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Measurement and Structural Model Class Separation in Mixture CFA: ML/EM Versus MCMC

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Parameter recovery was assessed within mixture confirmatory factor analysis across multiple estimator conditions under different simulated levels of mixture class separation. Mixture class separation was defined in the measurement model (through factor loadings) and the structural model (through factor variances). Maximum likelihood (ML) via the EM algorithm was compared to a Markov chain Monte Carlo (MCMC) estimator condition using weak priors and a condition using tight priors. Results indicated that the MCMC weak condition produced the highest bias, particularly with a weak Dirichlet prior for the mixture class proportions. Specifically, the weak Dirichlet prior affected parameter estimates under all mixture class separation conditions, even with moderate and large sample sizes. With little knowledge about parameters, ML/EM should be used over MCMC weak. However, MCMC tight produced the lowest bias under all mixture class separation conditions and should be used if tight and accurate priors can be placed on parameters.

Keywords: Bayesian estimation, confirmatory factory analysis, class separation, parameter recovery, finite mixture models

Class separation within mixture models is defined when there is true unobserved heterogeneity modeled through multiple latent mixture classes of individuals. The separation between these mixture classes can vary from high separation to extremely poor separation. In cases of poor mixture class separation, it can become more difficult to decipher one mixture class from another because the true parameters are quite similar for each of the mixtures. In some cases, this can lead to poor or inaccurate classification of individuals into latent mixture classes. Moreover, the magnitude of the class separation might also have an impact on the accuracy of the parameter estimates. This is especially the case under poor conditions of separation and with smaller sample sizes (Tueller & Lubke, 2010).

Perhaps the most common estimator used for mixture models has been maximum likelihood via the EM algorithm (ML/EM; see, e.g., Enders & Tofighi, 2008; Jedidi, Jagpal, & DeSarbo,

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1997; McLachlan & Peel, 2000; Tueller & Lubke, 2010). In fact, much of the research on class separation has been conducted within the ML/EM estimation framework. However, with the recent increase in usage and the accessibility to software implementing the Markov chain Monte Carlo (MCMC) estimator, such as R (R Development Core Team, 2008), WinBUGS (Lunn, Thomas, Best, & Spiegelhalter, 2000), and M*plus* (Muthén & Muthén, 1998–2010), it is important to examine the impact of the MCMC estimator on mixture class separation as well.

This study aims at assessing class separation in the context of ML/EM and MCMC to provide general recommendations for mixture model parameter estimation. Specifically, this study examines parameter recovery bias under two different forms of class separation within the context of finite mixture confirmatory factor analysis (mixture CFA). The first form of class separation is defined in the measurement model (via factor loadings) and the second form of separation is defined in the structural model (via factor variances).

This article is organized as follows. In the next section, finite mixture modeling is briefly introduced. This is followed by the specification of the mixture CFA model used for this investigation. Next is a presentation of the recent literature focused on mixture class separation in the context of finite mixture models. Issues surrounding the estimation of mixture models through ML/EM are briefly discussed. This is followed by an introduction to the MCMC estimation algorithm as well as the specification of prior distributions for this study.

Next, the study design is presented, separated into two main parts. First, a measurement model class separation study is described that compares parameter recovery under various conditions of class separation within the measurement model. Second, a structural model class separation study is described that compares parameter recovery for various conditions of structural model class separation. Results of both portions of this study are presented and this is followed by a summary of the findings, a discussion of benefits and risks of using the ML/EM and MCMC estimators under different class separation conditions, and implications for applied mixture class research.

FINITE MIXTURE MODELS

In the social and behavioral sciences, mixture modeling has proved to be an important tool for accounting for unobserved heterogeneity within a population, and the flexibility of these models has allowed for some innovative modeling techniques (see, e.g., Jedidi et al., 1997; Muthén & Shedden, 1999). In the case of unobserved heterogeneity, the variables that cause heterogeneity in the data are not known prior to data analysis. As a result, the class membership of each participant is unknown to the researcher and membership must be determined based on observed data patterns (Lubke & Muthén, 2005).

One common class of models used to assess this form of unknown heterogeneity is finite mixture modeling. The basic set-up of finite mixture models assumes that $\mathbf{x} = (\mathbf{x}_1', \mathbf{x}_2', \ldots, \mathbf{x}_n')'$ denotes a vector of $j = (1, 2, \ldots, J)$ random variables for $i = (1, 2, \ldots, n)$ cases. The density for a vector of random variables for an observation is denoted by $f(\mathbf{x}_i)$ and can be written in the general form of

$$f(\mathbf{x}_i; \boldsymbol{\Psi}) = \sum_{c=1}^{C} \pi_c f_c(\mathbf{x}_i; \boldsymbol{\theta}_c), \quad (1)$$

where π_c is a nonnegative mixing proportion used to model heterogeneity with

$$\sum_{c=1}^{C} \pi_c = 1 \tag{2}$$

and

$$\boldsymbol{\Psi} = (\pi_1, \pi_2, \ldots, \pi_c, \boldsymbol{\Theta}')' \tag{3}$$

represents a vector containing the unknown mixture proportions and $\boldsymbol{\Theta}$, which is a vector containing the $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_C$ model parameters. Additionally, $\boldsymbol{\theta}_c = (\theta_{1c}, \theta_{2c}, \ldots, \theta_{Jc})'$ represents a vector of unknown parameters for the cth mixture class. The density in Equation 1 is a common approach to setting up the density of an observation in a mixture analysis where variables are conditionally independent given their membership in mixture class c. In the case of mixture models with C classes, it is common to assume the densities follow a finite mixture multivariate Bernoulli distribution[1] specified as

$$f_c(\mathbf{x}, \boldsymbol{\theta}_c) = \prod_{j=1}^{J} \theta_{jc}^{x_j} (1 - \boldsymbol{\theta}_{jc})^{1-y_j}, \tag{4}$$

where $\boldsymbol{\theta}_{jc}$ represents the conditional probability that $\mathbf{x}_c = 1$ given membership in the cth mixture class.

MIXTURE CFA SPECIFICATION

This study specifically addresses the finite mixture CFA model. The mixture CFA measurement model used here is

$$\mathbf{x}_i = \boldsymbol{\nu}_x + \boldsymbol{\Lambda}_x \boldsymbol{\eta}_i + \boldsymbol{\epsilon}_i, \tag{5}$$

where $\mathbf{x}$ is a $q \times 1$ vector of observed responses on q items, $\boldsymbol{\nu}_x$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a $q \times k$ matrix of factor regression weights (loadings) for q items and k factors, $\boldsymbol{\eta}$ is a $k \times 1$ vector of common factors underlying the data, and $\boldsymbol{\epsilon}$ is a $q \times 1$ vector of unique variables that contain both measurement error and specific error. The structural model is

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha}_c + \mathbf{B}_c \boldsymbol{\eta}_i + \boldsymbol{\zeta}_i, \tag{6}$$

where $\boldsymbol{\eta}$ still represents a vector of the common factors, $\boldsymbol{\alpha}$ is a vector of factor means, $\mathbf{B}$ is a null matrix that can be used to relate the common factors to one another, and $\boldsymbol{\zeta}$ is a vector of deviations of the parameters from their respective population means. The c subscripts in this equation indicate that the factors are allowed to vary across mixture classes such that $c =$

[1]Although the multivariate Bernoulli density is discussed here, note also that many of the mixture distributions encountered in practice are Gaussian (univariate or multivariate). There are also other mixture distributions, such as exponential and Weibull, that are used in biostatistics and related fields to model the distribution of survival time. For a discussion of some of the common finite mixture distributions used, see Everitt (1996).

$(1, 2, \ldots, C)$ represents the mixture class variable, and the i subscripts allow the parameters to vary across individuals. Individuals are assigned to the mixture classes based on the posterior probabilities found in the following:

$$\tau_c(\mathbf{x}_i; \boldsymbol{\Psi}) = \frac{\pi_c f_c(\mathbf{x}_i; \boldsymbol{\theta}_c)}{\sum_{c=1}^{C} \pi_c f_c(\mathbf{x}_i; \boldsymbol{\theta}_c)} \tag{7}$$

which represents the posterior probability that $\mathbf{x}_i$ belongs to the cth mixture class. Often, cases will be assigned to a mixture class based on the highest corresponding posterior probability produced. This is the method used for class assignment in this investigation.

CLASS SEPARATION FOR FINITE MIXTURE MODELS

The magnitude of class separation is an important consideration when studying mixture models. Class separation is an indication of the true substantive differences between mixtures. If separation is low, it questions whether or not there are substantively different mixture classes. With low class separation, there is a risk of mislabeling in that cases can be placed in the "wrong" mixture class. This can create problems in convergence and produce estimates that are much different from the true model. As a result, it is important to have an understanding for the impact that class separation can have on estimation within different modeling situations. Very few studies have examined class separation directly within the context of structural equation modeling, but two such studies are highlighted here.

Tueller and Lubke (2010) conducted a study that assessed estimates and class assignment recovery for various conditions of a structural equation mixture model, which is a generalization of a factor mixture model. The conditions varied within this study were mixture class size, total sample size, and class separation. The findings of this study in relation to class separation were quite pronounced. Overall, there was smaller bias in the estimates when larger sample sizes were used with a greater degree of class separation; a study by Tolvanen (2008) found comparable results under similar design conditions. Correct class assignment also improved when class separation increased, indicating that class assignment becomes easier to decipher when separation is high. Higher class separation conditions were also able to tolerate lower total sample sizes that still produce adequate convergence and therefore dependable parameter estimates. However, if the smallest class (the minority class) only contains very few cases, results might not be as reliable even with increased class separation. The overall result that Tueller and Lubke (2010) found was that class recovery was poor for this study. They argued that this was due to the complexity of the model being assessed. More complex models appear to create additional difficulties in proper class recovery. However, it is important to note that this study was conducted in the context of ML/EM and that some of these issues might not arise when using alternate estimators.

