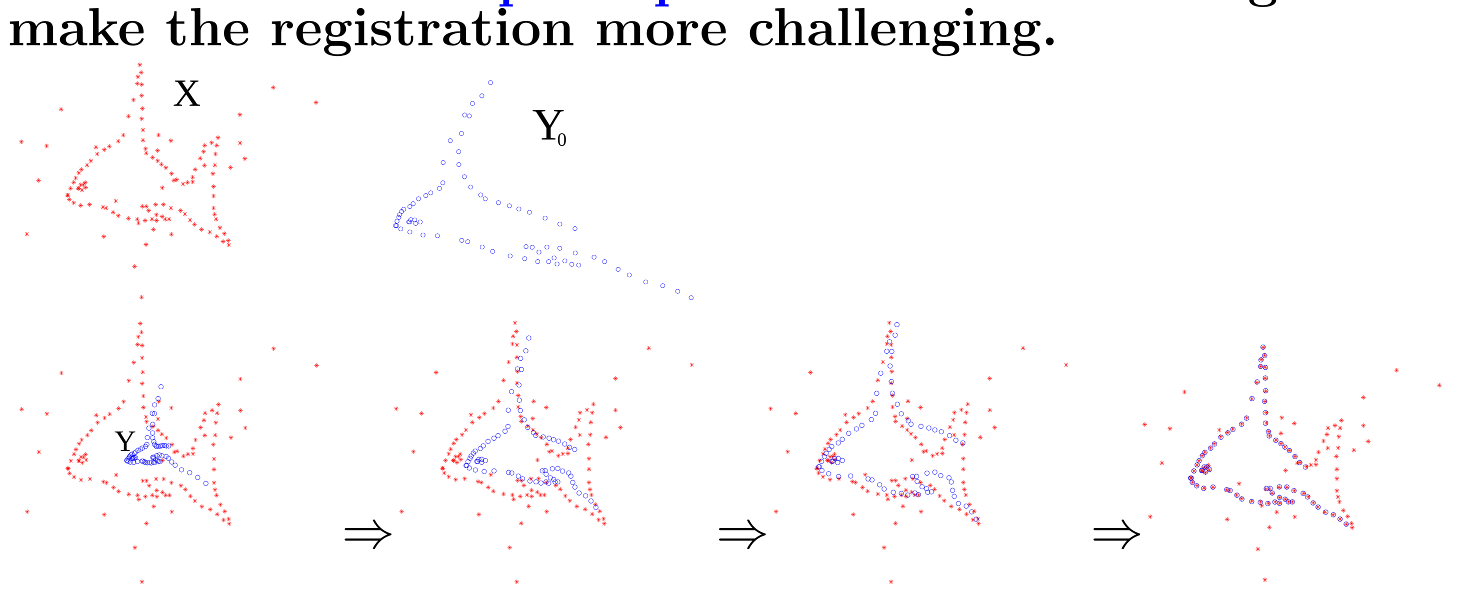
- Initialize parameters λ, β, σ
- Construct $\mathbf{G}$ matrix, initialize $\mathbf{Y} = \mathbf{Y}_0$
- Deterministic annealing:
  - ⋆ EM optimization:
    - ∗ E-step: Compute $\mathbf{P}$;
    - ∗ M-step: Solve for $\mathbf{W}$ from Eq. (2);
    - ∗ Update $\mathbf{Y} = \mathbf{Y}_0 + \mathbf{G}\mathbf{W}$.
  - ⋆ Anneal $\sigma = \alpha * \sigma$
- Compute the velocity field: $v(\mathbf{z}) = \mathbf{G}(\mathbf{z},\cdot)\mathbf{W}$
- Find the correspondences from posterion probabilities $\mathbf{P}$

4 RESULTS

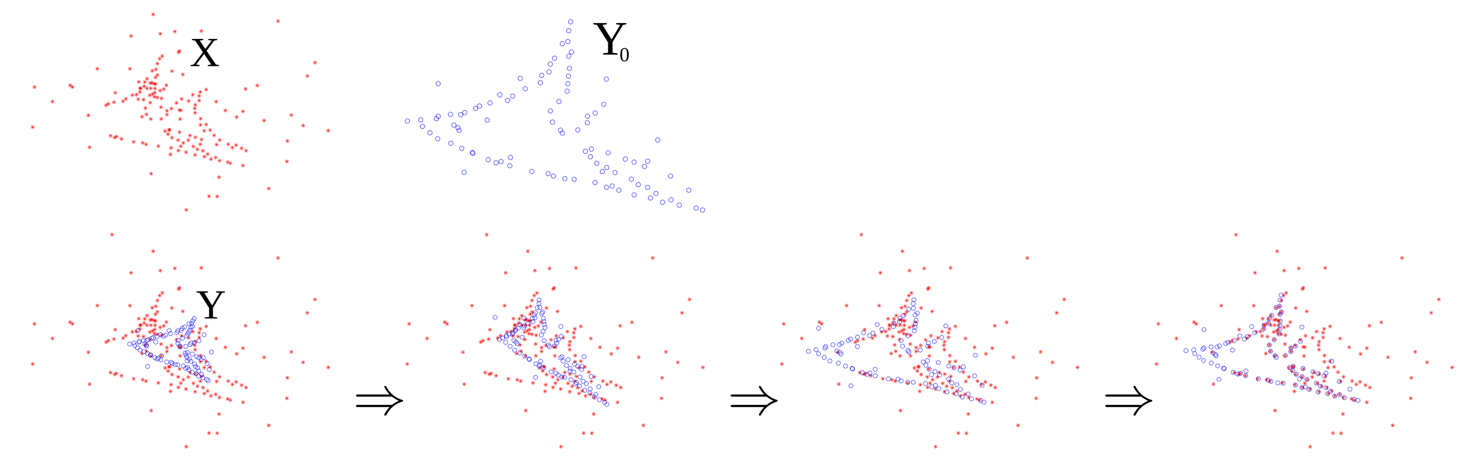
Experiment1. 2D contour alignment: blue onto red.



