Dot product: $\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z = ||\vec{u}||\,||\vec{v}||\cos\theta_{uv}$

Cross product: $\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$ and $||\vec{u}\times\vec{v}|| = ||\vec{u}||\,||\vec{v}||\,|\sin\theta_{uv}|$

Line through P in direction $\vec{v}$: $\vec{x}(t) = P + \vec{v}\,t$

Plane through P with normal $\vec{n}$: $\vec{n}\cdot(\vec{x}-P) = 0$

Projection of $\vec{v}$ onto $\vec{w}$: $\text{Proj}_{\vec{w}}\,\vec{v} = \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||}\frac{\vec{w}}{||\vec{w}||}$

General Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

General Hyperboloid: $\pm\frac{x^2}{a^2} \pm \frac{y^2}{b^2} \pm \frac{z^2}{c^2} = \pm 1$ not all signs the same

General Paraboloid: $z = \pm(\frac{x^2}{a^2} + \frac{y^2}{b^2}) + h$

General Hyperbolic paraboloid: $z = \pm(\frac{x^2}{a^2} - \frac{y^2}{b^2}) + h$

Position: $\vec{x}(t)$, Velocity: $\frac{d\vec{x}}{dt}$, Acceleration: $\frac{d^2\vec{x}}{dt^2}$, speed: $\left|\left|\frac{d\vec{x}}{dt}\right|\right|$

Arclength: $s = \int_C ds = \int_{t_i}^{t_f}\left|\left|\frac{d\vec{x}}{dt}\right|\right| dt$

When plotting $z = f(x,y)$:

$C = f(x,y)$ is a contour line in the domain,

$z = f(x,C)$ is a vertical trace in space

Partial derivative of $f(x,y)$: $\frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$

Plane tangent to $z = f(x,y)$ at (x_0,y_0,z_0): $\Pi(x,y) = z_0 + (x-x_0)\left(\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}\right) + (y-y_0)\left(\frac{\partial f}{\partial y}\Big|_{(x_0,y_0)}\right)$

Chain Rule for $f(x,y)$ with $x(s,t,u)$, and $y(s,t,u)$: $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$

Gradient of $f(x,y)$: $\text{grad} f = \nabla f = <\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}>$

Min/Max of $f(x,y)$: $P = (x_0,y_0)$ is a critical point if $\frac{\partial f}{\partial x}(x_0,y_0) = 0$ and $\frac{\partial f}{\partial y}(x_0,y_0) = 0$

If, in addition, $D = f_{xx}(P)f_{yy}(P) - (f_{xy}(P))^2 < 0$, then P is a saddle-point

if $D > 0$ and $f_{xx}(P) > 0$, then P is a local minimum

if $D > 0$ and $f_{xx}(P) < 0$, then P is a local maximum

For a double integrals: $\int\int f(x,y)\mathrm{d}A$ the area element $\mathrm{d}A$ is

$\mathrm{d}A = \mathrm{d}x\mathrm{d}y$ in Cartesian coordinates

$\mathrm{d}A = r\,\mathrm{d}r\mathrm{d}\theta$ in polar coordinates

For a triple integrals: $\int\int\int f(x,y,z)\mathrm{d}V$ the volume element $\mathrm{d}V$ is

$\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z$ in Cartesian coordinates

$\mathrm{d}V = r\,\mathrm{d}r\mathrm{d}\theta\mathrm{d}z$ in cylindrical coordinates

$\mathrm{d}V = \rho^2\,\sin\phi\,\mathrm{d}\rho\mathrm{d}\phi\mathrm{d}\theta$ in spherical coordinates.

Coordinate conversion:

For polar and cylindrical: $x = r\cos\theta$, $y = r\sin\theta$

For spherical: $r = \rho\sin\phi$, $z = \rho\cos\phi$

Line Integral $\int_C f(\vec{x})\mathrm{d}s = \int_{t_i}^{t_f} f(\vec{x}(t))\left|\left|\frac{d\vec{x}}{dt}\right|\right|\mathrm{d}t$

Fundamental Theorem. of Line Integrals: if $\vec{F} = \nabla\phi$ and C goes from A to B, then

$\int_C \vec{F}(\vec{x})\cdot\frac{d\vec{x}}{dt}\mathrm{d}t = \phi(B) - \phi(A)$

Green's Theorem: $\int_C \vec{F}(\vec{x})\cdot\frac{d\vec{x}}{dt}\mathrm{d}t = \int\int_D\left(\frac{dF_2}{dx} - \frac{dF_1}{dy}\right)dA$

Curl $\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix}$

div $\vec{F} = \nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

Surface Integrals of a surface S parametrized with $\vec{r}(u,v)$: $\int\int_S f(\vec{x})\mathrm{d}S,$ with $\mathrm{d}S = ||\vec{r}_u \times \vec{r}_v||\mathrm{d}u\mathrm{d}v$

Flux of $\vec{F}$ across a surface S parametrized with $\vec{r}(u,v)$:

$$\text{Flux } = \int\int_S \vec{F} \cdot \hat{n}\, \mathrm{d}S = \int\int_S \vec{F} \cdot (\vec{r}_u \times \vec{r}_v)\mathrm{d}u\mathrm{d}v$$

Divergence theorem: Outward Flux $= \int\int_S \vec{F} \cdot \hat{n}\, \mathrm{d}S = \int\int\int_V \mathrm{div}\vec{F}\, \mathrm{d}V$