Lecture 26, Section 6.1: Vector Fields

So far, we have dealt with scalar function of several variables, like $g(x, y, z)$. To each point in space, these functions associate a number.
Before that, we also saw vector-valued functions of one variable, like $\vec{r}(t) =< x(t), y(t), z(t) >$. To a single number (time) associate a point in space.

Now we want to associate a VECTOR to each point in space. In 2D, that gives:

$$\vec{F}(x, y) = F_1(x, y)\vec{i} + F_2(x, y)\vec{j} =< F_1(x, y), F_2(x, y) >$$

with $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$ and both F_1 and F_2 are scalar functions.
For example: $\vec{F} =< xy, e^y \sin x >$.

To visualize vector fields, we draw vectors $\vec{F}$ starting at point (x, y). So for $\vec{F}(x, y) = x\vec{i} + y\vec{j}$

Quiz 8, last Friday or Tuesday
Quiz 9, taken Friday or next Tuesday

Today: Lecture notes 26
Homework 26

Exam 2 is on April 24
Also posted:
Old Exam 2
List of topics
Cheat Sheet

Office hours today: François, ACS-362, 1-2pm

A 2D vector field.

Today: Vector Fields

We get a kind of explosion, or a source.

Similarly, we can go to 3D $\vec{F}(x, y, z) =< F_1(x, y, z), F_2(x, y, z), F_3(x, y, z) >$.
Those are harder to see though. Try

$$\vec{F}(x, y, z) =< \frac{-x}{(x^2 + y^2)^{1/2}}, \frac{-y}{(x^2 + y^2)^{1/2}}, z >=< -\cos\theta, -\sin\theta, z >$$

which points to the z-axis, upward in the upper plane and downward in the lower plane.

A 3D vector field.

In 3D, straight hair is a good example. Other examples come from science:
1) Force fields (like gravity)

Gravity force field.

2) Velocity fields (wind)

A velocity field.

3) Gradient fields $\nabla f = < f_x, f_y, f_z >$
Say $f(x,y) = \frac{M}{\sqrt{x^2+y^2}}$, $\nabla f = M < \frac{-x}{(x^2+y^2)^{1/2}}, \frac{-y}{(x^2+y^2)^{1/2}} >$

A special kind of vector field is a CONSERVATIVE vector field. A vector field $\vec{F}(x,y)$ is conservative if there exists a scalar function $\phi(x,y)$ such that $\vec{F}(x,y) = \nabla\phi(x,y)$. We call this scalar function the POTENTIAL ϕ (from physics).

How do you check if $\vec{F}$ is conservative? Look for $f(x,y)$. If is exists, you should have that $f_{xy} = f_{yx}$ (if they are both continuous). That would mean that

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$$