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My own viewpoint on mental measurement (1887)*, **

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Summary. The paper presents a translation of excerpts from Fechner's (1887) paper "On the principles of mental measurement and on Weber's law", which was his last and "most perfect" (Wundt) statement of the assumptions underlying his outer psychophysics. Fechner maintains that all measurement, including mental measurement, rests on the principle that n magnitudes that are judged equal may be added and result in a magnitude n times as large as the individual magnitudes. He concedes that bisection methods fulfill this principle as well as just noticeable differences. Weber's law is not a necessary precondition of mental measurement; its validity is an empirical question rather than a matter of principle. The differential threshold is not an inherent property of sensation or attention, but depends on the unavoidable spatio-temporal noncoincidence of stimuli and of the sensations corresponding to them. Given this presupposition and assigning a value of zero to the absolute threshold, it is possible to arrive at a scale of sensation differences, and thus of sensations, from a scale of difference sensations.

1. My starting point is a principle which, I think, taken as a general principle of measurement will not be found objectionable to a physicist or to a philosopher.

Given several values, in any field, which may be taken to be magnitudes inasmuch as they can be thought of as increasing or decreasing; given the possibility of judging the occurrence of equality and inequality in two or more of these values when they are observed simultaneously or successively; and given that n values have been *found* equal or, if they can be varied freely, have been *made* equal: then it is self-evident (because it is a matter of definition and therefore a tautology) that their total magnitude, which coincides with their sum, equals $n \times$ their individual magnitudes. It follows that each single value, or each definite fraction or each definite multiple of the magnitudes that have been found equal (no matter which), can be taken as the unit according to which the total magnitude, or every fraction of it, can be measured. The n equal parts that can be thought of as composing a total magnitude of course have the same magnitude as the n equal parts into which the total magnitude can be thought to be decomposable.

All physical measurement is based on this principle. All mental measurement will also have to be based on it, provided that the concept of magnitude is applied to it in the above sense; and this is true, at any rate, for the mental magnitudes which will be dealt with in what follows. Whether or not it has wider application need not concern us here, as long as we are merely interested in the possibility and justification of mental measurement *as such.*

Now, even in the field of physics the use of this principle encounters the difficulty that the equality of two or more magnitudes can never be observed or produced with *absolute* precision. It follows that even the most accurate physical, geodetical, astronomical measurements, which in the last instance depend on determinations of equality, are always open to a so-called probable error and perhaps also to a constant error. But this does not prevent all exact physical measurement from being based on the above principle. All that needs to be done in its empirical application is to mimimize the error of single observations by using the highest possible resolving power of the means of observation to compensate for the constant error whenever the field of observation permits, and to increase precision by the multiplication and averaging out of the observations – procedures which need not be considered in detail at the present moment. All this will not give us absolutely exact measurements, but still the sort of measurements that not only will be sufficient in practice, where the measurement of single objects is concerned, but also can lead to more general regularities on which a more general insight into the connection and succession of physical objects is founded. And all this can be transferred to the mental field. Only the empirical application of the general measurement principle is more difficult on the mental side and therefore the actual performance of measurement according to the the existing methods of psychophysical measurement is much less simple. In general, mental measurement is not particularly relevant to practical life. But it has enormous scientific importance and far-reaching implications. First, because of the common subordination of both the mental and the physical realms to the principle of mathematical determination; and second, because of the lawful relation between mental and physical magnitudes which automatically obtains when a mental measure is found. In-

* *Excerpts from:* G. T. Fechner, Über die psychischen Massprincipien und das Weber'sche Gesetz. *Philosophische Studien,* 1887, *4,* 161–230. The translated sections are on pages 178–198 of the original.

** Footnotes indicated in the text by subscript numbers are translated from Fechner's original German text; those indicated by subscript lowercase letters are the translator's comments

stead of going ahead with generalities of this sort, we shall demonstrate the application of the principle in a particularly instructive and historically remarkable case ....

