Physics 9 Fall 2009
Homework 5 - Solutions

The electric potential along the $x-$ axis is $V = 100x^2$ V, where $x$ is in meters. What is $E_x$ at (a) $x = 0$ m and (b) $x = 1.0$ m?

Solution

We know that $E_x = -\frac{\partial V}{\partial x}$. So, taking the derivative gives

$$E_x = -\frac{\partial}{\partial x} V = -\frac{\partial}{\partial x} (100x^2) = -200x.$$

Thus, $\vec{E} = -200x\hat{i}$ V/m.

(a) At $x = 1$, the electric field is $\vec{E} = -200 (0) \hat{i} = 0$ V/m.
(b) At $x = 2$, the electric field is $\vec{E} = -200 (1) \hat{i} = -200\hat{i}$ V/m.
2. Chapter 30 - Problem 24.
You need a capacitance of $50 \mu F$, but you don’t happen to have a $50 \mu F$ capacitor. You do have a $30 \mu F$ capacitor. What additional capacitor do you need to produce a total capacitance of $50 \mu F$? Should you join the two capacitors in parallel or series?

Solution

We want to connect the capacitors in parallel, since $C_{\text{net}} = C_1 + C_2$ in parallel, and we can get a larger net capacitance. If we connected the capacitors in series, then 

$$\frac{1}{C_{\text{net}}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{\text{net}} = \frac{C_1 C_2}{C_1 + C_2},$$

which is always less than or equal to $C_1$, since $\frac{C_2}{C_1 + C_2} \leq 1$, and so we wouldn’t get a larger capacitance.

So, we start with a $30 \mu F$ capacitor, and we need a $50 \mu F$ capacitance. This means that we just need to add a $20 \mu F$ capacitor in parallel.
Chapter 30 - Problem 39.
An infinitely long cylinder of radius $R$ has linear charge density $\lambda$. The potential on the surface of the cylinder is $V_0$, and the electric field outside the cylinder is $E_r = \lambda/2\pi\epsilon_0 r$. Find the potential relative to the surface at a point that is distance $r$ from the axis, assuming $r > R$.

Solution

For a given electric field, the change in potential is

$$\Delta V = -\int_{r_i}^{r_f} \vec{E} \cdot d\vec{s},$$

where $\Delta V = V(r_f) - V(r_i)$. With our electric field we have, after taking $r_f = r$, and $r_i = R$,

$$V(r) - V(R) = -\int_r^R \frac{\lambda}{2\pi\epsilon_0} \frac{dr}{r} = -\frac{\lambda}{2\pi\epsilon_0} \ln \left( \frac{r}{R} \right).$$

But, $V(R)$ is the potential when $r = R$; in other words, it is the potential on the surface of the sphere, which we’ve been told is $V_0$. So, we find that

$$V(r) = V_0 - \frac{\lambda}{2\pi\epsilon_0} \ln \left( \frac{r}{R} \right).$$
The electric potential in a region of space is \( V = \frac{200}{\sqrt{x^2 + y^2}} \) V, where \( x \) and \( y \) are in meters. What are the strength and direction of the electric field at \((x, y) = (2.0 m, 2.0 m)\)? Give the direction as an angle cw or ccw (specify which) from the positive \( x \)-axis.

Solution

For a potential of more than one variable, \( V(x, y) \), then

\[
\vec{E} = -\nabla V = - \left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} \right).
\]

Now, \( \frac{\partial V}{\partial x} = -\frac{200x}{(x^2 + y^2)^{3/2}} \), while \( \frac{\partial V}{\partial y} = -\frac{200y}{(x^2 + y^2)^{3/2}} \). Thus, the electric field is

\[
\vec{E}(x, y) = 200 \frac{(x\hat{i} + y\hat{j})}{(x^2 + y^2)^{3/2}}.
\]

Now, at \((x, y) = (2, 1)\), then

\[
\vec{E}(2, 1) = 200 \frac{(2\hat{i} + 1\hat{j})}{(2^2 + 1^2)^{3/2}} = 17.9 \left( 2\hat{i} + \hat{j} \right),
\]

which has a magnitude \( E = 17.9\sqrt{4+1} = 40 \) V/m. The angle, measured counterclockwise from the \( x \)-axis is \( \theta = \tan^{-1} \left( \frac{E_y}{E_x} \right) = \tan^{-1} \left( \frac{1}{2} \right) = 27^\circ. \)
5. Chapter 30 - Problem 84.