A study by Tofighi and Enders (2008) viewed class separation in the context of growth mixture models via ML/EM estimation. Overall, they found that various likelihood ratio-based fit tests and information criteria had a difficult time identifying the class structure properly within the low class separation conditions of the study. Similar results were obtained in this study compared to Tueller and Lubke (2010) in that the sample size in each mixture had a substantial impact on proper mixture class recovery. Specifically, the accuracy of class recovery decreased when only a small proportion of cases were in one of the classes (e.g., only 7% of the cases).

ML/EM ESTIMATION ISSUES

As mentioned, the previous work presented on class separation was completed in the context of the ML/EM estimator. In general, higher sample sizes are needed to ensure adequate estimates are produced when working with this asymptotic theory-based estimator. Similarly, higher class separation crossed with higher sample sizes is optimal when using the ML/EM estimator. However, these conditions might not be particularly realistic in actual research settings. Large sample sizes are not always practical to obtain. Likewise, the magnitude of class separation is not something that can be controlled or predetermined in an applied research setting. It is also the case that ML/EM can be incredibly sensitive to starting values, particularly within the context of mixture models (Bollen & Curran, 2006; McLachlan & Peel, 2000; Muthén, 2004). These issues have led researchers to consider alternative estimation procedures in search of an estimator that can be reliable under conditions more typical of real data (i.e., smaller samples and less defined class separation). The following section introduces one of the options for an alternative estimator, namely, the MCMC estimation algorithm.

MCMC AND FINITE MIXTURE MODELING

A description of the initial developments of finite mixture models within the MCMC framework was initially provided by Diebolt and Robert (1994). This work presented finite mixture models through a Gibbs sampler, which relies on the missing data structure of mixture models akin to the EM algorithm. MCMC further allows researchers to specify prior distributions of varying degrees of informativeness on the mixing proportions and other model parameters. This modeling flexibility is especially useful for analyzing mixture models with an unknown number of mixture components (Lee, 2007).

One of the main advantages of utilizing MCMC is the framework itself. This framework is much different from an estimator such as ML/EM because the focus is on converging to a distribution that carries certain distributional properties rather than converging to a point estimate. This occurs as the MCMC chain converges with the stationary (or target) distribution. A larger number of iterations within the chain is a better approximation of the stationary distribution because the iterations produce an independent and identically distributed sample from the marginal distribution (via conditional distributions), rather than directly computing the marginal density (Casella & George, 1992).

Although there are several different MCMC samplers that can be used to build an MCMC chain, this study uses the Gibbs sampler (Geman & Geman, 1984). This technique samples each parameter individually with respect to its conditional distribution and treats all other parameters as known. This updating process typically occurs in a particular fixed order for the parameters and is sometimes referred to as *scanning* (Geyer, 1991). This study uses Gibbs sampling as implemented through the MCMC framework recently added to the M*plus* software program (Muthén & Muthén, 1998–2010).

Relevant MCMC Convergence Diagnostics

Although there are several different convergence diagnostics used to assess parameter convergence within MCMC estimation, one of the of the most common diagnostics to employ is the Brooks, Gelman, and Rubin diagnostic. This originated with the work by Gelman and Rubin (see, e.g., Gelman, 1996; Gelman & Rubin, 1992a, 1992b), who designed a diagnostic based on analysis of variance that was intended to assess convergence among several parallel sequences (or chains) with varying starting values.[2] Specifically, they proposed a method where an overestimate and an underestimate of the variance of the target distribution were formed. The overestimate of variance was represented by the between-sequence variance and the underestimate was the within-sequence variance (Gelman, 1996). The theory was that these estimates would be approximately equal at the point of convergence. This comparison of between and within variances is referred to as the *potential scale reduction factor* (PSRF) and larger values typically indicate that the chains have not fully explored the target distribution. Specifically, a variance ratio is computed with values approximately equal to 1.0 indicating convergence. Brooks and Gelman (1998) added an adjustment for sampling variability in the variance estimates and also proposed a multivariate extension (MPSRF), which did not include the sampling variability correction. The changes by Brooks and Gelman reflect the diagnostic as implemented in the M*plus* software that is utilized for this investigation. Although the Brooks, Gelman, and Rubin diagnostic is the only assessment used to determine parameter convergence in this investigation, note that there are several other diagnostics commonly used in the MCMC literature (see, e.g., Sinharay, 2004).

Specification of Priors for the Mixture CFA Model

As is true with any type of model, the key to specifying a mixture CFA model within an MCMC framework is properly setting up the prior distributions on the model parameters. This section describes the conjugate priors (of the same family as the likelihood) for each of the parameters estimated in a mixture CFA model as presented in Lee (2007) and Asparouhov and Muthén (2010). Note, however, that priors need not be from the same distributional family as the likelihood.[3]

For any mixture model, we first assume that the data are generated from a mixture distribution represented by the following mixture density function for mixture class c:

$$f(\mathbf{x}_i|\boldsymbol{\Theta}) = \sum_{c=1}^{C} \pi_c f_c(\mathbf{x}_i|\boldsymbol{\Theta}), \tag{8}$$

[2]Note that this diagnostic can also be used to assess convergence in a single MCMC chain by comparing the first portion of the post burn-in iterations to the last portion of the chain. See Muthén and Muthén (1998–2010) for more details of how this is implemented in M*plus*.

[3]Although it is true that nonconjugate priors can be specified for any model, this is typically not encouraged for mixture models (see, e.g., Diebolt & Robert, 1994; Lee, 2007). The use of fully noninformative priors (e.g., uniform) can lead to improper posterior distributions. As a result, it is common for mixture models to be specified with conjugate priors to avoid this problem altogether.

where π_c represents the unknown mixture class proportion for the cth mixture class. Specific to mixture CFA, this represents the proportion of individuals in each of the C mixture classes.

Next, the prior distributions can be specified for the unknown parameters in the model. The parameters estimated in this model are the mixture class proportions (π_c), the factor loadings ($\mathbf{\Lambda}$), the factor means ($\boldsymbol{\alpha}$), the variance of the observed variables ($\mathbf{\Omega}_{\epsilon_{jj}}$), and the factor variance–covariance matrix ($\mathbf{\Omega}_{\zeta}$). To begin, the process assigning individuals to particular mixture classes is assumed to follow a multinomial distribution with a sample size parameter n and a class proportion parameter π. The conjugate prior for this class proportion parameter π is the Dirichlet distribution denoted as:

$$\pi_c \sim \mathcal{D}[\delta_1 \ldots \delta_C],$$

with the hyperparameter(s) $\delta_1 \ldots \delta_C$ that control how uniform the distribution will be.[4] Specifically, these parameters represent the proportion of cases in the C mixture classes.

The next set of model parameters to receive prior distributions are the factor loadings. The conjugate prior for the factor loadings is the normal distribution denoted as:

$$\mathbf{\Lambda} \sim \mathcal{N}[\boldsymbol{\mu}_{\Lambda}, \mathbf{\Omega}_{\Lambda}],$$

with hyperparameters $\boldsymbol{\mu}_{\Lambda}$ representing the mean and $\mathbf{\Omega}_{\Lambda}$ representing the variance term. Similarly, the factor means are also distributed normally, which can be denoted as:

$$\boldsymbol{\alpha} \sim \mathcal{N}[\boldsymbol{\mu}_{\alpha}, \mathbf{\Omega}_{\zeta}],$$

where $\boldsymbol{\mu}_{\alpha}$ represents the mean term for the factor means and $\mathbf{\Omega}_{\zeta}$ represents the factor variances and covariances.

The next prior to specify is for the variances on the observed variables denoted as $\mathbf{\Omega}_{\epsilon_{jj}}$, which represents a single cell in the variance–covariance matrix $\mathbf{\Omega}_{\epsilon_{ij}}$. The conjugate prior for the observed variable variances is assumed to follow an inverse gamma (IG) distribution and can be seen as:

$$\mathbf{\Omega}_{\epsilon_{jj}} \sim \mathcal{IG}[a_{\Omega_{\epsilon_{jj}}}, b_{\Omega_{\epsilon_{jj}}}],$$

where the hyperparameters a and b represent the shape and scale parameters for the IG distribution, respectively.

The last prior distribution to be specified is for the matrix of factor variances and covariances denoted as $\mathbf{\Omega}_{\zeta}$. Recall that from Equation 6, $\boldsymbol{\zeta}_i$ represents a vector of deviations of the parameters from their respective population means. The conjugate prior for the factor variance–covariance matrix is assumed to be inverse Wishart (IW) distributed and is denoted as:

$$\mathbf{\Omega}_{\zeta} \sim \mathcal{IW}[\mathbf{\Omega}, d],$$

[4] *Hyperparameters* are the parameters of a prior distribution. For example, the hyperparameters for the normal distribution are the mean and variance terms.

where $\mathbf{\Omega}$ is a positive definite matrix of size p and d is an integer that can vary depending on the informativeness of the prior distribution.[5] Note that for setting up an IW prior distribution, each term in the variance–covariance matrix can get a prior. As a result, the $\mathbf{\Omega}$ hyperparameter can actually be replaced with a constant.

The Impact of Priors (Informative Versus Noninformative)

Priors are viewed by some researchers as a key advantage for the MCMC estimation framework because the uncertainty of all parameter estimates is taken into account (Lambert, Sutton, Burton, Abrams, & Jones, 2005). However, priors have also been pinpointed as being one of the main drawbacks to this framework as a result of the inherent subjectivity that is coupled with choosing prior distributions and the corresponding hyperparameters. It is this duality in beliefs that makes the actual impact of prior distributions an important research topic to explore within MCMC.