2. Leaving aside the first class of stellar magnitudes, which contains only the brightest stars of very different brightnesses, astronomers find an equal differene in brightness between stars of second and third, third and fourth, fourth and fifth magnitude; in fact, they standardize the classes of magnitude in this way. To avoid the consequences of this fundamental fact, one might say the following. Either it is not equal *mental*, but equal *physical* (photometric), differences by which the different classes of magnitude are separated from each other; or it is only according to an old tradition whereby stars of different brightness are ordered into classes, the equality of perceived differences playing no role in a matter in which sensation is not qualified to pass judgment. Granted that the same difference in brightness is perceived between two stars of the physical brightnesses A and B as between two other stars of the same physical brightnesses A and B; nevertheless, given three or more stars $A, B, C \ldots$ in an increasing order of physical brightness, it will be impossible to tell whether the perceived difference in brightness between B and C is or is not equal to the difference between B and A, and in general it will be impossible to establish the equality of *perceived* differences or *difference sensations* (expressions which I use synonymously) in different parts of the brightness scale.

It is indeed necessary to make this distinction, and *the entire question as to whether a mental measure is totally impossible revolves around this very impossibility.* However, the possibility of finding that the perceived difference between brightnesses in one part of the brightness scale is equal to the perceived difference between brightnesses in another part of it is proved by the *fact* of the astronomical judgment of equality of such differences, and the two objections labelled *'either', 'or'* above are equally and decisively replaced by a *'neither','nor'*.

In fact, the first objection – which says that the magnitude classes of stars are ordered according to equal physical or photometric, rather than according to equal mental or perceived, brightness differences – is definitely contradicted by the following. Long after the order of magnitudes had been established by astronomers in accordance with the equality of perceived differences without any photometric measurement, photometric examination carried out much later showed that what corresponds to equal differences in classes of magnitude are not equal *differences*, but, in the sense of *Weber's* well-known law, equal *ratios* of photometric brightness, which renders the course of stellar magnitude classes quite disparate from the course of their photometric values. Because this discovery was made much later than the ordering of stellar magnitudes, the other assumption (to which astronomers themselves would object strongly[1]) was also rejected, viz., that the magnitude classes were initially distinguished by arbitrary convention only, barring any estimation of the equality of their distances according to the judgment of sensation (i.e., according to equally perceived differences) and that they were maintained by pure habit only. For it would have been a most unlikely coincidence if the fundamental law of psychophysics (*Weber's* law) had been established precisely in this way.

Meanwhile we can do more, in this respect, than appeal to observations in the sky and mere improbability, because indoor experiments with voluntary manipulation of physical brightnesses furnish direct proof that difference sensations may be found to be equal in different regions of the brightness scale where the same fundamental law of psychophysics that was deduced from estimations of stellar magnitudes is corroborated within the limits in which it can presumed to be valid – this being one of the ways to establish *Weber's* law. It is easy to guess that I am referring to the *Plateau-Delboeuf* procedure which *Wundt* has named the *'bisection method'*, the essence of which consists in the following. Given two surfaces A, B, of different physical brightness, where B is brighter than A, then on the mental side some difference in brightness will be perceived. Whatever it may be, it will be possible to add an even brighter surface C and to modify its brightness until the difference in brightness between B and C appears equal, on the mental side, to the difference in brightness between A and B. Accordingly, we find in the three surfaces A, B, C the circumstances obtaining for three stars of successively ordered magnitudes. Or alternatively: between two surfaces, A, C, of different physical brightness a third surface B may be inserted in such a way that the total perceived difference in brightness between A and C may be fractionated into two equally perceived differences between A and B and between B and C, and the total difference between A and C can be considered to be measured as 2 if both of the two components are assigned the value of 1. Correspondingly, the perceived, and thus the mental difference between the second and the fourth stellar magnitude classes will be decomposable into the two differences, perceived to be equal, between the second and the third, and between the third and the fourth magnitude classes, and thus it must be taken to be twice as large as each of these differences.