Consider a uniformly charged sphere of radius $R$ and total charge $Q$. The electric field $E_{\text{out}}$ outside the sphere ($r \geq R$) is simply that of a point charge $Q$. In Chapter 28, we used Gauss’s law to find that the electric field $E_{\text{in}}$ inside the sphere ($r \leq R$) is radially outward with field strength

$$E_{\text{in}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^3} r.$$ 

(a) Graph $E$ versus $r$ for $0 \leq r \leq 3R$.

(b) The electric potential $V_{\text{out}}$ outside the sphere is that of a point charge $Q$. Find an expression for the electric potential $V_{\text{in}}$ at position $r$ inside the sphere. As a reference, let $V_{\text{in}} = V_{\text{out}}$ at the surface of the sphere.

(c) What is the ratio $V_{\text{center}}/V_{\text{surface}}$?

(d) Graph $V$ versus $r$ for $0 \leq r \leq 3R$.

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Solution

(a) The graph is shown in the figure to the right. Inside the sphere the electric field grows proportional to the distance from the center, $E \propto r$. Outside the sphere, the electric field falls away with the usual inverse square law, $E \propto 1/r^2$. Note that even though the electric field is continuous, it has a kink in it.

(b) Since $E = -\frac{dV}{dr}$, this means that $V (r) = -\int \vec{E} \cdot d\vec{s}$. Integrating the electric field inside the sphere gives

$$V_{\text{in}} (r) = -\frac{1}{4\pi\varepsilon_0} \frac{Q}{R^3} \int r dr = -\frac{Q}{8\pi\varepsilon_0 R^3} r^2 + \text{const.},$$

where we have added an arbitrary integration constant, which we need to solve for. We know that the potential inside and outside the spheres have to match. So, when $r = R$, $V_{\text{in}} (R) = V_{\text{out}} (R)$, and then

$$V_{\text{in}} (R) = -\frac{Q}{8\pi\varepsilon_0 R^3} R^2 + \text{const.} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = V_{\text{out}} (R)$$

Comparing gives const. $= \frac{3Q}{8\pi\varepsilon_0 R}$. So, inside the sphere,

$$V_{\text{in}} (r) = \frac{Q}{8\pi\varepsilon_0 R} \left( 3 - \frac{r^2}{R^2} \right).$$
(c) The ratio of $V_{\text{center}}/V_{\text{edge}}$ is

$$\frac{V_{\text{center}}}{V_{\text{edge}}} = \frac{1}{4\pi \varepsilon_0} \frac{3Q}{2R} = \frac{3}{2}.$$ 

(d) The potential is graphed to the right. Inside the sphere the potential grows $V \propto r^2$. Outside the sphere, the potential falls away, $V \propto 1/r$. Notice here that the potential is continuous and has no kinks.
6. **Chapter 31 - Exercise 5.**

1.44 × 10^{14} electrons flow through a cross section of a 2.00 mm × 2.00 mm square wire in 3.0 µs. the electron drift speed is 2.00 × 10^{-4} m/s. What metal is the wire made of?

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**Solution**

We know the number of electrons passing through a unit area of a wire is given by

\[ n = \frac{N_e}{A v_d \Delta t}, \]

where \( N_e \) is the number of electrons, \( A \) is the area of the wire, \( v_d \) is the drift velocity of the electrons in the metal, and \( \Delta t \) is the amount of time it takes for the electrons to travel a given distance. Inserting the numbers gives,

\[ n = \frac{N_e}{A v_d \Delta t} = \frac{1.44 \times 10^{14}}{(2 \times 10^{-3})^2 (2.0 \times 10^{-4}) (3 \times 10^{-6})} = 6.0 \times 10^{28} \text{ electrons/m}^2. \]

Consulting Table 31.1 in the textbook tells us that this material is aluminum.
7. Chapter 31 - Exercise 39.
A circuit calls for a 0.50 mm-diameter copper wire to be stretched between two points. You don’t have any copper wire, but you do have aluminum wire in a wide variety of diameters. What diameter wire will provide the same resistance?

