MATH 23 – Unit Exam 1 VERSION C *Spring Semester 2026*

Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work. The total number of points is 100.

1. (24pts: 8,8,8) Consider the trajectory of a projectile given as a function of time by:
   $\vec{r}(t) =<\ 10t,\ 5t,\ 10t - 5t^2 >$, where t stands for time.

   (a) Compute the velocity, $\vec{v}$, and acceleration, $\vec{a}$, of a point traveling on this curve.

   (b) Sketch the curve $\vec{r}(t)$ and add to your sketch $\vec{v}(0)$ and $\vec{a}(0)$.

   (c) Give an expression describing the distance travelled by a point following the curve between time $t = 0$ and time $t = \frac{1}{3}$. Do not evaluate.

2. (24pts: 8,8,8) Consider the function $f(x, y) = 16x^2 + 16y^2$.

   (a) Sketch at least 2 traces (x = constant) and 2 level curves (z = constant) for this function.

   (b) Without calculating exact values, sketch the gradient of $f(x, y)$ at a point on one of your level curves.

   (c) Sketch or describe the surface $z = f(x, y)$.

3. (24pts: 4,10,10) Consider the plane Π given by $0 = x + y - 3z + 7$

   (a) Give a point P on this plane.

   (b) Give the equation of a line $\vec{r}(t)$ going through P and that is perpendicular to the plane.

   (c) Give a non-zero vector that is parallel to Π.

4. (28pts: 7,7,7,7)
   Answer the following questions in no more than two lines of text or computations.

   (a) Draw a vector $\vec{a}$ of length 1 and a vector $\vec{b}$ of length 3 in a different direction. Draw the vector $\vec{z} = \text{Proj}_{\vec{b}}\, \vec{a}$.

   (b) The function $f(x, y)$ is linearly approximated by $L(x, y)$ at the point (4,1). Give a constant approximation AND a linear approximation of $f(4.01, 0.9)$ in terms of $L(x, y)$.

   (c) Use the chain rule to find $\frac{\partial z}{\partial u}$ if $z = xe^y$, and $x = u^2 + v^2,\ y = 3u - 4v$.

   (d) The total number of views of a video is given by $n(t, d)$ as a function of the time t since it was posted and of the budget spent on the video d. The meaning of $\frac{\partial n}{\partial t}$ is:

   (A) The rate of change of the respect in the number of views.
   (B) The rate of change of the number of views with respect to time.
   (C) The rate of change of the number of views with respect to dollars spent.
   (D) The average number of views per days.
   (E) The average number of views per dollar spent.

CHEATSHEET

Dot product: $\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z = ||\vec{u}||\,||\vec{v}||\cos\theta_{uv}$

Cross product: $\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$ and $||\vec{u}\times\vec{v}|| = ||\vec{u}||\,||\vec{v}||\,|\sin\theta_{uv}|$

Line through P in direction $\vec{v}$: $\vec{x}(t) = P + \vec{v}\,t$

Plane through P with normal $\vec{n}$: $\vec{n}\cdot(\vec{x}-P) = 0$

Projection of $\vec{v}$ onto $\vec{w}$: $\text{Proj}_{\vec{w}}\,\vec{v} = \frac{\vec{v}\cdot\vec{w}}{\|\vec{w}\|}\frac{\vec{w}}{\|\vec{w}\|}$
General Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$

General Hyperboloid: $\pm\frac{x^2}{a^2}\pm\frac{y^2}{b^2}\mp\frac{z^2}{c^2} = \pm 1$ not all signs the same

General Paraboloid: $z = \pm(\frac{x^2}{a^2}+\frac{y^2}{b^2}) + h$

General Hyperbolic paraboloid: $z = \pm(\frac{x^2}{a^2}-\frac{y^2}{b^2}) + h$

Position: $\vec{x}(t)$, Velocity: $\frac{d\vec{x}}{dt}$, Acceleration: $\frac{d^2\vec{x}}{dt^2}$, speed: $\left\|\frac{d\vec{x}}{dt}\right\|$

Arclength: $s = \int_{t_i}^{t_f}\left\|\frac{d\vec{x}}{dt}\right\|dt$

When plotting $z = f(x,y)$:
$C = f(x,y)$ is a contour line in the domain,
$z = f(x,C)$ is a vertical trace in space

Partial derivative of $f(x,y)$: $\frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$

Plane tangent to $z = f(x,y)$ at (x_0,y_0,z_0): $\Pi(x,y) = z_0 + (x-x_0)\left(\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}\right) + (y-y_0)\left(\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}\right)$

Chain Rule for $f(x,y)$ with $x(s,t,u)$, and $y(s,t,u)$: $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$