One feature of data analysis that is closely tied to the impact of prior distributions is the size of the sample used for estimation. Theory indicates that with large amounts of data, the priors specified in the model do not have a significant impact because the data swarms the prior, thus making it irrelevant (Ghosh & Mukerjee, 1992). However, research has indicated that even noninformative priors (used interchangeably with *weak* or *diffuse* priors) can impact estimates using larger sample sizes. One such study examining the impact of noninformative priors on sample size was conducted by Natarajan and McCulloch (1998) in the context of a probit model with a single random effect. Findings indicated that weak priors impacted the posterior distribution even for moderate and large sample sizes. Likewise, Lambert et al. (2005) conducted a similar study that addressed the impact of different prior distributions (all noninformative) on a random effects meta-analysis model. This study found that the choice of the prior distribution had a large impact on the estimated posterior distribution, especially when the number of studies in the simulated meta-analysis data was small (akin to a small sample size).

This study aims at addressing the direct impact of both informative (used interchangeably with *tight* priors) and noninformative prior distributions in the context of different sample sizes. This will, in part, be a replication of the work by Natarajan and McCulloch (1998) and Lambert et al. (2005) in that different levels of informativeness (noninformative vs. informative) of the priors will be directly assessed in relation to varying degrees of sample size. However, this study extends the investigation to mixture models, as well as varying degrees of mixture class separation. There has been little research done specifically about the impact of prior distributions on mixture models, especially regarding the Dirichlet distribution, which is a conjugate prior for mixture class proportions.

Mixture models add another layer of modeling complexity that can create additional concerns with respect to priors. If prior distributions are not carefully determined, then additional problems might arise within estimation (e.g., poor parameter recovery and poor mixture class recovery). As a result, this study aims at addressing the direct impact of different conjugate priors

[5] Although this article deals with priors in terms of the variance–covariance matrix by using an IW distribution, there are some software programs (e.g., WinBUGS) that can only place prior distributions on the precisions (inverse variance–covariance matrix). In this case, the prior distribution on the precision matrix would be the Wishart distribution.

on model parameter recovery under various conditions of mixture class separation and sample sizes. The results from the different conditions of prior distributions will also be compared to the more conventional ML/EM estimator in the context of mixture CFA class separation.

DESIGN

This article is made up of two different simulation studies focused on class separation within the context of mixture CFA. The first simulation study focuses on class separation defined in the measurement model and the second simulation study focuses on class separation within the structural model. First, the design features that were held constant across the measurement and structural model portions of this article are outlined. This is followed by design details specific to each of these studies.

For this study, a simple structure (no cross-loadings) two-factor mixture CFA model was specified with a total of five indicators per factor. The first item loading on each factor had a loading weight fixed at 1.00 to set the metric. There were two mixture classes specified for all class separation conditions. A depiction of the data generating model can be found in Figure 1.

The relative size of the two mixture classes was manipulated such that mixture class 1 (C1) and mixture class 2 (C2) had sample size proportions of 0.80/0.20, 0.20/0.80, and 0.50/0.50,

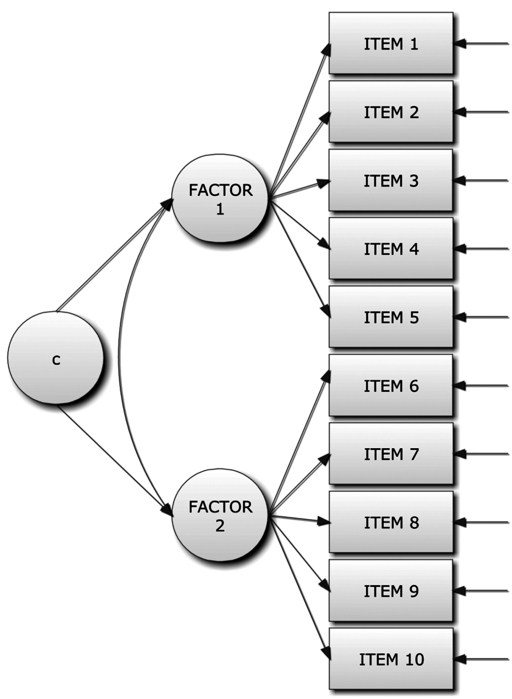

FIGURE 1 Mixture CFA data generating model.

respectively. Likewise, sample sizes typical of applied research of 100, 300, and 800 were used to examine whether there would be an impact of the minority class sample size on parameter recovery under different conditions of class separation.

For both the measurement and the structural parts of this article, the M*plus* software program was used. Three different estimation conditions were compared within each class separation condition: ML/EM using multiple starting values, MCMC using weak priors (MCMC weak), and MCMC using tight priors (MCMC tight); specifics of the priors used for the measurement and structural models are detailed later.[6]

Each MCMC analysis consisted of a single chain with the first 25,000 iterations discarded as the burn-in phase and 25,000 post burn-in iterations. Each cell of this design requested 100 replications using the Monte Carlo feature of the M*plus* software.[7] Bias was then computed for each parameter based on the average estimate produced by the simulations.[8] Bias values less than 10% were deemed as representing low or negligible parameter bias, whereas anything greater than or equal to 10% was considered to be exhibiting high and therefore problematic parameter bias. The design conditions specific to the measurement and structural portions of this article are presented next.

Design Specifics for the Measurement Model

Class separation for the measurement model was defined by the factor loadings for the two mixtures. The design conditions can be found in Table 1. Factor loadings of 0.80 were used for all 10 items (five in each of the two factors) for C1. The measurement model class separation was then defined in relation to C1 by setting the loadings in C2 as either representing poor class separation (0.70 factor loadings), moderate class separation (0.50 factor loadings), or high class separation (0.30 factor loadings). Note that to prevent within-class factor loading variation from contributing to the parameter recovery, the true values for factor loadings were the same across both factors within each mixture. Specifically, this was used to prevent *factor* separation from affecting the parameter recovery within each mixture. The factor variances, factor means, and residual variances were all held equal across the mixture classes to isolate the effects of the factor loading conditions.

As mentioned previously, three different estimators were used to compare each class separation condition: ML/EM, MCMC weak, and MCMC tight. For MCMC weak, the default noninformative priors provided by M*plus* (see Muthén & Muthén, 1998–2010) were used for all of the parameters except for the factor loadings. The factor loadings are distributed normally and weak priors for each loading were defined by setting the true loading value as the mean term of the prior and by having a variance of 100 for the variance term of the prior (e.g., for

[6]For the ML/EM estimator, the number of random starts for the models specified in this study was 100. The number of final stage optimization steps was 10. These settings were used to ensure that estimates were not the result of problems with local maxima. Likewise, there was no evidence of any class label switching, a potential problem in mixture simulation studies, in any of the estimator conditions presented here.

[7]For the ML/EM condition and both MCMC estimator conditions, the number of replications that converged properly under the poor and moderate class separation conditions were as low as 58%. These results are similar to those obtained in Tueller and Lubke (2010), which indicated poor convergence under lower class separation conditions. ML/EM and MCMC appeared to have equal difficulty in obtaining convergence under these separation conditions.

[8]Bias is computed by using the following equation: 100*([estimate − true value]/true value).

TABLE 1
Data Generating Parameters: Measurement and Structural Model Conditions

	Measurement Model			*Structural Model*		
Parameter	*High Separation*	*Moderate Separation*	*Low Separation*	*High Separation*	*Moderate Separation*	*Low Separation*
Loadings: Mixture 1						
Item 1	0.80	0.80	0.80	0.80	0.80	0.80
Item 2	0.80	0.80	0.80	0.80	0.80	0.80
Item 3	0.80	0.80	0.80	0.80	0.80	0.80
Item 4	0.80	0.80	0.80	0.80	0.80	0.80
Item 5	0.80	0.80	0.80	0.80	0.80	0.80
Item 6	0.80	0.80	0.80	0.80	0.80	0.80
Item 7	0.80	0.80	0.80	0.80	0.80	0.80
Item 8	0.80	0.80	0.80	0.80	0.80	0.80
Item 9	0.80	0.80	0.80	0.80	0.80	0.80
Item 10	0.80	0.80	0.80	0.80	0.80	0.80
Loadings: Mixture 2						
Item 1	0.30	0.50	0.70	0.30	0.30	0.30
Item 2	0.30	0.50	0.70	0.30	0.30	0.30
Item 3	0.30	0.50	0.70	0.30	0.30	0.30
Item 4	0.30	0.50	0.70	0.30	0.30	0.30
Item 5	0.30	0.50	0.70	0.30	0.30	0.30
Item 6	0.30	0.50	0.70	0.30	0.30	0.30
Item 7	0.30	0.50	0.70	0.30	0.30	0.30
Item 8	0.30	0.50	0.70	0.30	0.30	0.30
Item 9	0.30	0.50	0.70	0.30	0.30	0.30
Item 10	0.30	0.50	0.70	0.30	0.30	0.30
Variances: Mixture 1						
Factor 1	1.00	1.00	1.00	1.00	1.00	1.00
Factor 2	1.00	1.00	1.00	1.00	1.00	1.00
Cov(F1,F2)	0.40	0.40	0.40	0.40	0.40	0.40
Variances: Mixture 2						
Factor 1	1.00	1.00	1.00	5.00	3.00	1.00
Factor 2	1.00	1.00	1.00	5.00	3.00	1.00
Cov(F1,F2)	0.40	0.40	0.40	0.40	0.40	0.40
Means: Mixtures 1,2						
Factor 1	2.00	2.00	2.00	2.00	2.00	2.00
Factor 2	1.00	1.00	1.00	1.00	1.00	1.00

Note. All conditions were fully crossed with the three mixture class proportion conditions (0.80/0.20, 0.20/0.80, and 0.50/0.50), the three sample size conditions (100, 300, and 800 cases), and the three estimators (maximum likelihood via the EM algorithm, MCMC using weak priors, and MCMC using weak priors). MCMC = Markov chain Monte Carlo.

a loading with a true value of 0.80, the weak prior on this loading would be $\mathcal{N}(0.80, 100)$). This produced a standard deviation of 10 for each loading, which represented a weak prior on the factor loading.