Now it might be said that if the perceived difference between A and B has been found equal to the perceived difference between B and C, then it by no means follows that the total perceived difference between A and C, if compared *directly*, will exactly equal double the two partial differences perceived separately; it might be a totally indefinite function of both. But in fact this could not be said, because to do so would mean contradicting a tautological sentence. Nor would it be possible to say in physics: If three weights A, B, C are given and if the weight difference between A and B has been found equal to the weight difference between B and C, then it by no means follows that the weight difference between A and C is twice as large as those two partial differences. We simply *call* a total difference twice as large as each of the two equal partial differences of which it is composed, in the above sense, or into which it can be thought to be decomposable; and I do not see any reason why in this respect there should be any difference between the mental and the physical fields. We also obtain the same advantages of definiteness and computability in the mental, as in the physical, field. Indeed, if we have an entire series of stars or surfaces $A, B, C, D \ldots$ whose physical brightnesses, as well as

[1] In fact, for the purpose of finer determinations, astronomers divide the differences among magnitude classes in tenths, without photometric assistance, and accordingly they add a decimal to the integer value of a star's magnitude

whose perceived brightnesses, are ordered in an increasing sequence, and if the differences between A and B and between B and C, and so on are perceived to be equal, then – if the objection is correct – one can only say that the mental differene between two more distant terms of the series would be greater than the difference between two nearer terms; but what is to be done with such a vague statement? However, following our conception, which is based on the general principles of measurement, every mental difference can be expressed as a definite multiple of the difference between the two nearest terms, and we shall know that if a perceived brightness difference is described as two or three times as large as another, then we shall find the former by interleaving two or three brightness differences equal to the latter; and this is a fact which admits conclusions that can be used in experimentation and computation.

In order to discuss how a measure of difference sensations or of perceived differences can be found, I have appealed to the estimation of stellar magnitudes and to the bisection method, whereby the equality of differences greater than the just noticeable difference is estimated. This has been because the discussion in this case seemed to me most easily comprehensible; however, one can also appeal to the method of just noticeable differences (which was my starting point in the "Elements"), because it proves for small perceived differences the same that is proved for greater differences by estimations of stellar magnitude and the bisection method, viz., that it is possible to estimate the equality of mental differences in different regions of the stimulus and sensation scale; and, as was stated above, the possibility of measuring perceived differences depends on this precondition. In this respect the methods of bisection and of just noticeable differences (j.n.d.) complement each other to good advantage. True, it has been denied that it is possible to estimate the equality of very small, i.e., just noticeable differences; but, I think, to no avail. *First, Weber's* law can be tested only if this possibility is conceded, and if that is denied, the law could not have been established (within the limits for which it is valid) according to this method. *Second,* because the bisection method has demonstrated that it is possible to judge the equality of larger perceived differences, certain theoretical or empirical reasons should be given to show why this would not be possible beyond certain limits of smallness; however, such reasons have not been given so far. *Third,* I can appeal to my own observations, where in order to establish *Weber's* law I have used the j.n.d. method, taking the utmost care in judging equal the small difference in various regions of the stimulus and sensation scales. Of course, certain mistakes (such as observation errors) can occur in this procedure, but this is also true for the bisection method and in general for methods of observation in any field; it is always necessary to reduce errors by multiplying observations and taking their average. ... *Finally,* even if all these reasons showing why equality judgments about just noticeable differences are possible were not found convincing, any remaining doubt would remain inconsequential, because estimations of stellar magnitude and the bisection method, where this possibility undoubtedly exists with respect to greater differences, are already sufficient to prove that a mental measure can be obtained. I have taken these proofs as my present starting point precisely because the objections raised against just noticeable differences cannot be raised against them. Nevertheless, taken from a different angle just noticeable differences still have advantages because it is easier to make the transition to differentials, which leads to the elaboration of general measurement formulae such as the fundamental formula.

3. Thus it can be taken for granted that, based on equality judgments in different regions of the stimulus and sensation scales at least for mental (that is, perceived) differences or differene sensations, a measure can be found such that we can tell how often a smaller difference sensation (concerning more proximate physical magnitudes) is contained in a greater difference sensation (concerning more distant physical magnitudes). This is adequate for the general principle of measurement; and all philosophical counter-demonstrations are, I think, mere writing in the sand.

