Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work. The total number of points is 100.

1. (24pts: 8,8,8) The function $p(s,c)$ represents the price of a large pizza as a function of the price of tomato sauce per pound, s, and of the price of cheese per pound, c. At $s = 1$ and $c = 2.4$, the tangent plane to this function is $F(s,c) = 3s + c + 8$.
   (a) Explain in words the meaning of $p_s(1, 2.4)$.
   (b) Estimate the price of pizzas if the price of tomato sauce is $\$1.20$/pound and that of cheese is $\$2.30$ per pounds.
   (c) If the price of tomato sauce changes in time according to $s(t) = 1 + \sin t$ and that of cheese according to $c(t) = (t^2 + 4t + 4.8)/2$, give a formula for the time rate of change of the price of pizza and evaluate it at time $t = 0$.

2. (24pts: 8,8,8) Consider the line $\vec{r_1}(t) =< 3-2t, t, 1 >$ and the line $\vec{r_2}(t) =< 3+t, -t, 1+2t >$. These lines intersect at $P = (3, 0, 1)$.
   (a) Give an expression for the angle θ between those two lines.
   (b) Give the equation of the plane, Π, that contains those two lines.
   (c) Give a point that is at a distance of 4 from the plane Π (do not simplify numbers).

3. (24pts: 8,8,8) Consider the cycloid given by:
   $\vec{r}(t) =< 0, R(1 - \cos(\omega t)), R(\omega t - \sin \omega t) >$, where t stands for time.
   (a) Compute the velocity, $\vec{v}$, and acceleration, $\vec{a}$, of a point traveling on this curve.
   (b) Sketch the curve $\vec{r}(t)$ and add to your sketch $\vec{v}(0)$ and $\vec{a}(0)$.
   (c) Give an expression describing the distance travelled by a point following the curve between time $t = 0$ and time $t = 2\pi$. Do not evaluate.

4. (28pts: 7,7,7,7)
   Answer the following questions in no more than two lines of text or computations.
   (a) A vector $\vec{a}$ has length 4 and makes an angle of $\theta = \pi/6$ with a vector $\vec{b}$. Give an expression for the projection of $\vec{a}$ onto $\vec{b}$.
   (b) Sketch or describe a surface whose traces for $x = k$ or $y = k$ are parabolas opening downward, and whose contours are concentric circles.
   (c) Sketch the domain of $f(x,y) = \log(x^2 - y^2)$.
   (d) Give a direction in which the function $f(x,y) = x^3 - y^2$ remains constant at the point $(x_0, y_0) = (1, 2)$.

CHEATSHEET

Dot product: $\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z = ||\vec{u}||\,||\vec{v}||\cos\theta_{uv}$

Cross product: $\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$ and $||\vec{u}\times\vec{v}|| = ||\vec{u}||\,||\vec{v}||\,|\sin\theta_{uv}|$

Line through P in direction $\vec{v}$: $\vec{x}(t) = P + \vec{v}\,t$

Plane through P with normal $\vec{n}$: $\vec{n}\cdot(\vec{x}-P) = 0$

General Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

General Hyperboloid: $\pm\frac{x^2}{a^2} \pm \frac{y^2}{b^2} \pm \frac{z^2}{c^2} = \pm 1$ not all signs the same

General Paraboloid: $z = \pm(\frac{x^2}{a^2} + \frac{y^2}{b^2}) + h$

General Hyperbolic paraboloid: $z = \pm(\frac{x^2}{a^2} - \frac{y^2}{b^2}) + h$

Position: $\vec{x}(t)$, Velocity: $\frac{d\vec{x}}{dt}$, Acceleration: $\frac{d^2\vec{x}}{dt^2}$, speed: $\left|\left|\frac{d\vec{x}}{dt}\right|\right|$

Arclength: $s = \int_{t_i}^{t_f} \left|\left|\frac{d\vec{x}}{dt}\right|\right| dt$

When plotting $z = f(x,y)$:
$C = f(x,y)$ is a contour line in the domain,
$z = f(x,C)$ is a vertical trace in space

Partial derivative of $f(x,y)$: $\frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$

Plane tangent to $z = f(x,y)$ at (x_0, y_0, z_0):
$\Pi(x,y) = z_0 + (x-x_0)\left(\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}\right) + (y-y_0)\left(\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}\right)$

Chain Rule for $f(x,y)$ with $x(s,t,u)$, and $y(s,t,u)$: $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$

Gradient of $f(x,y)$: $\text{grad} f = \nabla f = <\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}>$