Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work. The total number of points is 100.

To complete this exam, you are allowed to use materials from this class ONLY.
You are NOT allowed to use

1. Calculators
2. Websites unrelated to this class
3. The help of any other person.

Academic integrity is the foundation of an academic community and without it none of the educational or research goals of the university can be achieved. Academic integrity applies to research as well as undergraduate and graduate coursework/exams.Existing policies forbid cheating on examinations, plagiarism and other forms of academic dishonesty. UC Merced students are held to high standards of personal and professional conduct in compliance with the UC Merced Academic Honesty Policy and the UCM Code of Student Conduct.UCM Code of Student Conduct can be found here: http://studentconduct.ucmerced.edu

By completing this exam, I acknowledge and confirm that I will not give or receive any unauthorized assistance on this assignment/examination. I will conduct myself within the guidelines of the university academic integrity guidelines.

Please begin your exam by writing: "**I am aware of the rules governing this exam**" and signing your next name next to the statement.

1. (20pts: 6,7,7)
Consider the double integral

$$\int_0^4 \int_{\sqrt{y}}^2 xy^2 \, dx \, dy$$

   (a) Sketch the domain over which this integral is taken.

   (b) Rewrite this integral by changing the order of integration.

   (c) Evaluate the integral you obtained in (b). If you could not do (b), evaluate the integral in the original order. (DO NOT SIMPLIFY NUMBERS).

2. (20pts: 10,10)
The density of a spherical ball of radius 3 centered at the origin is given by $g(x, y, z) = x^2 + y^2$ in kg/m^3. You need to compute the mass of the portion of the ball where $x \geq 0$, $y \geq 0$, and $z \geq 0$.

   (a) Set up an integral to compute the mass in cylindrical coordinates (DON'T EVALUATE).

   (b) Set up an integral to compute the mass in spherical coordinates (DON'T EVALUATE).

3. (25 pts: 10,7,8)
Consider a vector field $\vec{F} =< 6x^2y,\ 2x^3 - e^y >$ and the line segment going from $P = (6, 4)$ to $Q = (4, -1)$.

   (a) Set up an integral of one variable to calculate the work done by $\vec{F}$ on a particle traveling along this line segment by parametrizing the line segment. (DO NOT EVALUATE)

   (b) Verify if $\vec{F}$ is conservative.

   (c) Explain in words how having a conservative vector field could provide you with an alternative way to compute the work mentioned in (a).

4. (35pts: 7,7,7,7,7)
Answer the following questions in no more than two lines of text or computations.

   (a) Give all the conditions you need to check to verify that the point (1,3) is a saddle-point of a function $h(x, y)$.

   (b) How is the Jacobian of a transformation $x(u, v)$ and $y(u, v)$ defined, and when would you use it? not covered in 2026

   (c) The integral $\int \int_D f(x, y)dA$ give the total mass of CO_2 emitted in California in a year. Give (in words) likely interpretations of D and $f(x, y)$.

   (d) Give in words a physically relevant example of a two-dimensional vector field and sketch it.

   (e) Explain when using Green's theorem might be beneficial compared to computing a line integral using a parametrization.

5. (This question multiplies your grade by 1 if correct and by 0 if incorrect).
If it is true, write the statement: "**I have abided by the rules of this exam**" and sign your name next to it.