It can be remarked, though, that a measure of perceived differences does still not provide a measure of the sensations between which the difference is perceived; but we shall be able to show how we can arrive at the latter from the former. And even if it were not possible to reach the latter, still the former would provide us with a mental measure and thus the task of demonstrating its mere possibility would have been fulfilled. At any rate, even if we do not arrive at a measure of sensation, and totally independent of it, a measure of perceived differences presents some interest even for the sake of practical applications, as has been demonstrated by the estimates of stellar magnitude which are founded on it: at the same time a proof that it not only exists in theory, but has been realized, as it were, on the largest existing scale in the world.

True, because sensations (with which we are dealing here) are evoked by physical stimuli and are specified by their relation to this source, their measure as well as the measure of the differences perceived between them can also be established solely with respect to this source. Thus, when talking about perceived differences between stellar magnitude classes (on which their gradation depends) we must refer to the stars that either fall into these classes or are specified by their physical light intensity.

Indeed, if we know that equal difference sensations belong to equal stimulus ratios, then all we have to do is to take as a unit the difference sensation corresponding to an arbitrary stimulus ratio in an arbitrary region of the stimulus and sensation scale, and to arrive at the n-times difference sensation (d.s.) by multiplying the stimulus ratio n times by itself; which means that the d.s. referring to the initial and the final stimulus, at which we arrive by raising the power with the exponent n, is n times as great as the unit in question ...

Meanwhile it is imperative to stress that the measurability of difference sensations is by no means restricted to the validity of *Weber's* law, indeed, that in principle it has nothing to do with the law. Granted, for outer psychophysics, which includes all experimentally based mental measurement, beyond certain limits *Weber's* law is not valid enough to serve as the foundation of the mental measure; and for very small and very large stimulus magnitudes it seems to fail altogether. Well then, if it fails, it has to be discarded; and mental measurements have to be based on some other relation between mental differences and stimulus differences or ratios in the relevant parts of the stimu-

lus and sensation scales corresponding to each other, provided that it is revealed experimentally.

Nevertheless, within the limits of its approximate validity – which are fairly wide for vision and very wide for hearing, especially as far as the ordinary use of the senses is concerned – *Weber's* law still remains the simplest and most important basis for the use of the principles of mental measurement and for the derivation of the psychophysical formulae. Its importance increases if it is conceded that, as I myself maintain, the law can be transferred from external stimuli, where it is only approximate, to the psychophysical excitation that depends on external stimuli (and on such internal stimuli as are present); that is, if one concedes that it can be transferred from outer to inner psychophysics. This is not conceded by all; but here we need not touch upon the relevant controversy though in other respects it is extremely important; for the question – with which we are dealing here – whether a mental measure is possible, has to be decided exclusively on the basis of facts from outer psychophysics.

4. In order to attain a measurement of sensation from the measurement of difference sensations or perceived differences, some preliminary considerations are necessary. We must deal with the existence and nature of the so-called *difference threshold* and of the *sensation differences* that occupy a middle place between difference sensations and sensations.

In fact, a difference between sensations, in brief a *sensation difference*, may exist without the difference entering into consciousness; or, as I am wont to express it, without being *perceived*, without giving a *perceived difference* or a *difference sensation*. The need for a distinction between sensation differences and difference sensations or perceived differences is immediately revealed by the following observation. When two sensations occur in different people, or when in one person one of the two sensations has been forgotten while the other is arising, the real difference between them, i.e., the sensation difference, however great it may be, still will not serve as the basis for a difference sensation, because not even a *comparison* of the sensations is possible. But even when we can make a comparison – for instance when two surfaces of physically different brightnesses are simultaneously present in the visual field, or even when they are contiguous and when they produce mentally different brightnesses corresponding to their physical difference – even with the highest degree of attention we shall not perceive a difference between them as long as the difference of the physical brightnesses does not exceed a certain limit: the so-called *difference threshold*; or as long as their ratio does not exceed a certain limit: the so-called *ratio threshold*. We may refer to either of the two thresholds because the one entails the other if the stimuli are known between which the difference or the ratio in question occurs. Let us take, as usual, the difference threshold. One of the most instructive and universally accessible examples is the fact that the stars are not seen in daylight, i.e., that they are not distinguished from the surrounding brightness of the sky, as long as the difference in physical brightness from the surrounding brightness does not exceed the difference threshold.

