FREE-FORM NON-RIGID IMAGE REGISTRATION: USING GENERALIZED ELASTIC NETS

OGI SCHOOL OF SCIENCE & ENGINEERING

OREGON HEALTH & SCIENCE UNIVERSITY

Andriy Myronenko, Xubo Song and Miguel Á. Carreira-Perpiñán

Department of Computer Science & Electrical Engineering, OGI, OHSU

Portland, OR, USA

1 ABSTRACT

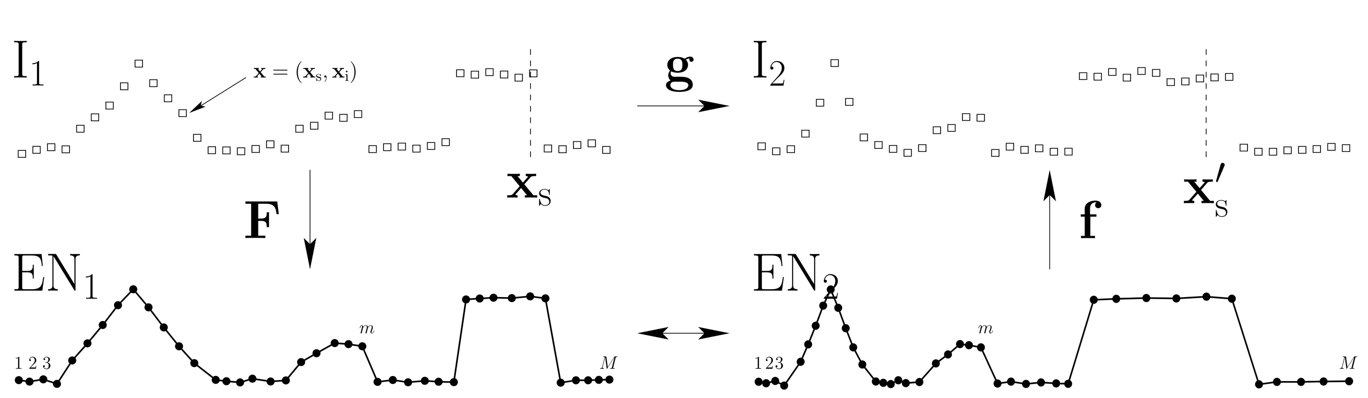
We introduce a novel probabilistic approach for nonrigid image registration using generalized elastic nets, a model previously used for topographic maps. The idea of the algorithm is to adapt an elastic net (a constrained Gaussian mixture) in the spatial-intensity space of one image to fit the second image. The resulting net directly represents the correspondence between image pixels in a probabilistic way and recovers the underlying image deformation. We regularize the net with a differential prior and develop an efficient optimization algorithm using linear conjugate gradients. The nonparametric formulation allows for complex transformations having local deformation.

2 METHOD

The *elastic net* is a Gaussian mixture model (GMM) with a quadratic prior on its centroids. The centroids implicitly represent a nonlinear manifold that probabilistically models an image in spatial-intensity space.

- We represent two images I_1 and I_2 in the spatial-intensity space.
- The elastic net $\mathbf{Y}$ is initialized with each centroid representing the spatial-intensity value of one pixel in I_1.
- The net is adapted by adjusting the centroids to fit data $\mathbf{X}$ (image I_2 in spatial-intensity space) by MAP estimation.
- the final centroid locations ($\mathbf{Y}$), when E is minimized, directly show the displacement of each pixel in I_1 deformed into I_2.

Illustration of the alignment method (for 1D images, for simplicity).



