On the Statistical Mistreatment of Index Numbers

In a recent comment in this journal Lord [1] presented an interesting parable concerning some statistical adventures with "football numbers." The purpose of the paper was not expressly stated; however, certain clues were given. The following statements appeared:

> "But you can't multiply 'football numbers,'" the professor wailed. "Why, they aren't even ordinal numbers, like test scores."
>
> "The numbers don't know that," said the statistician. "Since the numbers don't remember where they came from, they always behave just the same way. . . ."
>
> He [the professor] is happy because, when he has added together a sample of 1,600 "football numbers," he finds that the resulting sum obeys the same laws of sampling as they would if they were real honest-to-God cardinal numbers (p. 751).

The moral of Lord's tale appears to be: "He [the professor] will no longer lock his door when he computes the means and standard deviations of test scores."

If Lord's comment had a point to make, and if the point is reasonably reflected by the quotations (both being my assumptions), then the point seems to be in error. Unfortunately it is not an uncommon error when wandering in the realm of the theory of numbers.

Neither purely nominal nor purely ordinal numbers can be manipulated by measurement statistics. However, numbers for which there happens to be a set of lawful cardinal axioms *can* be manipulated by measurement statistics. Furthermore, these numbers *can* be used for nominal or ordinal purposes. In addition, having been used for index purposes by someone who has forgotten how he used them, they *can* then be attacked with measurement statistics. However, having assigned a set of numbers in a one-to-one relation to a physical system either as index or ordinal numbers, and then having manipulated these numbers in terms of their cardinality, one *can* obtain some very precise, but precisely meaningless, information about the physical system.

Consider a set of numbers capable of identifying football players to TV audiences: Joe Faguda No. **Γ**, Paul Blunt No. **Δ**, Bob Smyth No. **Λ**. Perfectly good "football numbers!" These numbers have attached to them a set of rules that do not extend into cardinality or ordinality. However, when we use more familiar and traditional numbers, Joe Faguda No. 6, Paul Blunt No. 9, Bob Smyth No. 2, these numbers do have a set of historical rules attached, the rules of cardinality; $2 + 2 + 2 = 6$ for example. When we use these numbers as football numbers, it is easy to start using these handy, but irrelevant, cardinal rules. Bob Smyth + Bob Smyth + Bob Smyth = Joe Faguda.

To treat Bob Smyth's "2" in a traditional counting (cardinal) manner is to fail to recognize that the 2, 6, and 9 have merely been borrowed as convenient symbols for identifying football players.

If a statistician sets to discussing these numbers in terms of bigness or smallness, highness or lowness, and with the further assumption that $2 + 2 + 2 = 6$, etc., then he is making an analysis of the numbers on the basis of their available cardinality. Pure index numbers do not have highness or lowness. When we use only the numbers **Γ**, **Δ**, **Λ**, a test of significance of difference borders on the idiotic.

If we had a large number of **Γ**'s, **Δ**'s, and **Λ**'s, and we counted them and found 27**Γ**'s, 44**Δ**'s, and 16**Λ**'s, we might apply measurement statistics to our 27, 49, and 16. Counting is the prime characteristic of cardinality. However, now we are not considering football numbers, but counts of football numbers. We certainly cannot obtain means and standard deviations. What is

$$\overline{X} = \frac{27\boldsymbol{\Gamma} + 44\boldsymbol{\Delta} + 16\boldsymbol{\Lambda}}{87}?$$

If we randomly used more conventional numbers, like 6 for **Γ**, 9 for **Δ**, and 2 for **Λ**, and we slip off the deep end and obtain a mean

$$\overline{X} = \frac{27 \times 6 + 44 \times 9 + 16 \times 2}{87} = 6.8$$

this mean makes no more sense than the previous one.

To make the mean meaningful, we must insist upon some cardinal aspects of lowness to highness in respect to our 2, 6, and 9. When such is the case, we have imposed a lot more meaning than just football-player identification. We *have* cardinality.

So it is with Lord's parable. The freshman-sophomore argument settled by the statistician was one of cardinal highness or lowness in a set of numbers used in an entirely different context to identify football players. Our Professor X had best re-retire; his helpful statistical friend had best return to his TV set. I at least shall continue to lock my door when computing the means and standard deviations of test scores.

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[1] LORD, F. M. On the statistical treatment of football numbers. *Amer. Psychologist*, 1953, 8, 750–751.

Further Comment on "Football Numbers"

It is only fair to any prospective critic to state clearly the final conclusions reached in my note, "On the Statistical Treatment of Football Numbers," which appeared in the December 1953 *American Psychologist*. The conclusion is that nominal and ordinal numbers (including test scores) may be treated by the usual