The existence of the difference threshold certainly implies an *estimation error*. A difference between sensations depending on stimuli of different intensity ... does exist, but it does not seem to us to exist. One may ask the reason for this error. Without having spoken directly on the topic, *Köhler*[a] ... can only find it in conditions of apperception (attention). If I am not mistaken, he would state the reason roughly as follows. A stronger stimulus difference (contrast) is apperceived more strongly, i.e., it (involuntarily) provokes stronger attention. If the stimulus difference falls below a certain limit, attention is weakened to such an extent that the actual difference is no longer perceived. But this conception is not in line with the fact that the stars in the sky in daylight remain invisible even if we voluntarily reinforce our attention (which involuntarily is but weakly engaged) towards certain places in the sky as strongly as possible. Thus, I believe that the reason must be found elsewhere. First, however, I shall mention other cases where the sensation difference is estimated erroneously and where the reason for the error is very obvious; this will make it easier to arrive at the reason for the error in the difference threshold.

Let two stimuli *A*, *B* be applied, in succession, to the same place in a sensory organ. Depending on whether *A* precedes *B* or vice versa, the size of their difference will be perceived differently. First, because we can compare the earlier sensation to the later one only in accordance with a memory of the former which is not entirely permanent; and second, because the state of the sensory organ on which its sensitivity depends has been modified by the action of the preceding stimulus, and this in a different way when either the stronger or the weaker stimulus comes first. If these circumstances occur in a regular fashion in methodical experiments, they form the basis of the so-called *time error*. In addition, we have the so-called *space error*, when the different stimuli act on different places of the sensory organ and the different places are endowed with different sensitivity. In general the circumstance that the stimuli on which sensations of different magnitude depend, and thus the sensations themselves do not coincide with respect to space and time and thus do not directly superimpose upon each other, leads to errors in estimating their difference. About these errors I say, in short, that they depend on a spatio-temporal difference.[2]

Now, among the influences of constant time and space errors upon the estimation error we have to distinguish, in accordance with their *direction*, an influence upon the direction of the estimation error, and in accordance with their *size*, an influence upon the size of the estimation error. If we proceed methodically with a combination of ex-

[a] Reference is made to a paper by Alfred Köhler (1886) containing a criticism of formulations of the psychophysical law from a Wundtian viewpoint. Whenever Fechner engages in a discussion of Köhler's paper, he is in fact criticizing Wundt's "psychological" interpretation of psychophysical measurement

[2] In speaking of spatio-temporal differences of sensations and of the stimuli on which they depend, I employ a shorthand expression meaning that there is *either* a spatial *or* a temporal difference, or a difference in the sensations and stimuli both in space and in time. Now, to speak of spatially localized mental activities or phenomena may be thought to be incompatible with a stringent metaphysical conception (if there is any at all) of the mind and of space. Well, it does not matter here. Just take spatially distinct sensations to mean such sensations that depend on spatially distinct psychophysical excitations. But then you also ought to deny that the thought and sensations of different people and of creatures on different planets are localized differently in space

periments involving opposite time and space positions, the former influence may be made good to such an extent that it may be considered to have been removed in the averaged results. But even then the second kind of error remains and it cannot be made good as long as the spatio-temporal difference continues to exist. However, such a difference *is bound* to exist with sensations of the intensive class (with which we are here concerned all the time), if the sensations are to be distinguished at all, because otherwise they would merge indiscriminately. Thus, the *possibility* of discrimination at the same time is responsible for the *error* of discrimination.

