Quiz 2 - Solutions

Make sure your name is on your quiz, and please box your final answer. Because we will be giving partial credit, be sure to attempt all the problems, even if you don’t finish them!

1. A line of positive charge is formed into a semicircle of radius \( R \) as shown in the figure to the right. The charge per unit length along the semicircle is given by \( \lambda \), and is constant. The total charge on the semicircle is \( Q \).

(a) Determine the constant, \( \lambda \), in terms of the Coulomb constant \( k \), total charge \( Q \), and radius \( R \).

(b) What is the electric field, \( \vec{E} \) (magnitude and direction), at the origin (the center of curvature)?

Note: Recall that the arclength subtended by an angle \( \theta \) in radians along a circle of radius \( R \) is \( s = R\theta \). Furthermore, you might find the following integral useful:

\[
\int_{-\pi/2}^{\pi/2} \cos \theta \, d\theta = 2.
\]

Solution

(a) Because the linear charge density is constant, the charge per unit length is \( \lambda = Q/L \). Because the line is a half-circle, the total length is just half the overall circumference of a full circle of radius \( R \). Thus,

\[
\lambda = \frac{Q}{\pi R}.
\]

(b) From the diagram, and from the fact that the charge density is constant, and symmetric about \( \theta = 0 \), it’s clear that the net electric field should point down,
along the $-y$-axis. So, $\vec{E} = -E_y \hat{j}$. Now, considering a little charge $dq$ on the line at angle $\theta$, then the $y$-component of the electric field it creates at the origin is

$$dE_y = \frac{k \ dq}{R^2} \cos \theta.$$ 

Now, $dq = \lambda ds = \lambda R d\theta = \frac{Q}{\pi} d\theta$, such that

$$dE_y = \frac{kQ}{\pi R^2} \cos \theta d\theta.$$ 

To get the total electric field we just need to integrate over the angle,

$$E = \frac{kQ}{\pi R^2} \int_{-\pi/2}^{\pi/2} \cos \theta \ d\theta = \frac{2kQ}{\pi R^2},$$

which gives the total electric field as

$$\vec{E} = -\frac{2kQ}{\pi R^2} \hat{j}.$$