Minimize: $E(\mathbf{Y}) = -\sum_{n=1}^{N} \log \sum_{m=1}^{M} e^{-\frac{1}{2}\left\|\frac{\mathbf{x}_n - \mathbf{y}_m}{\sigma}\right\|^2} + \frac{1}{2}\operatorname{tr}\left(\mathbf{Y}^T\mathbf{S}\mathbf{Y}\right)$

where $\mathbf{S} = \beta_1\mathbf{D}_1^T\mathbf{D}_1 + \beta_2\mathbf{D}_2^T\mathbf{D}_2$, and $\mathbf{D}_1$, $\mathbf{D}_2$ are first- and second-order derivatives. Weights β_1 and β_2 control tearing and folding regularization. $\|\mathbf{DY}\|^2 = \operatorname{tr}\left((\mathbf{DY})^T(\mathbf{DY})\right) = \operatorname{tr}\left(\mathbf{Y}^T\mathbf{D}^T\mathbf{D}\mathbf{Y}\right)$

$$\mathbf{D}_1 = \begin{pmatrix} \cdots\cdots\cdots\cdots\cdots \\ \cdots 1 \;\; \text{-}1 \;\; 0 \;\; \cdots \\ \cdots 0 \; 1 \;\; \text{-}1 \;\; 0 \cdots \\ \cdots\cdots\cdots\cdots\cdots \end{pmatrix} \quad \mathbf{D}_2 = \begin{pmatrix} \cdots\cdots \;\; \cdots \;\; \cdots\cdots \;\; \cdots \;\; \cdots \;\;\; \cdots \\ 1 \;\; \cdots \;\; 0\;1 \;\; \text{-}4 \;\; 1\;0 \;\; \cdots \;\; 0\;1\;0 \;\; \cdots \\ 0\;1 \;\; \cdots \;\; 0\;1 \;\; \text{-}4 \;\; 1\;0 \;\; \cdots \;\; 0\;1\;0 \cdots \\ \cdots\cdots \;\; \cdots \;\; \cdots\cdots \;\; \cdots \;\; \cdots \;\;\; \cdots \end{pmatrix}$$

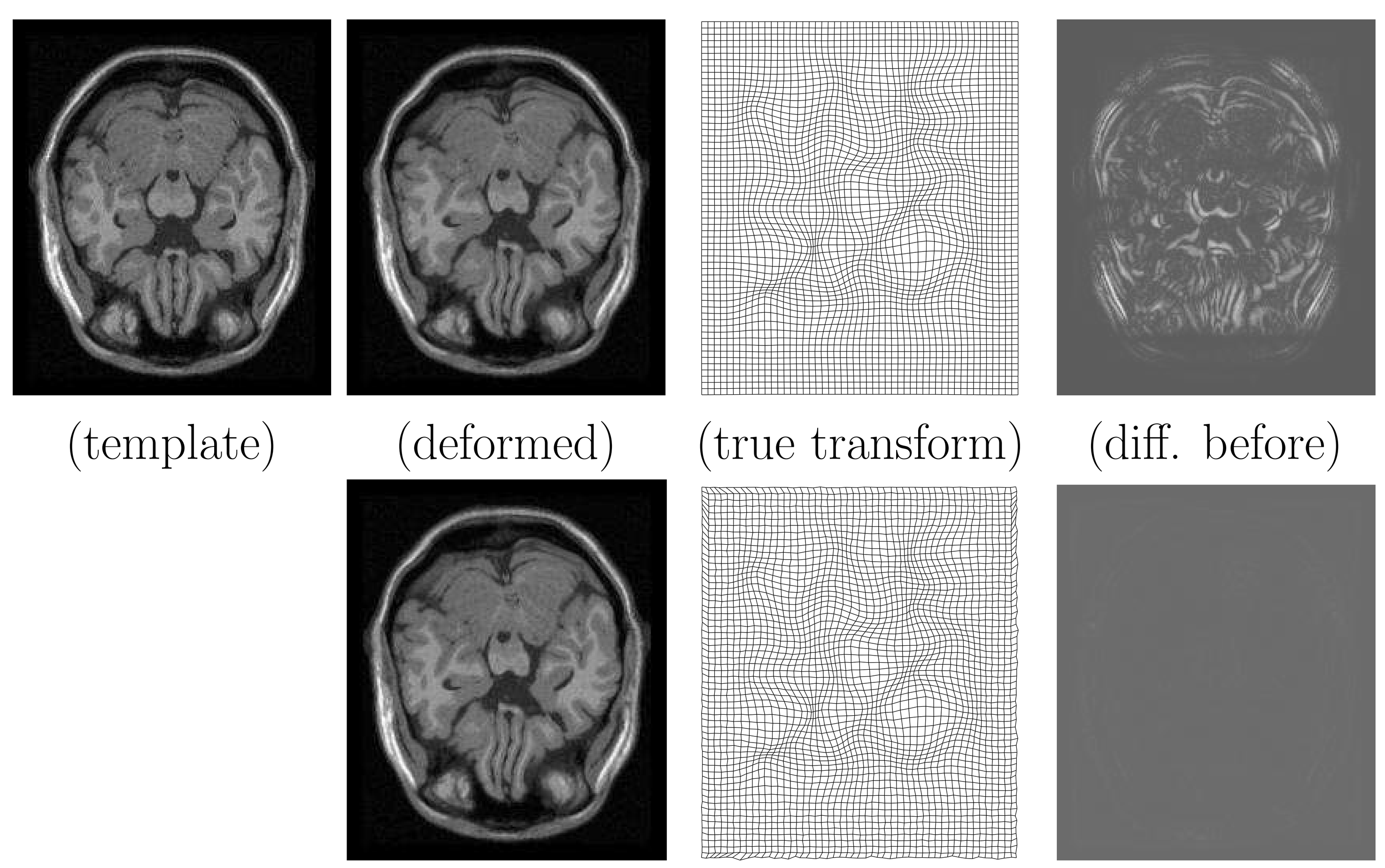
Taking the derivative of $E(\mathbf{Y})$ and equating it to zero, we obtain non-linear system of equations:

Solve: $(\operatorname{diag}(\mathbf{P1}) + \sigma^2\mathbf{S})\mathbf{Y} = \mathbf{PX}$

where $\mathbf{P}$ matrix has elements $p_{mn} = e^{-\frac{1}{2}\left\|\frac{\mathbf{x}_n - \mathbf{y}_m}{\sigma}\right\|^2} / \sum_{k=1}^{M} e^{-\frac{1}{2}\left\|\frac{\mathbf{x}_n - \mathbf{y}_k}{\sigma}\right\|^2}$. We use fixed-point iteration to solve the system. On each iteration we use linear conjugate gradient (CG) method, which has following advantages: (1) we can initialize the linear CG from the previous $\mathbf{Y}$ value; (2) we can run only a few linear CG steps.

3 EXPERIMENTAL RESULTS

GEN registration with known deformation.

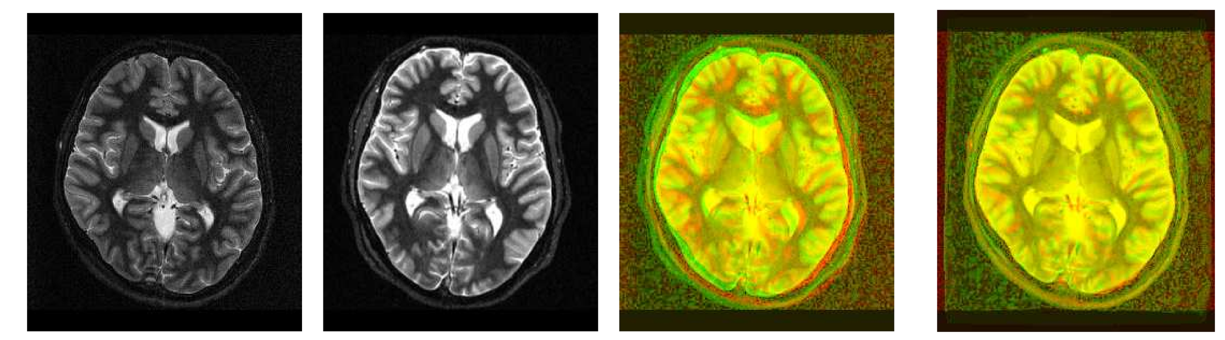


(template) (deformed) (true transform) (diff. before)