Accordingly, my opinion[3] is that the existence of the difference threshold (or of the ratio threshold) is based on the non-removable spatio-temporal non-coincidence of the stimuli and thus of the psychophysical excitations on which the different sensations considered here depend; and my opinion rests on the following three reasons. *First*, it is connected, in the way described, with the unassailable conception of the constant time and space errors, and serves only to make it complete. While, depending on the different time and space positions which are possible only in virtue of their non-coincidence, the constant time and space errors give rise to deviations from the true sensation differences in opposite *directions*, both positions have this in common, that a non-coincidence exists *at all*. And this leads to another common factor, viz., that a deviation from the true *magnitude* of the difference occurs, where the difference threshold comprises just the special limiting case that a very small real difference appears to us to be *altogether absent*, while greater differences . . . appear only diminished. *Second*, in fact the size of the difference threshold essentially depends, among others, on the spatio-temporal circumstances of the stimuli and, all other things being equal, it increases with the spatio-temporal difference. In extreme cases, where different sensations occur in different people, or where one sensation has been forgotten when the other arises, this leads to the impossibility of *discrimination* because of the impossibility of *comparison*. *Third*, our opinion must be based on the direct testimony of experiments in the area of extensive sensations. Here relevant experiments are directly possible, and they may be transferred naturally to the area of intensive sensations where relevant experiments are not feasible. In the extensive area, we can observe two rulers either separately or superimposed without their being fused indiscriminately, which latter is not feasible in the intensive field. Thus, in order to start from definite ideas, let there be two undivided rulers, one of them 100 lines[b] and the other 101 lines long. Let us first think of them as being kept apart and preferably, in order to avoid any approximation to coincidence, as being arranged in such a way that one of them forms the prolongation of the other. In general, it will then be impossible to notice the small difference existing between them by moving the eyes back and forth between them.[4] If our opinion is correct, the difference must be noticed at once if the two rulers are superimposed in such a way that one of them stretches out 1 line beyond the other; and in fact even differences much smaller than 1 line are noticed, the difference between the rulers being noticed as easily as if it were a spatial magnitude in itself.

Perhaps the following objection will be made. When, looking at two separate rulers with little or no length difference between them, I am uncertain whether or not one of them is longer than the other, it does not follow that I judge them to be *equal*, and the analogy mentioned above is not appropriate inasmuch as the difference threshold depends on a real *judgment of equality* about stimuli and the sensations depending on them which differ little from each other. But in fact in this respect intensive stimulus magnitudes present a state of affairs no different from that of extensive lengths. When I lift two weights of little or no difference either simultaneously or successively, or when I observe two surfaces of little or no difference with respect to physical brightness either after each other or one close to the other, there is as little chance of *absolute equality judgment* of magnitude as with separate rulers, but only an *uncertainty* corresponding to that with length comparisons; naturally enough, because the same cause is present, that is, spatio-temporal noncoincidence. From this point of view we may take it as a case of infinitely large difference threshold when, in relation to two sensations occurring in two different people, each of them remains uncertain whether his sensation is the same as the other's sensation, or whether it deviates from it in one direction or the other.

Incidentally, as was mentioned above, even in the field of physics our most accurate measurements depend on equality determinations and are still prone to probable errors, which are present though they may not have been computed, and which prove an uncertainty in the determination of equality corresponding to that found in the mental field.

Having set out the above three reasons, I am inclined to believe that some confidence can be given to my view about the nature of the difference threshold . . .

5. It has been shown above that in order to obtain a measure of the difference sensations we need not know *Weber's* law or any other psychophysical law in advance; however, once *Weber's* law has been established for some region of a mutually related stimulus and sensation scale (briefly, of a psychophysical scale), then within the relevant region of the scale the measure can be based on it. This will be assumed in all that follows.

Now, if *Weber's* law could be transferred from difference sensations to sensation differences, then in the same way as was previously shown for the former we could obtain a measure for the latter. Such a transference seems to be permissible from the following viewpoint. The validity of *Weber's* law presupposed the elimination of constant time and space errors; and under this presupposition *Web-*

[3] This viewpoint was not clear to me earlier, and therefore it has been formulated and justified here for the first time

[b] A "line" is an antiquated length measure amounting to 1/144 of a foot; in the "Parisian" system probably followed by Fechner it measures 2.25 mm. See Scheerer (1987) for another use of the "line" measure

[4] That length or distance differences that are too small are not recognized is proved most strictly by experiments with the adjustment method, where the task is to *make* equal two immediately adjacent lengths or distances. Here, the so-called pure average error remaining after the elimination of random and constant errors can arise only because it has not been recognized; otherwise it would not be committed

er's law is valid for difference sensations *independently* of the size of the difference threshold.[5] Thus, it can also be claimed in the special case where the difference threshold falls to zero (and the ratio threshold falls to 1), the estimation error dependent on it is abolished, the difference sensation coincides with the sensation difference, and the measure applicable to difference sensations becomes applicable in the same way to sensation differences . . .