The prior distributions for MCMC tight were determined in a similar fashion. For the factor loadings, the true value for the loading was set as the mean term of the normal prior. However, the variance term was set at 0.01 (e.g., $\mathcal{N}(0.80, 0.01)$), thus creating a standard deviation of 0.10 for each standardized factor loading. Likewise, tighter priors were also placed on the class proportions through the Dirichlet prior distribution. Specifically, rather than using the default weak prior distribution for two mixtures ($\mathcal{D}(10, 10)$), priors reflecting the sample sizes and the mixture class proportions were used. For example, the prior distribution for a sample size of 100 cases with a mixture class proportion of 0.80/0.20 for the two mixture classes would be ($\mathcal{D}(80, 20)$). The condition employing tighter priors for the measurement model portion of this article only used tighter priors on the factor loadings and the class proportions. Default weak prior distributions were used for all of the other model parameters.

Design Specifics for the Structural Model

For the structural model conditions, the factor loadings were fixed at 0.80 and 0.30 (representing high measurement separation) for the two mixture classes, respectively. This was to ensure that class separation was only being defined by the structural model in this part of the study. Likewise, the factor means and residual variances were held equal across the mixture classes.

For simplicity, class separation for the structural model was only defined through the factor variances for the two mixtures.[9] Factor variances for C1 were fixed at 1.00 for both factors across all conditions. The structural class separation conditions were then created by manipulating the factor variances for C2. Factor variances for both factors in C2 were set as either 1.00, 3.00, or 5.00 to represent either poor, moderate, or high structural class separation, respectively. Akin to Tueller and Lubke (2010), these factor variances, in effect, created conditions of 1:1, 1:3, and 1:5 variance ratios for the two mixture classes. For example, if the factor variance for C1 is 1.00 and the factor variance for C2 is 5.00, this represents less overlap and therefore a greater degree of class separation in the structural model. Figure 2 shows a graphical representation of variance separation conditions for Factor 1, as an example. The full design conditions for the structural model portion of this study can be found in Table 1.

The same three estimators were used for all of the conditions in the structural portion of this article. ML/EM and MCMC weak were implemented the same as described for the measurement model portion of this study. The conditions employing MCMC tight were similar to the measurement model conditions in that the priors for the factor loadings and the mixture class proportions were defined exactly the same. However, this portion of the study also utilizes tighter priors on some of the structural parameters. Specifically, the factor means were distributed normally with a mean equal to the true parameter value and a variance equal to 0.10 (e.g., for a factor mean with a true value of 2.00 the prior would be ($\mathcal{N}(2.00, 0.10)$), creating a standard deviation of 0.32 for each factor mean.

Likewise, the elements in the factor variance–covariance matrix were also given tighter priors that follow an IW distribution. To tighten priors on the elements of a matrix, each element was given its own prior distribution. To create an informative IW prior on the factor variances and covariance terms, the two hyperparameters of the IW distribution must be specified appropriately. First, Ω represents the true parameter value specified for the corresponding factor variance or covariance. Second, d represents the dimension of the variance–covariance matrix plus 1 ($d = \text{dimension} + 1$). For example, if the true value of a factor variance was 1.00, and there were two factors in the model thus creating a two-dimensional variance–covariance

[9]Note, however, that structural model class separation could have also been defined through the factor means or factor covariances or correlations. Nevertheless, it was deemed sufficient, for the purposes of this study, to define structural class separation in terms of the factor variances.

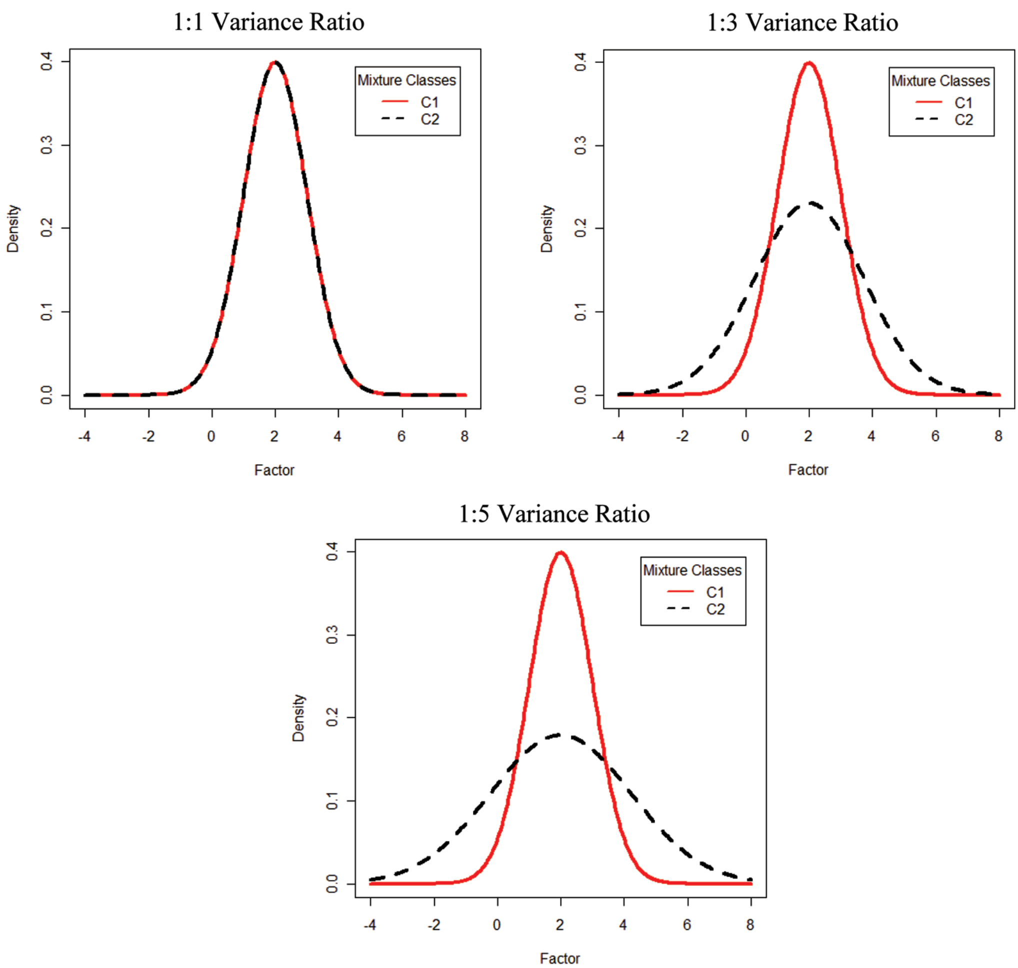

FIGURE 2 Mixture class factor variance ratio for Factor 1. (color figure available online)

matrix, the tight IW prior on this parameter would be ($\mathcal{IW}(1, 3)$). All of the variance and covariance parameters received tight prior distributions using this method.

MEASUREMENT MODEL RESULTS

Results presented in the tables for the measurement model portion of this study are only based on the sample size condition of $n = 100$.[10] Results from the $n = 300$ and 800 conditions are summarized in the following and the tables are available on request.

[10]Each condition took less than 2 minutes to run for the ML/EM and MCMC estimators.

Results for the 0.80/0.20 mixture class proportions where the minority class (C2) had lower factor loadings can be found in Table 2. Results indicated that the ML/EM estimator condition showed high bias for the minority mixture class (C2) class proportion in the moderate separation condition. Likewise, bias was high for both class proportion estimates under the poor separation condition. However, with high class separation, the ML/EM estimator did not produce problematic bias for the class proportions. The MCMC weak conditions showed problematic bias in both of the class proportions for all three class separation conditions. The bias increased as class separation worsened. The MCMC tight condition showed relatively low bias for all separation conditions.

The factor loadings show a similar pattern across estimators in Table 2. ML/EM only yielded higher bias for Item 9 in C2 (the minority class) for the high separation condition. However, the MCMC weak condition shows higher bias for all of the items in the moderate and poor class separation conditions for C2. There was an improvement under the high separation condition in that only two items produced higher bias for the MCMC weak conditions. Finally, none of the factor loadings produced problematic bias under the MCMC tight conditions.

The structural parameters (factor variances, covariances, and means) showed higher bias for both of the MCMC conditions. The ML/EM estimator condition produced lower bias compared to the MCMC conditions. However, note that the priors on these structural parameters were weak under both MCMC conditions. This was done to assess the impact of tight priors only on the measurement model parameters and the class proportions. Bias in the structural portion of the model is examined in more detail later.

Table 3 presents results for the 0.20/0.80 mixture class proportions where the minority class (C1) is now represented by the larger factor loadings. Results showed similar patterns with the MCMC weak condition compared to the 0.80/0.20 conditions presented in Table 2. Specifically, the class proportion bias was the highest compared to the other estimator conditions. Both of the mixture classes had high bias under the moderate and poor separation conditions; the minority class C1 had the highest bias. ML/EM showed bias in the class proportion corresponding to C1 under the moderate separation condition. In addition, ML/EM also produced high bias for both classes under the poor separation condition. None of the estimators produced problematic bias for the high separation condition here and the MCMC tight condition did not show high bias for any of the separation conditions.

