

1. (3,3,4 pts) Above the point  $(0, 1)$ , the plane  $5 + x - 3y + 2z = 0$  is tangent to the surface given by  $z = f(x, y)$ .
  - (a) What is the value of  $f(0, 1)$ ?
  - (b) Find the gradient vector  $\vec{\nabla} f(0, 1)$ .
  - (c) What is the directional derivative of  $f(x, y)$  at  $(0, 1)$  in the direction of  $\vec{v} = -\vec{i} - \vec{j}$ ?
2. (4,3,3 pts) Consider the function  $f(x, y) = \frac{1}{x^2 + y^4}$ .
  - (a) Find the direction in which the function  $f(x, y)$  decreases the fastest at the point  $(1, 1)$ .
  - (b) What is the rate of change of  $f$  in that direction?
  - (c) Use a linear approximation to estimate  $f(x, y)$  at a distance of  $1/2$  unit from  $(1, 1)$  in the direction you found.
3. (5,5 pts) Consider the function  $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$ .
  - a) Find all the critical points of  $f(x, y)$ .
  - b) Classify all the critical points as maximum, minimum or saddle-points.