For example, when *Weber's* law is confirmed for three stars of the physical brightnesses *A, B, C* such that given equality of *B/A* and *C/B*, the difference sensation relative to *A, B* is found equal to the difference sensation relative to *B, C*, then after transferring *Weber's* law from difference sensations to sensation differences it is permissible to conclude that the sensation difference relative to *A, B* is also equal to the sensation difference relative to *B, C*. Thus, we can obtain a measure for sensation differences in exactly the same way as has been obtained for difference sensations, and it does not seem necessary to elaborate on this again. However, an occasion for an elaboration of this sort will automatically present itself in Section 6. It can easily be seen that a unit of its own kind underlies the measure of sensation differences in the same way as it underlies the measure of difference sensations; only the two are different. Yet in order to arrive at a measure of sensation differences it is by no means necessary to know the *relation* in which the unit of sensation differences stands to the unit of the difference sensations. For both it is sufficient to be able to determine the relation of the single measurements to the arbitrarily chosen unit of their kind.

Nevertheless, I take this occasion to note that according to a process not to be reproduced here. . . . I find the following. If R_1, R_2 are two different stimuli,[c] and if v means the ratio threshold depending on the spatiotemporal conditions of stimulation, and k a constant which is independent of the absolute stimulus magnitudes and dependent on the arbitrary sensation unit, then one obtains for the sensation difference U and the corresponding difference sensation u the following equations:
$U = k \log (R_2/R_1)$; $u = k \log (R_2/R_1 v)$; $U-u = k \log v$.
The formula for U corresponds to my difference formula, and the formula for u corresponds to my difference measurement formula.

It follows from the equation for $U-u$ that in general the sensation difference U exceeds the difference sensation u by a certain quantity which is proportional to the logarithm of the ratio threshold v, but coincides with the latter when the ratio threshold equals 1, which means that the difference threshold equals 0.

It should be noted that the alteration undergone by the real sensation differences in their transformation into perceived differences under the influence of the spatio-temporal non-coincidence of the stimuli, and thus by the sensations, does not apply to the *sensations themselves.* Their difference being judged erroneously, the sensations remain what they are at the moment of their formation and throughout their duration; and with respect to them we need not make a corresponding distinction between difference sensations and sensation differences – naturally enough, because the motive for such a distinction, the spatio-temporal non-coincidence, ceases to exist for every sensation in particular. Every stimulus, as well as the psychophysical excitation and the sensation depending on it, coincides with itself throughout its duration. As long as we do not explicitly compare two sensations with each other, which only happens in exceptional cases in ordinary life, every sensation exists correctly for itself as it is, and thus their difference exists correctly as it is. The *comparison* of two sensations, in order to achieve awareness of their equality or difference, cannot happen at once during perception of them, and in memory each of them may be judged incorrectly against the other; for we must go right beyond the second sensation in memory, at the very moment when we pass on from its immediate perception to its comparison with the first sensation.

Now one could say the following. The sensations may be modified, if not by spatio-temporal circumstances, by apperceptive (attentional) circumstances. Consequently, it is necessary to distinguish apperceived sensations, that is, sensations that have been called into consciousness by attention and have thus increased in clarity and intensity, from sensations that are not yet apperceived, but do exist in the mind. If we do not pay attention to a stimulus capable of provoking a sensation, the sensation can vanish entirely from consciousness, and if we do pay attention, the sensation may enter consciousness with various clarity and intensity, depending on the degree of attention. Well, first, in the case when the sensation entirely vanishes from consciousness, I say: it is not there; though there may be an incomplete psychophysical condition for its appearance; and in the second case I say: it is present with exactly that clarity and intensity bestowed on it by the degree of attention paid to it. However, to speak of a sensation which, owing to insufficient attention, vanishes in such a way from consciousness that we neither can say anything of its quality, nor can compare it according to its quantity with other sensations of the same kind, as a sensation nevertheless existing in the mind: this seems to me to be using language which leads to obscurity and the incomprehensible . . .