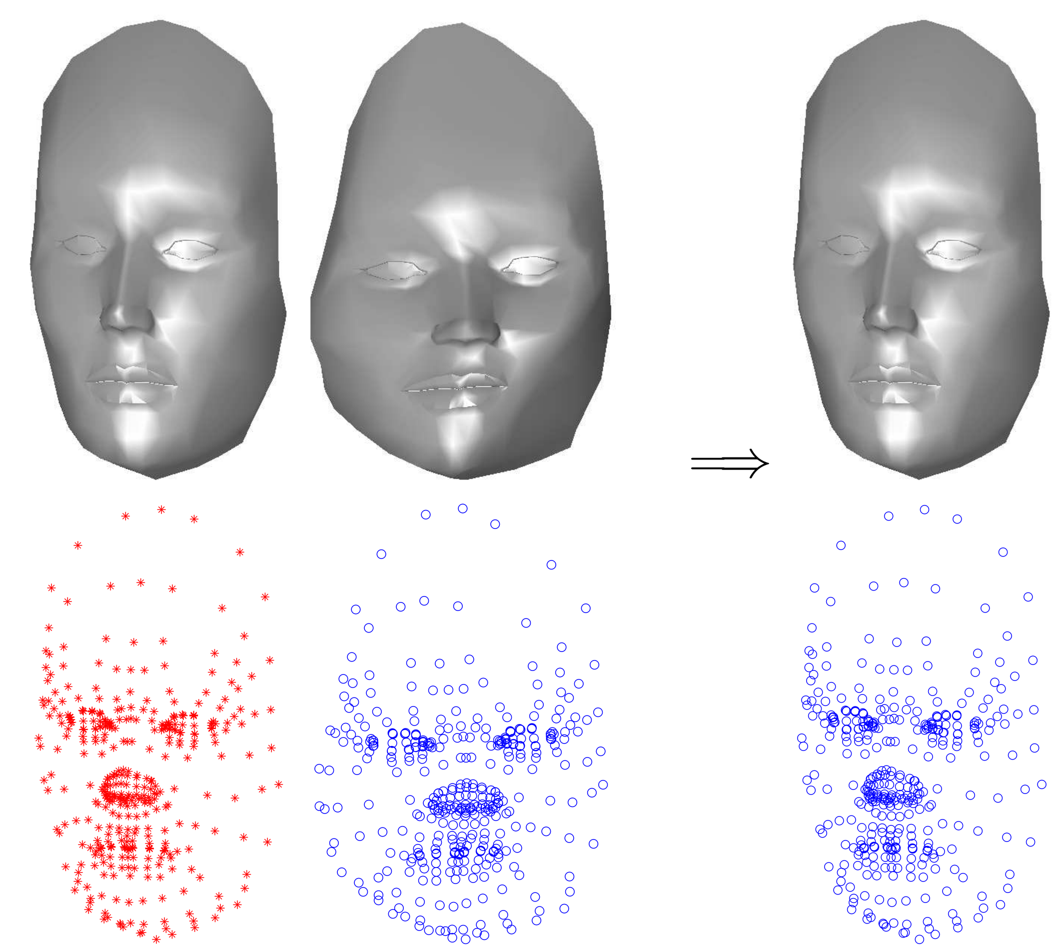
Experiment2. The reference point set is corrupted by noise and the template point set has a missing tail to make the registration more challenging.



Experiment3. The reference point set has a missing head and the template point set has a missing tail. Both points sets are corrupted by noise.



Experiment4. Non-rigid alignment of 3D surfaces using its control points: blue onto red.



5 CONCLUSIONS

We introduce Coherent Point Drift (CPD), a new probabilistic method for non-rigid point set registration. The registration is considered as a GMM fitting, where one point set represents centroids and the other represents the data. We regularize the velocity field to enforce coherent motion. We derive the form of the solution for the penalized ML estimation and estimate it with EM and deterministic annealing. The estimated velocity field represents the underlying non-rigid transformation. The correspondence between the two point sets is inferred through the posterior probability of the GMM components.

6 REFERENCES

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`Matlab code is available at www.csee.ogi.edu/~myron/matlab/cpd`