Lecture 33, Section 6.6: Surface parametrization

We want to do calculus on general surfaces, not just the xy-plane. So far, we used $z = f(x, y)$ to describe a surface. What it really mean is given 2 parameters, x and y, your position in space, and on the surface is: $x = x, y = y, z = f(x, y)$.
This is a surface parametrization. It needs TWO parameters.

In general, we can use the parameters u and v, and define
$x(u, v)$
$y(u, v)$
$z(u, v)$
These are the parametric equations of a surface. Given u and v, you are at a point in space, on the surface. All the values of u and v describe the entire surface.

If we fix $u = u_0$, and let v vary. You get a curve (like $\vec{r}(t)$), a trace.
If we fix $v = v_0$, and let u vary. You get a curve (like $\vec{r}(t)$), a trace.
Varying both u, v, you get the whole surface.

Example, the simplest kind: $x = u, y = v, z = f(x, y) = f(u, v)$.

We can do that based on other coordinates: (r, θ, z)
$x = r \cos \pi/3$
$y = r \sin \pi/3$
$z = z$.
This is a half plane.

Other example
$x = 2 \cos \theta = 2 \cos u$
$y = 2 \sin \theta = 2 \sin u$
$z = v$ describe a cylinder of radius 2, along the z-axis.

Based on spherical coordinates
$x = 2 \cos \theta \sin \phi$
$y = 2 \sin \theta \sin \phi$
$z = 2 \cos \phi$.
is a sphere of radius 2.

$x = \rho \cos \theta \sin \pi/4$
$y = \rho \sin \theta \sin \pi/4$
$z = \rho \cos \pi/4$.
is a cone.

In general, we can do surfaces of revolution in a systematic manner: If you rotate a curve $y = f(x)$ around

Quiz 11 is posted, taken Friday or next Tuesday

Notes 33-34 today: parametrizing surfaces
Homework 33-34 (6.6a)
Notes 34-35 today: parametrizing surfaces
Homework 34-35 (6.6b)

Office hours today: Hannah, 1:30-3:30pm, ACS-312

Final exam is 2 weeks from today

Parametrization of a surface of revolution.

the x-axis, you are rotating in the yz-plane, and $f(x)$ acts like a radius. So $y^2 + z^2 = r^2 = (f(x))^2$.
So our parametrization is
$x = x$
$y = f(x)\cos\theta$
$z = f(x)\sin\theta$
So when $\theta = 0$ you are in the xy-plane, and if $\theta = \pi/2$, you are in the xz-plane.

One more example: a plane, if you know that $\vec{v}_1$ and $\vec{v}_2$ are in the plane, and P_0 is a point on the plane. Then we have $\vec{n} = \vec{v}_1 \times \vec{v}_2$ and $\vec{n} \cdot (< x, y, z > -P_0) = 0$. If $\vec{v}_1$ and $\vec{v}_2$ are not parallel, you can describe the whole plane through

$$\vec{r}(u, v) = P_0 + u\vec{v}_1 + v\vec{v}_2$$

Parametrization of a plane.

And one more rotation example.
Consider the curve $x^2 - z^2 = 1$, rotated about the z axis. This can be written as $x = f(z) = \sqrt{1 + z^2}$.
In the xy-plane, we have $x^2 + y^2 = (f(z))^2 = 1 + z^2$ with $f(z)$ the radius. So we find
$x = (1 + z^2)^{1/2}\cos\theta$
$y = (1 + z^2)^{1/2}\sin\theta$
$z = z$

Surface of revolution, rotated about the z-axis.