High bias for the factor loadings was only produced by ML/EM and the MCMC weak condition. Note that ML/EM did not produce high bias for any of the items in the majority class (C2), but that the MCMC weak condition showed higher bias in both classes. Strangely, the high bias in loadings was only experienced in the high and moderate class separation conditions. Akin to the 0.80/0.20 condition in Table 2, the structural parameters for both MCMC conditions exhibited higher and more frequent bias in Table 3 as well.

Table 4 includes the results for the 0.50/0.50 mixture class proportion condition. None of the estimator conditions showed problematic bias for the mixture class proportions under this condition. High bias for the factor loadings was only exhibited by the MCMC weak condition for C2. Finally, the structural model parameters still produced higher bias for some of the parameters under each of the estimator conditions.

Tables 2 through 4 also include information about bias as sample sizes were increased to either $n = 300$ or 800. Most of the parameters with problematic bias at $n = 100$ experienced a substantial decrease in bias as sample sizes increased, especially when increased to $n = 800$.

TABLE 2
Measurement Model Separation: 80/20 Mixture Class Proportions, 100 Cases

	High Separation			*Moderate Separation*			*Poor Separation*		
Parameter	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*
C1 proportion	−2.32	1.38	−3.39	−9.31	0.88	**−22.42**[a]	**−15.90**[a]	0.88	**−32.18**
C2 proportion	9.29	−5.53	**13.56**[a]	**37.23**[b]	−3.54	**89.68**[b]	**63.61**	−4.35	**128.74**
Loadings: C1									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−0.43	1.06	4.54	−0.25	0.71	5.71	1.25	−0.41	0.99
Item 3	−0.04	0.36	3.37	−0.10	1.30	5.72	1.51	−0.70	1.15
Item 4	−0.13	0.26	3.45	−0.33	1.19	5.47	0.99	−0.61	0.92
Item 5	−0.91	0.67	4.10	−0.28	0.54	5.39	−0.76	−0.23	1.05
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−0.65	0.32	4.54	−0.80	0.21	3.49	−0.74	−0.36	2.18
Item 8	0.29	0.59	4.53	−0.14	−0.01	3.09	4.45	0.04	2.21
Item 9	0.31	0.25	4.10	0.30	0.46	3.36	0.63	−0.81	2.10
Item 10	−1.19	0.01	3.80	−0.15	0.11	3.69	−0.03	−0.53	2.21
Loadings: C2									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	4.37	0.47	0.67	−6.26	−0.60	**36.70**[b]	−4.50	0.34	**13.10**
Item 3	3.57	−1.80	−2.17	−2.84	0.02	**38.94**[b]	8.16	−0.27	**12.77**
Item 4	6.93	−1.60	−3.13	1.86	−3.42	**35.94**[b]	3.73	0.04	**12.66**
Item 5	2.27	0.30	1.27	−1.90	−1.04	**36.68**[b]	−3.67	0.00	**12.46**
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−1.10	0.33	**15.33**[a]	−1.48	−2.06	**28.88**[a]	−7.89	−0.61	**14.83**
Item 8	0.73	0.47	**11.50**[a]	−2.10	−2.16	**28.00**[a]	−2.80	−1.00	**14.39**
Item 9	**10.73**[a]	−1.10	8.90	3.84	−2.40	**30.50**[b]	−5.49	−0.46	**15.00**
Item 10	5.63	−1.30	8.80	−6.42	−1.02	**31.22**[b]	−6.97	−0.99	**14.59**
Variances: C1									
Factor 1	−2.50	9.44	1.51	−1.06	6.48	8.64	−7.72	0.38	7.73
Factor 2	−0.30	4.03	0.22	3.15	5.91	**16.39**[a]	4.02	−0.29	7.61
Cov(F1,F2)	−6.35	6.98	−3.08	−6.13	9.00	7.60	−6.23	3.72	3.65
Variances: C2									
Factor 1	−5.11	**85.12**[b]	**66.96**[b]	1.72	**84.89**[b]	**11.90**[a]	−8.24	**90.91**[b]	7.30
Factor 2	−7.61	**86.72**[b]	**42.76**[b]	−4.06	**93.03**[b]	**18.96**[a]	−7.36	**106.91**[b]	7.19
Cov(F1,F2)	**−11.55**[a]	**84.43**[b]	**57.10**[b]	**−13.68**[a]	**82.05**[b]	5.53	**−18.93**	**83.68**[b]	−6.68
Means: C1									
Factor 1	**24.63**[a]	0.12	−0.03	**232.33**	−3.76	0.11	**70.00**	−4.91	−5.07
Factor 2	**20.67**[a]	−3.14	−5.93	**171.00**	−7.06	**−10.38**[a]	**53.15**[b]	−3.31	**−13.56**[a]
Means: C2									
Factor 1	**26.52**[a]	3.54	**−10.65**[a]	**229.19**	−2.36	**−13.21**[a]	**57.62**	0.87	**−12.02**[a]
Factor	**21.98**[a]	−3.14	**−19.48**[a]	**165.45**	−1.46	**−29.16**[b]	**48.10**	−0.96	**−26.74**

Note. ML/EM = maximum likelihood via the EM algorithm; Tight = Markov chain Monte Carlo estimation using tight priors; Weak = Markov chain Monte Carlo estimation using weak priors; C1 = mixture class 1; C2 = mixture class 2. Bold values indicate problematic bias levels greater than 10.00%.

[a]Bias decreased below 10.00% when sample size increased to 300.

[b]Bias decreased below 10.00% when sample size increased to 800.

TABLE 3
Measurement Model Separation: 20/80 Mixture Class Proportions, 100 Cases

	High Separation			*Moderate Separation*			*Poor Separation*		
Parameter	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*
C1 proportion	−2.93	−8.38	9.90	**22.74**[a]	**−11.22**[a]	**78.89**[b]	**68.76**	−4.89	**136.63**
C2 proportion	0.73	2.09	−2.47	−5.68	2.80	**−19.72**[a]	**−17.19**	1.22	**−34.16**
Loadings: C1									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	1.94	0.79	**10.24**[a]	**19.94**[a]	1.40	0.24	6.36	−0.25	−4.70
Item 3	3.84	−0.03	8.68	**24.16**[a]	−0.55	−3.25	5.59	−0.65	−4.63
Item 4	2.00	−0.36	8.31	**22.58**[a]	0.63	−3.76	2.31	−0.99	−4.69
Item 5	3.01	0.13	8.70	**21.59**[a]	0.08	−1.65	9.34	−1.04	−4.86
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−0.24	−1.13	9.04	−1.41	−0.38	−6.09	0.75	−0.76	−3.16
Item 8	1.09	0.16	**12.96**[a]	−1.40	0.83	−0.66	5.41	−1.04	−2.51
Item 9	0.75	−0.63	**10.99**[a]	0.71	−0.34	−6.16	2.30	−1.16	−3.23
Item 10	−0.39	−1.90	8.11	−1.96	−1.23	**−10.08**[a]	−1.59	−0.74	−2.70
Loadings: C2									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−1.27	2.60	8.23	−3.42	1.02	**14.02**[a]	0.24	0.80	4.41
Item 3	0.33	0.30	5.17	−2.24	0.38	**10.00**[a]	1.19	0.51	4.44
Item 4	0.43	−0.23	4.13	−2.96	−1.30	8.14	−1.04	1.04	4.46
Item 5	−1.43	1.80	7.30	−3.98	−0.32	**10.16**[a]	−2.81	0.80	4.03
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−4.17	3.83	6.63	−1.98	1.80	3.96	−3.57	0.59	5.03
Item 8	−1.97	2.67	5.73	−1.66	1.76	4.78	−5.29	−0.04	5.10
Item 9	−1.30	2.10	6.27	−3.52	0.92	3.54	−4.93	−0.11	5.19
Item 10	−5.73	2.53	5.13	−4.28	1.94	6.96	−2.33	1.09	4.94
Variances: C1									
Factor 1	−6.46	**86.84**[b]	**67.69**[b]	**−14.37**[a]	**176.62**[b]	**39.00**[b]	**−19.54**[a]	**−82.50**[a]	6.48
Factor 2	−3.09	**76.91**[a]	**40.02**[b]	0.20	**91.36**[b]	**19.69**[a]	0.89	**−83.16**[a]	4.78
Cov(F1,F2)	**−17.53**[a]	**89.88**	**75.28**	**−19.85**[a]	**140.60**	**38.45**	**−13.35**[a]	**73.40**[b]	−3.43
Variances: C2									
Factor 1	0.77	4.66	−0.95	2.31	4.77	−1.56	1.12	−0.58	6.25
Factor 2	1.58	2.55	−1.11	−2.30	0.76	2.09	−7.76	1.01	7.39
Cov(F1,F2)	−4.15	1.23	−5.33	−2.48	−1.68	**−14.65**[a]	−2.58	0.45	−0.15
Means: C1									
Factor 1	**32.22**[a]	0.03	**−11.32**[a]	**62.48**[b]	0.16	**−20.38**[a]	**90.49**[b]	−2.13	**−14.71**[a]
Factor 2	**10.98**[a]	4.80	**−18.49**[a]	**354.83**[b]	4.26	**−32.26**[b]	**119.48**	3.72	**−28.57**
Means: C2									
Factor 1	**33.66**[a]	−1.29	−3.79	**64.03**[b]	−4.98	−9.75	**90.86**[b]	−3.31	−9.85
Factor 2	9.77	3.59	**−12.94**[a]	**349.68**[b]	−4.66	**−21.62**[a]	**130.18**	−4.12	**−21.06**[a]

Note. ML/EM = maximum likelihood via the EM algorithm; Tight = Markov chain Monte Carlo estimation using tight priors; Weak = Markov chain Monte Carlo estimation using weak priors; C1 = mixture class 1; C2 = mixture class 2. Bold values indicate problematic bias levels greater than 10.00%.