(result) (found transform) (diff. after)

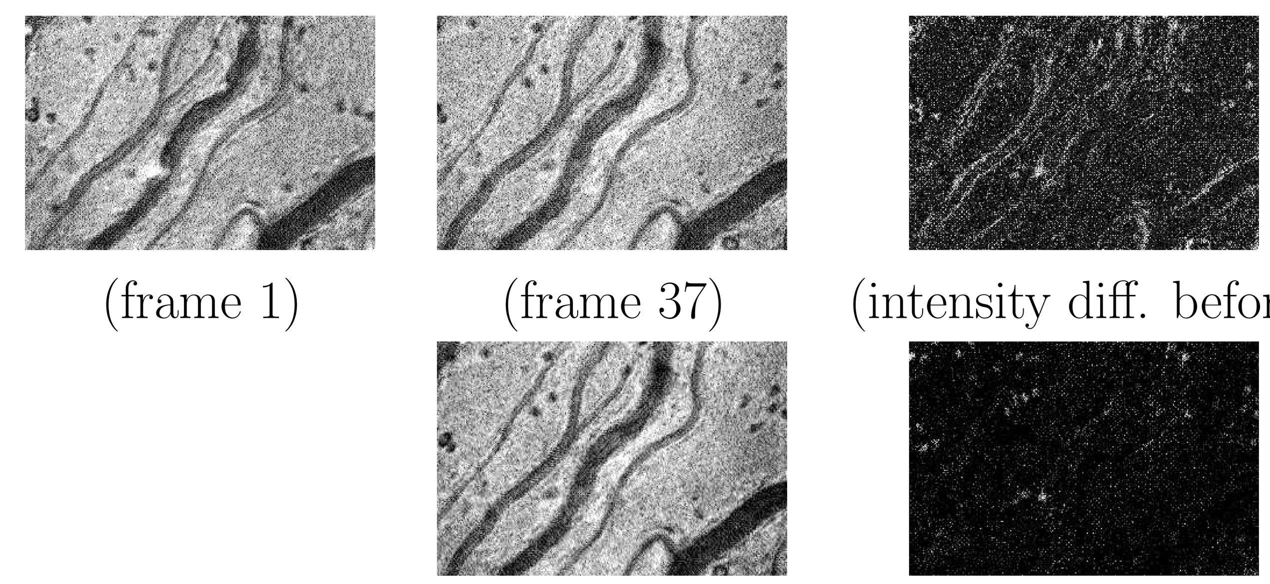
Deform. STD	GEN: Transform RMSE (pixels)	GEN: Intensity RMSE	ITK: Intensity RMSE
1.0	0.3135	0.0044	0.0094
2.0	0.9753	0.0053	0.0117
3.0	1.0962	0.0059	0.0143

Intersubject brain registration.



Person 1 Person 2 before after GEN reg.

Stabilization of microscopic iris videos.



(frame 1) (frame 37) (intensity diff. before)

(registration result) (intensity diff. after)

4 CONCLUSION

We have developed a probabilistic nonrigid image registration method based on the generalized elastic net. The resulting formulation is a penalized maximum likelihood problem. The nonparametric transformation allows to model complex and localized deformations flexibly without prior knowledge about the type of transformation required, and to use sophisticated regularizers (e.g. high-order derivatives and linear combinations of them). The structured, sparse nature of the regularizer matrix allows an efficient optimization with linear conjugate gradients, faster than thin-plate splines. The method accurately registers images with nonlinear local deformations, and has robustness to image intensity distortion. The method accommodates arbitrary features (e.g. gradient information and color components), spatial dimensions (e.g. 3D, 4D), and images of different spatial resolutions.

ACKNOWLEDGMENT: this work is partially supported by NIH grant NEI R01 EY013093, NSF grant IIS–0313350 (awarded to X. Song) and NSF CAREER award IIS–0546857 (awarded to M. Á. Carreira-Perpiñán).