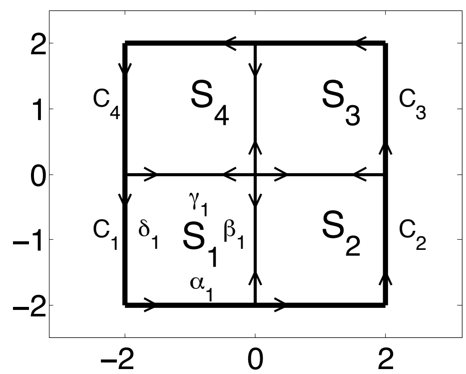
1. Consider the square S of side 4 centered at the origin, as shown in the figure below. S is split into four smaller squares, S_1, S_2, S_3, and S_4, of side 2, each with boundary C_1, C_2, C_3, and C_4, respectively. We label each side of the small squares as α, β, γ, and δ, starting from the bottom. Consider
$$W = \int_{C_1+C_2+C_3+C_4} \vec{F} \cdot \frac{d\vec{r}}{dt} dt$$

   (a) Parametrize β_2.

   (b) Identify the sides where the integral above will cancel.

   (c) Using the idea of Green's theorem, we approximate $\int_{C_n} \vec{F} \cdot \frac{d\vec{r}}{dt} dt \approx A_n = \Delta A \text{ curl}\vec{F}|_{P_n}$ for $n = 1, 2, 3, 4$. Determine what the points P_n should be, and what should be the value of ΔA.

   (d) Bonus (3pts) Explain in words why the sum $A_1 + A_2 + A_3 + A_4$ corresponds to as an approximation of Green's theorem.



2. The vector field $\vec{F} = < 3xy,\ -yz,\ z^2 >$ describes the velocity of the air flowing in a room. Compute the curl and divergence of $\vec{F}$ and describe how a small net held at the point $(1, 2, 1)$ would react to that velocity field (would it inflate, deflate, rotate, translate, etc)?

3. Parametrize the surface obtained by rotating the curve $x = y^4$ around the x-axis and find its normal in terms of your parameters.