[a]Bias decreased below 10.00% when sample size increased to 300.

[b]Bias decreased below 10.00% when sample size increased to 800.

TABLE 4
Structural Model Separation: 50/50 Mixture Class Proportions, 100 Cases

	High Separation			*Moderate Separation*			*Poor Separation*		
Parameter	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*
C1 proportion	−1.61	−0.71	0.16	−1.86	0.71	4.18	2.09	0.40	0.97
C2 proportion	1.61	0.71	−0.16	1.86	−0.71	−4.18	−2.09	−0.40	−0.97
Loadings: C1									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−0.44	0.75	7.61	−1.53	0.74	7.95	1.65	−0.15	−2.61
Item 3	0.16	−0.33	5.98	−1.82	−0.46	5.60	1.60	−1.18	−2.41
Item 4	−0.40	−0.55	5.34	−1.59	−0.91	5.76	1.74	−2.45	−2.11
Item 5	−0.95	−0.03	6.44	−3.54	−0.16	5.71	2.26	−1.33	−5.09
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−1.25	−0.03	5.94	−1.93	0.06	2.44	0.95	−0.50	−8.38
Item 8	−0.54	−0.04	6.11	0.31	0.11	3.40	5.40	−0.29	−2.21
Item 9	−0.05	−0.58	5.00	0.35	−0.73	1.88	2.06	−1.81	0.04
Item 10	−1.78	−0.85	4.61	−2.05	−0.85	1.58	−3.46	−1.21	1.63
Loadings: C2									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	3.47	0.20	5.17	−1.14	1.44	**16.18**[a]	1.36	1.73	**10.89**[a]
Item 3	5.30	−0.63	2.43	1.06	1.22	**15.68**[a]	3.21	1.54	**10.41**[a]
Item 4	5.90	−0.77	1.63	1.32	0.40	**14.52**[a]	0.53	2.04	**10.00**[a]
Item 5	3.07	−0.03	4.10	−1.48	0.72	**16.66**[a]	−1.94	1.81	**13.44**[a]
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−2.27	0.02	7.77	−0.16	0.46	**11.16**[a]	−6.67	−0.11	**12.51**[a]
Item 8	1.90	2.53	8.97	0.60	0.66	3.86	−3.56	1.20	**12.81**[a]
Item 9	−1.20	1.93	6.17	−3.22	0.56	**11.54**[a]	−3.93	1.24	**10.07**[a]
Item 10	−2.47	0.90	7.70	−2.26	0.82	**12.24**[a]	−6.24	0.56	**10.54**[a]
Variances: C1									
Factor 1	−1.96	**16.86**[a]	7.41	−5.78	**14.25**	4.73	−4.83	6.57	4.23
Factor 2	3.15	**14.23**[a]	6.08	3.66	**11.36**	**10.84**	4.33	3.04	7.43
Cov(F1,F2)	−2.38	**14.65**[a]	4.03	−7.28	**15.00**	**11.45**[a]	**−11.13**[b]	8.65	−2.38
Variances: C2									
Factor 1	−1.06	**15.29**[a]	**11.05**	0.97	**12.77**	1.47	−8.65	**13.67**[a]	5.82
Factor 2	−3.94	7.33	1.37	−4.90	8.74	5.66	**−10.00**[a]	**11.33**[a]	**10.89**[a]
Cov(F1,F2)	**−10.73**[a]	**12.23**[a]	7.45	**−11.85**	**11.23**	0.45	**−19.75**[a]	9.85	3.23
Means: C1									
Factor 1	**13.38**[a]	−0.61	−6.66	**105.49**[b]	−5.19	−7.66	**203.93**	−9.19	−6.17
Factor 2	8.20	−0.93	−8.04	**72.31**[b]	−9.91	**−11.42**[a]	**404.75**	−6.44	**−23.66**[a]
Means: C2									
Factor 1	**14.81**[a]	−0.69	−7.76	**103.86**[b]	−7.60	**−10.04**[a]	**199.31**	−1.65	**−12.59**[a]
Factor 2	**10.80**[a]	−0.96	**−11.16**[a]	**66.75**[a]	**−14.06**[a]	**−17.50**[a]	**401.95**	−2.06	**−21.24**[b]

Note. ML/EM = maximum likelihood via the EM algorithm; Tight = Markov chain Monte Carlo estimation using tight priors; Weak = Markov chain Monte Carlo estimation using weak priors; C1 = mixture class 1; C2 = mixture class 2. Bold values indicate problematic bias levels greater than 10.00%.

[a]Bias decreased below 10.00% when sample size increased to 300.

[b]Bias decreased below 10.00% when sample size increased to 800.

However, there were still some parameters that showed higher bias even with a relatively large sample size. For example Tables 2 and 3 show that the bias remained high for the mixture class proportions for ML/EM and MCMC weak under the poor class separation condition as sample sizes increased.

STRUCTURAL MODEL RESULTS

Akin to the measurement model, results presented in the tables for the structural model portion of this study are only based on the sample size condition of $n = 100$. Results from the $n = 300$ and 800 conditions are summarized and the tables are available on request.

Results for the 0.80/0.20 mixture class proportion for the structural model conditions can be found in Table 5. The only mixture class proportion exhibiting high bias is under the poor separation condition for the MCMC weak condition; note that it was the minority class C2 experiencing high bias. All other separation and estimator conditions produced acceptable bias levels for mixture class proportions. The poor structural model separation condition produced higher bias for Item 9 under ML/EM and for Items 7 and 8 for the MCMC weak condition. All of the other conditions had acceptable bias levels for factor loadings.

The structural parameters for the majority mixture class (C1 in this case) only showed high bias for a few of the parameters. Specifically, the Factor 1 variance under the high separation condition for the MCMC weak condition showed high bias. Likewise, the factor means for ML/EM exhibited higher bias in the moderate and poor separation conditions for C1.

In contrast, the results for the minority class (C2 in this case) showed high bias for many more parameters compared to the majority class (C1). Specifically, all of the factor variance and covariance parameters showed higher bias for the MCMC weak condition. Likewise, ML/EM exhibited higher bias for these parameters under the moderate and high separation conditions as well as for the factor covariance in the poor separation condition. The MCMC tight condition only produced higher bias for the factor covariances for the minority class (C2). Finally, ML/EM produced higher bias for the factor means for C2 under all separation conditions. The MCMC weak condition only produced higher bias for the C2 factor means under the poor separation condition.

Results from the structural model conditions for the 0.20/0.80 conditions are presented in Table 6. Mixture class proportions showed acceptable bias with the exception of the minority class (C1 in this case) for moderate separation under the MCMC weak condition. Likewise, the only problematic bias exhibited for the factor loadings can be found in C1 under the MCMC weak condition. The ML/EM and MCMC tight conditions both produced low bias for the class proportions and factor loadings.

The structural model parameters show higher bias for many of the parameters under ML/EM and the MCMC weak condition; note that bias is higher and more frequent for the minority mixture class (C1). The MCMC tight condition only exhibited higher bias for the factor covariance for the minority class (C1) under the poor and moderate separation conditions. Likewise, only one factor mean showed higher bias under this estimator condition.

Finally, the structural model condition results for the 0.50/0.50 mixture class proportions can be found in Table 7. Being that there were an ample number of cases in each of the mixture classes under this condition, the factor loadings and mixture class proportions did not exhibit

TABLE 5
Structural Model Separation: 80/20 Mixture Class Proportions, 100 Cases

	High Separation			*Moderate Separation*			*Poor Separation*		
Parameter	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*
C1 proportion	−0.36	0.51	0.19	−0.17	0.79	−0.62	−2.32	1.35	−3.39
C2 proportion	1.43	−2.03	−0.76	0.66	−3.16	2.48	9.29	−5.41	**13.56**
Loadings: C1									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−0.69	1.84	2.56	−0.29	1.91	3.22	−0.43	2.03	4.54
Item 3	−0.13	0.76	1.16	0.17	0.91	1.99	−0.04	1.31	3.37
Item 4	−0.53	1.07	1.65	−0.18	1.19	2.75	−0.13	1.29	3.45
Item 5	−0.98	0.90	1.56	−0.70	0.97	2.60	−0.91	1.22	4.10
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−0.91	0.91	2.21	−0.56	0.88	3.03	−0.65	0.86	4.54
Item 8	0.75	1.21	2.32	1.07	1.19	3.74	0.29	1.10	4.53
Item 9	0.05	0.76	2.15	0.44	0.70	3.35	0.31	0.67	4.10
Item 10	−1.25	0.65	1.35	−0.89	0.69	2.90	−1.19	0.65	3.80
Loadings: C2									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	2.30	1.40	0.30	4.37	1.13	1.00	4.37	0.97	0.67
Item 3	1.00	−1.53	−4.37	2.97	−1.77	−4.00	3.57	−0.70	−2.17
Item 4	4.63	−0.93	−3.57	6.60	−1.47	−4.33	6.93	−1.03	−3.13
Item 5	2.20	0.57	−0.30	3.30	0.73	0.50	2.27	1.20	1.27
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−2.53	2.33	3.47	−3.33	2.43	6.17	−1.10	1.97	**15.33**[a]
Item 8	−1.63	1.97	3.80	−2.67	2.00	6.47	0.73	1.73	**11.50**[a]
Item 9	2.50	0.77	1.40	3.60	0.67	2.37	**10.73**[a]	0.97	8.90
Item 10	−1.90	−0.43	0.93	−1.93	−0.97	1.03	5.63	0.03	8.80
Variances: C1									
Factor 1	0.97	1.89	**10.04**[a]	1.32	2.33	7.38	−2.50	1.36	1.51
Factor 2	0.99	−3.06	3.68	0.10	−2.87	2.03	0.30	−3.30	0.22
Cov(F1,F2)	−2.28	1.35	6.08	−1.75	2.37	4.87	−6.35	1.40	−3.08
Variances: C2									
Factor 1	**−10.70**[a]	3.75	**66.59**[a]	**−15.45**[a]	1.01	**61.47**[b]	−5.11	−3.50	**66.96**[b]
Factor 2	**−10.39**[a]	−0.40	**58.34**[b]	−9.23	−1.61	**63.53**[b]	−7.61	−6.32	**42.76**[b]
Cov(F1,F2)	**−56.93**[b]	**29.48**[a]	**44.23**[a]	**−46.50**[b]	**11.63**[a]	**87.50**[a]	**−11.55**[a]	7.33	**57.25**[b]
Means: C1									
Factor 1	5.30	−2.02	−2.64	**10.05**[a]	−2.71	−5.44	**24.63**[a]	−2.67	−3.47
Factor 2	2.44	−2.00	−0.90	1.94	−2.91	−4.17	**20.67**[a]	−2.45	−5.93
Means: C2									
Factor 1	**18.44**[a]	0.55	2.32	**23.50**[a]	0.75	−2.15	**26.52**[a]	0.41	**−10.65**[a]
Factor 2	**12.06**[a]	−0.99	−0.18	**10.76**[a]	−0.85	−7.62	**21.98**[a]	−0.58	**−19.48**[a]

