Lecture 6, Section 2.6: Plotting surfaces in 3D: cylinders and quadrics

We want to plot functions in 3D: z = f(x,y) = height over the xy-plane.

Quiz 2, taken Friday or Tuesday

Problem of the month

Lecture notes 6_7 Wed.
Homework 6_7 posted

Office hours: Hannah, today, 1:30-2:30pm, 3:45-4:45pm ACS-312

Today: Finish Cylinders and Quadrics

Function of 2 variables over its domain.

More generally, we want to plot all points x, y, z satisfying some equation, not necessarily explicitly. We start here with first degree polynomials:

$$ax + by + cz + d = 0$$

This is a plane, with normal $\vec{n} =< a, b, c >$.
O.K. then on to second degree polynomials:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

That leaves lots of possibilities, so let's simplify things a little. By completing squares, and using a other axes, we can eliminate D, E and F. So for now, we set $D = E = F = 0$.
If $A \neq 0$, we can change $Ax^2 + Gx$ into $A(x + G/2A)^2 - G^2/4A^2$, which is a shift in x and a new value of J. So for now, if $A \neq 0$, we set $G = 0$. Similarly in y and z.

Simplest case: One variable doesn't appear at all $\rightarrow$ Cylinder. This is plotted as you would any 2D graph, and then extended, unchanged, in the direction of the missing variable. Example

$$z = y^2$$

For ANY x, we get the same parabola. Note that we draw a few curves and connect them.

Function of one variable seen in 3D, named a cylinder.

Second Example:

$$x^2 + \frac{z^2}{4} = 1$$

Elliptic cylinder and its contours

For any y, we get an ellipse. Fixing one variable yields a curve, which is called a trace, or a CONTOUR when we fix $z = k$ =constant.

A more complicated, but not so bad case, is one where A, B, and C all have the same sign.

$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

This should look ”spherish”. If we look at some traces, we get:

Ellipsoid

If $x = 0$, we have an ellipse.
If $y = 0$, we have an ellipse.
If $z = 0$, we have an ellipse.
So overall, we get an ELLIPSOID.

Slightly harder is the case where A, B, C don't all have the same sign, and none of them is 0.

$$z^2 - x^2 - y^2 = 1$$

If $x = 0$, we have a hyperbola.
If $y = 0$, we have a hyperbola.
If $z = 0$, we have.. nothing! but if $z = 2$ we get circles
This is called a hyperboloid, of two sheets.

Two-sheet hyperboloid

$$z^2 - x^2 - y^2 = -1$$

gives a hyperboloid of one sheet, as all the contours are circles, and they are all connected.

One sheet hyperboloid

The special case where $J = 0$ gives a CONE.

$$z^2 - x^2 - y^2 = 0$$

A cone

Finally, say $C = 0$, but A and B are non-zero.
Case 1, they have the same sign

$$z = x^2 + 4y^2$$

If $x = k$, we have a parabola.
If $y = k$, we have a parabola.
If $z = k$, we get an ellipse
so this is a paraboloid (elliptic).

Paraboloid

Case 2, they don't have the same sign

$$z = x^2 - 4y^2$$

If $x = k$, we have a parabola, downward.
If $y = k$, we have a parabola, upward.
If $z = k$, we get a hyperbola
so this is a saddle.

Saddle or hyperbolic paraboloid