6. On the basis of one fact and one simple mathematical theorem, the measurability of sensations follows from the measurability of sensation differences.

First, the fact of the simple *stimulus threshold,* which is analogous to the fact of the difference threshold, and according to which a stimulus capable of provoking a certain kind of sensations must exceed a certain magnitude, i.e., the stimulus threshold, before the sensation begins to be felt, and thus to exceed the zero value of a real sensation. We can investigate the reason for the stimulus threshold as we have investigated the reason for the difference threshold, and we shall have to come back to this question,[d] but

[5] In fact, in different people, under different conditions of sensitivity, and under different spatio-temporal circumstances the difference, as well as the ratio threshold assumes different values without excluding the validity of *Weber's* law in such circumstances

[c] The notation for stimuli is *R*, as an abbreviation of the German word *Reiz*

[d] The discussion of the stimulus threshold (i.e., the absolute threshold) is contained in Section 9 of the original paper (pp. 202–207), not translated here. It leads to the conclusion that the absolute threshold is a "mixture threshold" arising from the fact that the sensory system is never in a state of absolute rest; as a result, the excitation provoked by a stimulus mixes with, and must be separated from, the internal psychophysical excitation present at the moment when the stimulus is applied

at present we are dealing with a fact that certainly exists. The mathematical theorem is this: the differences of given values from zero coincide with the values themselves, i.e., $a - 0 = a$, $b - 0 = b$, and so forth.

Let there be, within the limits of the validity of *Weber's* law, an ascending series of arbitrarily modifiable stimuli

$$A, B, C, D, E \ldots$$

to which belong the modifiable sensations

$$a, b, c, d, e \ldots$$

Then by changing the values of the upper series we shall be able to make

$$B/A = C/B = D/C \text{ etc.},$$

that is, that according to *Weber's* law the relevant difference sensations are made equal. Because *Weber's* law can be transferred from difference sensations to sensation differences, the successive sensation differences

$$b - a, c - b, d - c, e - d$$

will also be equal. According to our general measurement principle, the sensation difference between any term of the series, say c, and the upper final term, in brief, the sensation difference $e - c$ will then be composed of two equal sensation differences $d - c$, $e - d$, and thus it will be twice as great as any of them. First, then, we obtain a measure of the sensation difference $e - c$, presupposing $d - c$ as unit; for we shall be able to say that the sensation difference $e - c = 2$, if the sensation difference $d - c$ is put at 1. The same will also remain true if, given the equality of the successive stimulus ratios on which the equality of the successive sensation differences depends, one of the sensations, say again c, has the value zero and thus C has the threshold value: a case which can always be realized if the stimuli are modifiable at will. In this case, too, $e - c$ will be twice as great as $d - c$, and consequently e will be twice as great as d, because $c = 0$. And if the series of capital and lower case letters should be extended, in the same manner, beyond E and e, following the same derivation the sensation f belonging to the stimulus F would be three times greater than the sensation d, and thus the principle of a sensation measure would be fulfilled.

Granted, this derivation of the sensation measure seems very complicated, while the measurement formula

$$E = k \log (R/\mathrm{R}),$$

which is implied by it and where R is the threshold value of the stimulus, is very simple. However, here we were essentially and exclusively concerned with a clarification of the principle of sensation measurement, and for this purpose we could not appeal to the measurement formula; the derivation of the formula had to be made on the basis of the measurement principle only.

[There follows a defense of negative sensations and their interpretation.]

References

Köhler, A. (1886). Über die hauptsächlichen Versuche einer mathematischen Formulierung des psychophysischen Gesetzes von Weber. *Philosophische Studien, 3,* 572–642.

Scheerer, E. (1987). Tobias Mayer: Experiments on visual acuity (1755). *Spatial Vision,* 3, in press.