Note. ML/EM = maximum likelihood via the EM algorithm; Tight = Markov chain Monte Carlo estimation using tight priors; Weak = Markov chain Monte Carlo estimation using weak priors; C1 = mixture class 1; C2 = mixture class 2. Bold values indicate problematic bias levels greater than 10.00%.

[a]Bias decreased below 10.00% when sample size increased to 300.

[b]Bias decreased below 10.00% when sample size increased to 800.

TABLE 6
Structural Model Separation: 20/80 Mixture Class Proportions, 100 Cases

	High Separation			*Moderate Separation*			*Poor Separation*		
Parameter	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*
C1 proportion	0.02	−7.84	7.98	−1.32	−7.93	**12.72**[a]	−2.93	−7.35	9.90
C2 proportion	0.00	1.96	−2.00	0.33	1.98	−3.18	0.73	1.84	−2.47
Loadings: C1									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	3.12	1.39	7.12	4.62	1.43	**11.64**[a]	1.94	1.89	**10.24**[a]
Item 3	5.71	0.80	6.74	6.65	0.80	**10.36**[a]	3.84	1.00	8.67
Item 4	3.18	0.57	6.34	4.40	0.46	**10.05**[a]	2.00	0.82	8.31
Item 5	4.45	0.67	6.80	5.37	0.66	**10.18**[a]	3.01	1.21	8.70
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−1.76	0.35	6.15	−1.86	0.49	4.89	−0.24	0.52	9.04
Item 8	−1.16	1.40	7.94	−0.48	1.51	7.59	1.09	1.72	**12.96**[a]
Item 9	−0.43	0.74	6.11	0.01	0.95	5.94	0.75	0.95	**10.99**[a]
Item 10	−2.25	−0.56	3.84	−2.24	−0.48	2.76	−0.39	−0.29	8.11
Loadings: C2									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−1.03	1.70	1.27	−1.57	2.63	1.77	−1.27	4.17	8.23
Item 3	−0.33	0.63	−0.13	−0.53	1.33	−0.17	0.33	1.97	5.17
Item 4	−0.33	0.20	−0.53	−0.43	0.67	−0.90	0.43	1.33	4.13
Item 5	−1.73	1.47	0.83	−2.37	2.20	1.33	−1.43	3.07	7.30
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−1.73	2.83	1.97	−2.40	3.83	3.00	−4.17	5.13	6.63
Item 8	−0.83	2.20	1.37	−0.70	2.93	2.10	−1.97	4.17	5.73
Item 9	−0.77	1.67	0.87	−0.77	2.50	1.37	−1.30	4.10	6.27
Item 10	−2.80	1.73	0.47	−3.47	2.50	1.27	−5.73	4.03	5.13
Variances: C1									
Factor 1	1.75	1.81	**199.86**	−5.44	4.26	**−66.73**[b]	−6.46	1.48	**67.69**[b]
Factor 2	4.94	−2.79	**197.70**	6.41	−2.50	**71.51**[b]	−3.09	−0.54	**40.02**[b]
Cov(F1,F2)	−0.53	9.08	**403.25**[b]	−7.93	**11.20**[a]	**86.85**	**−17.53**[a]	**11.13**[a]	**75.28**
Variances: C2									
Factor 1	1.75	1.81	**199.86**[a]	0.64	−1.88	**−39.37**[a]	0.77	−4.54	−0.95
Factor 2	4.94	−2.79	**197.70**[a]	0.05	−5.68	**−67.70**[a]	1.58	−7.51	−1.11
Cov(F1,F2)	**−18.03**[a]	−4.63	**−15.18**[a]	−8.55	−4.68	**267.93**[a]	−4.15	−3.43	−5.33
Means: C1									
Factor 1	6.40	**−49.45**[a]	**−13.51**[a]	**103.42**[a]	0.51	**−20.53**[a]	**32.22**[a]	0.02	**−11.32**[a]
Factor 2	**11.01**[a]	1.11	**−10.73**[a]	**19.90**[a]	1.04	**−14.53**[a]	**10.98**[a]	0.24	**−18.49**[a]
Means: C2									
Factor 1	**13.08**[a]	−0.23	−1.86	**107.48**[a]	−0.40	**60.35**[a]	**33.66**[a]	−2.24	−3.79
Factor 2	**17.24**[a]	−1.38	−2.74	**24.59**[a]	−1.71	**210.87**[a]	9.77	−2.09	**−12.94**[a]

Note. ML/EM = maximum likelihood via the EM algorithm; Tight = Markov chain Monte Carlo estimation using tight priors; Weak = Markov chain Monte Carlo estimation using weak priors; C1 = mixture class 1; C2 = mixture class 2. Bold values indicate problematic bias levels greater than 10.00%.

[a]Bias decreased below 10.00% when sample size increased to 300.

[b]Bias decreased below 10.00% when sample size increased to 800.

TABLE 7
Structural Model Separation: 50/50 Mixture Class Proportions, 100 Cases

	High Separation			*Moderate Separation*			*Poor Separation*		
Parameter	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*	*ML/EM*	*Tight*	*Weak*
C1 proportion	−1.21	−1.08	1.65	−1.07	−1.12	1.64	−1.61	0.03	0.16
C2 proportion	1.21	1.08	−1.65	1.07	1.12	−1.64	1.61	−0.03	−0.16
Loadings: C1									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−0.24	1.71	5.31	−0.69	1.72	5.46	−0.44	1.97	7.61
Item 3	0.49	0.30	3.46	0.15	0.38	3.61	0.16	0.82	5.97
Item 4	−0.13	0.40	3.37	−0.44	0.51	3.44	−0.40	0.59	5.34
Item 5	−0.54	0.90	4.08	−0.84	1.01	4.16	−0.95	1.25	6.44
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−1.41	1.03	3.79	−1.34	0.92	3.87	−1.25	0.95	5.94
Item 8	−1.19	1.00	3.20	−0.93	0.92	3.34	−0.54	1.09	6.11
Item 9	−0.28	0.38	2.36	−0.09	0.38	2.54	−0.05	0.41	5.00
Item 10	−2.04	0.24	2.00	−1.93	0.26	2.19	−1.78	0.20	4.61
Loadings: C2									
Item 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 2	−0.17	1.80	1.47	0.30	2.20	2.20	3.47	2.50	5.17
Item 3	1.33	−0.03	−0.97	1.97	0.30	−0.60	5.30	1.20	2.43
Item 4	1.40	0.37	−0.40	1.90	0.53	−0.13	5.90	0.87	1.63
Item 5	0.10	1.50	0.90	0.33	1.73	1.53	3.07	1.57	4.10
Item 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Item 7	−1.70	2.07	1.20	−1.87	2.97	2.40	−2.27	0.02	7.77
Item 8	0.23	2.37	1.73	0.47	3.20	3.07	1.90	2.53	8.97
Item 9	0.13	1.40	0.77	0.13	2.27	1.70	−1.20	1.93	6.17
Item 10	−2.40	1.40	0.57	−2.63	2.23	1.27	−2.47	0.90	7.70
Variances: C1									
Factor 1	−2.26	−0.12	**15.66**[a]	−0.98	0.30	**15.38**[a]	−1.96	0.76	7.41
Factor 2	5.37	−2.12	**10.86**[a]	5.09	−1.69	**10.10**[a]	3.15	−1.34	6.08
Cov(F1,F2)	0.32	−0.95	**13.18**[a]	0.55	−0.23	**12.60**[a]	−2.38	1.57	4.03
Variances: C2									
Factor 1	−1.19	1.95	**17.73**[a]	−1.78	0.69	**15.49**[a]	−1.06	−5.72	**11.05**[a]
Factor 2	−4.57	−5.82	8.57	−4.95	−7.23	7.48	−3.94	**−10.92**[a]	1.37
Cov(F1,F2)	**−39.78**[b]	**10.73**[a]	**14.78**[a]	**−27.48**[b]	4.97	9.77	**−10.73**[a]	−1.03	7.45
Means: C1									
Factor 1	4.60	−0.66	−5.07	6.56	−0.91	−5.06	**13.38**[a]	−2.05	−6.66
Factor 2	5.74	−0.67	−5.24	5.98	−0.61	−5.02	8.20	−1.72	−8.04
Means: C2									
Factor 1	7.96	−0.21	−0.86	9.99	−0.62	−1.29	**14.81**[a]	−1.74	−7.76
Factor 2	**12.43**[a]	−1.76	−4.03	**11.84**[a]	−1.73	−3.40	**10.80**[a]	−0.60	**−11.16**

Note. ML/EM = maximum likelihood via the EM algorithm; Tight = Markov chain Monte Carlo estimation using tight priors; Weak = Markov chain Monte Carlo estimation using weak priors; C1 = mixture class 1; C2 = mixture class 2. Bold values indicate problematic bias levels greater than 10.00%.

[a]Bias decreased below 10.00% when sample size increased to 300.

[b]Bias decreased below 10.00% when sample size increased to 800.

high bias under any of the separation or estimator conditions. However, several of the structural model parameters did show higher bias under certain conditions.

Specifically, the MCMC weak condition exhibited higher bias for several variance and covariance parameters in each of the mixture classes. ML/EM only showed higher bias in the factor covariance parameters for C2. Likewise, the MCMC tight condition only showed higher bias for two of the parameters in C2. In a similar fashion, the factor means produced some parameters with higher bias for ML/EM under all separation conditions. There was also one factor mean (Factor 2 under C2) where the MCMC weak condition exhibited higher bias for the poor separation condition. None of the factor means had high bias for the MCMC tight condition.

Tables 5 through 7 also illustrate the impact of sample sizes on the bias levels. For example, Table 6 indicates that bias is still problematic with larger sample sizes for the minority class (C1) factor variances in the high class separation condition under the MCMC weak condition. However, the results in Table 7 show that all of the parameters experienced an adequate decrease in estimate bias as sample sizes increased. This was likely a function of the larger sample sizes in each of the mixture classes. Specifically, in this 0.50/0.50 condition, there was not a small minority class as in the other class proportion conditions. An example of the M*plus* input code for an MCMC tight condition is provided in the Appendix.

CONCLUSIONS AND IMPLICATIONS FOR APPLIED RESEARCHERS

In general, results indicated that ML/EM and MCMC weak exhibited higher bias than MCMC tight. This held true for the measurement model and the structural model separation conditions, with bias being higher for poor separation conditions. Additionally, MCMC weak showed substantially higher and more prevalent bias than ML/EM in all conditions. The results also indicated that class separation in the structural model yielded higher bias when compared to the measurement model for all estimators included in the investigation. This implies that structural model class separation can produce a larger impact on the corresponding parameter estimates compared to measurement model separation.

As expected, the issue of sample size also had an influence on parameter estimates. Bias levels were higher and more prevalent in lower sample size conditions, particularly when the minority class size was smaller (e.g., the 0.80/0.20 condition for $n = 100$ had higher bias than the 0.50/0.50 condition).

However, there were some conditions that did not improve as sample sizes were raised, indicating that bias in some parameters will not decrease by merely increasing the sample size. Specifically, the mixture class proportions exhibited high bias even with a larger sample size in the measurement model separation conditions. The structural model results differed in that mixture class proportions rarely produced high bias levels. This finding implies that the mixture class separation defined in the measurement model has a larger impact on the class proportion estimates than mixture class separation defined in the structural model. In addition, this affect is not necessarily combated by simply increasing the total sample size because high levels of estimate bias were still present with larger sample sizes under the measurement model separation conditions.

In addition, when comparing results for the measurement and structural portions of this study, it appears that the structural model results produced higher and more prevalent estimate bias compared to the measurement model results. Although problematic bias levels were obtained for some measurement model parameters when class separation was defined in the context of the measurement model, the bias levels were far greater in the structural model parameters when class separation was defined in the structural model. This result indicates that poor mixture class separation in the structural model has a larger impact on the structural model parameters compared to the impact that poor measurement model separation has on measurement model parameters.

As a caveat for applied researchers, when using the MCMC estimation algorithm, it appears to be crucial to include accurate and informative priors on the mixture class proportions. If there is reason to believe that a smaller sized mixture class exists, the optimal estimator choice is to use MCMC with a tighter Dirichlet (or akin) prior distribution on the mixture proportions to reflect this belief. Results of this study showed that having a diffuse prior on the mixture proportions created mixture class estimates that neared 0.50/0.50, particularly in the measurement model portion of the study. This observation explains why the mixture proportions exhibited such high bias levels under the MCMC weak condition in Tables 2 and 3. In these conditions, the true proportions were 0.80/0.20 and 0.20/0.80 but the weak Dirichlet prior ($\mathcal{D}(10, 10)$) was pulling these proportions to 0.50/0.50 for the two mixtures; this happened even in larger sample sizes.

The results for the condition with true proportions of 0.50/0.50 found in Table 4 show very minimal bias for the mixture proportions under this same MCMC condition using weak priors. This is likely due to the true model parameters coinciding with the tendency for the diffuse Dirichlet prior to pull proportions to $1/C$ where C is the number of mixture classes. In other words, the true proportions specified for these conditions happened to also correspond with the weak Dirichlet prior specified. Overall, results indicate that a weak Dirichlet prior can impact proportion estimates even with larger sample sizes. Although theory suggests that higher sample sizes combat the influence of prior distributions (particularly diffuse priors), this finding is in agreement with the results presented in Natarajan and McCulloch (1998) where diffuse priors also affected estimates under moderate and large sample sizes. More research is needed in this area to determine in what (if any) circumstances the use of a weak Dirichlet prior is appropriate.

In summary, the findings reported here indicate that applied researchers should only use MCMC for finite mixture models when accurate and tighter priors can be placed on at least some of the parameters. Results indicate that this approach should include tighter priors on the mixture proportions and the structural model parameters, in particular. However, if sufficient knowledge is lacking and weak priors must be placed on these parameters, using an MCMC estimator might increase the estimate bias and therefore produce inaccurate model parameter interpretations; this is especially the case for the mixture class proportions. As a result, situations where there is minimal prior knowledge about model parameters would warrant using the ML/EM estimator with multiple starting values to ensure proper convergence. Although high bias was still present in the ML/EM conditions, it was much lower and less prevalent compared to MCMC weak.

Finally, although it was necessary for the design of this study to remain relatively succinct, one limitation here was that the informative prior conditions only explored very tight prior distributions that would perhaps be unrealistic under certain applied settings. Future research should focus on varying degrees of the informativeness of priors (especially for the mixture proportions), to better understand the impact of tighter priors on estimate bias.

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APPENDIX

#MCMC Tight M*plus* Input Code

```
title: High loading separation, 0.80/0.20
montecarlo:
     names are y1-y10;
     genclasses = c(2);
     classes = c(2);
     nobs = 100;
     nrep = 100;
analysis:
     type = mixture;
     estimator=BAYES;
     chains=1;
     distribution=50,000;
     point=mean;
model priors:
     a2∼N(.8,.01);
     a3∼N(.8,.01);
     a4∼N(.8,.01);
     a5∼N(.8,.01);
     b7∼N(.8,.01);
     b8∼N(.8,.01);
     b9∼N(.8,.01);
     b10∼N(.8,.01);
     c2∼N(.3,.01);
     c3∼N(.3,.01);
     c4∼N(.3,.01);
     c5∼N(.3,.01);
     d7∼N(.3,.01);
     d8∼N(.3,.01);
     d9∼N(.3,.01);
     d10∼N(.3,.01);
     e∼D(80,20);
     f1∼IW(1,3);
     f2∼IW(1,3);
     f3∼IW(.4,3);
     g1∼IW(5,3);
     g2∼IW(5,3);
     g3∼IW(.4,3);
     h1∼N(2,.1);
     h2∼N(1,.1);
     i1∼N(2,.1);
     i2∼N(1,.1);
```

```
model population:
%overall%
     y1-y10*.25;
     f1 by y1@1 y2-y5*.8;
     f2 by y6-y10*.8;
     [f1 - f2] @0;
     f1*1;
     f2*1;
     f1 with f2 *.4;
     [c#1 * 1.4225] (e);
%c#1%
     [f1 * 2] (h);
     [f2 * 1] (h);
     f1 by y1@1 y2-y5*.8(a);
     f2 by y6@1 y7-y10*.8(b);
     f1*1(f);
     f2*1(f);
     f1 with f2 *.4(f);
%c#2%
     [f1 * 2] (i);
     [f2 * 1] (i);
     f1 by y1@1 y2-y5*.3(c);
     f2 by y6@1 y7-y10*.3(d);
     f1*5(g);
     f2*5(g);
     f1 with f2 *.4(g);
model:
%overall%
     y1-y10*.25;
     f1 by y1@1 y2-y5*.8;
     f2 by y6-y10*.8;
     [f1 - f2@0];
     f1*1;
     f2*1;
     f1 with f2 *.4;
     [c#1 * 1.4225] (e);
%c#1%
     [f1 * 2] (h1);
     [f2 * 1] (h2);
     f1 by y1@1 y2-y5*.8(a2-a5);
     f2 by y6@1 y7-y10*.8(b7-b10);
     f1*1(f1);
     f2*1(f2);
     f1 with f2 *.4(f3);
%c#2%
     [f1 * 2] (i1);
     [f2 * 1] (i2);
     f1 by y1@1 y2-y5*.3(c2-c5);
     f2 by y6@1 y7-y10*.3(d7-d10);
     f1*5(g1);
     f2*5(g2);
     f1 with f2 *.4(g3);
```