Solution

We know that the resistance, \( R \), of a material is related to its resistivity, \( \rho \), by \( R = \frac{\rho L}{A} \), where \( L \) is the length of a wire, and \( A \) is its area. If we want the resistances of a copper and aluminum wire to be the same, then

\[
\frac{\rho_{\text{Cu}} L_{\text{Cu}}}{A_{\text{Cu}}} = \frac{\rho_{\text{Al}} L_{\text{Al}}}{A_{\text{Al}}}
\]

Since we have a fixed distance between the two points, then \( L_{\text{Cu}} = L_{\text{Al}} \), and so

\[
A_{\text{Al}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} A_{\text{Cu}}.
\]

For a wire, \( A = \pi r^2 \), and so \( r_{\text{Al}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} r_{\text{Cu}}} \), or in terms of diameters,

\[
D_{\text{Al}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} D_{\text{Cu}}}.
\]

Looking up the values of \( \rho_{\text{Al}} \) and \( \rho_{\text{Cu}} \), and and noting that \( D_{\text{Cu}} \), we have

\[
D_{\text{Al}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} D_{\text{Cu}}} = \sqrt{\frac{2.8 \times 10^{-8}}{1.7 \times 10^{-8}}} (0.5) = 0.64 \text{ mm}.
\]
8. Chapter 31 - Problem 42.

The electron beam inside a television picture tube is 0.40 mm in diameter and carries a current of 50 \( \mu \)A. This electron beam impinges on the inside of the picture tube screen.

(a) How many electrons strike the screen each second?
(b) What is the current density in the electron beam?
(c) The electrons move with a velocity of \( 4.0 \times 10^7 \) m/s. What electric field strength is needed to accelerate electrons from rest to this velocity in a distance of 5.0 mm?
(d) Each electron transfers its kinetic energy to the picture tube screen upon impact. What is the power delivered to the screen by the electron beam?

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**Solution**

(a) The current is just the number of electrons per second, times the electron’s charge. So, \( N_e = \frac{I}{e} \Delta t \), where \( \Delta t \) is the amount of time the electrons hit. So, for one second, \( N_e = \frac{I}{e} (1) = \frac{50 \times 10^{-6}}{1.602 \times 10^{-19}} = 3.1 \times 10^{14} \) electrons per second.

(b) The current density is \( J = I/A \), where \( A \) is the area. Thus, \( J = \frac{I}{\pi r^2} = \frac{50 \times 10^{-6}}{\pi (0.2 \times 10^{-3})^2} = 400 \) A/m\(^2\).

(c) Recall that, for a constant acceleration, for a given distance, \( v_f^2 = v_i^2 + 2a\Delta x \). For us, \( v_i = 0 \), and so \( a = \frac{v_f^2}{2\Delta x} \), which gives a force \( F = ma = \frac{mv_f^2}{2\Delta x} \). But, for an electric field, \( F = qE \), and so \( E = \frac{mv_f^2}{2q\Delta x} \). Plugging in values gives

\[
E = \frac{mv_f^2}{2q\Delta x} = \frac{(9.11 \times 10^{-31})(4.0 \times 10^7)^2}{2(1.602 \times 10^{-19})(5 \times 10^{-3})} = 9.1 \times 10^5 \) V/m.

(d) The power just measures how fast energy changes, \( P = \frac{\Delta E}{\Delta t} \). Since the kinetic energy is constant, this is just the energy times the rate at which the electrons hit the screen, \( P = KE \frac{N_e}{\Delta t} \), which we found in part a. So,

\[
P = \frac{1}{2} mv_f^2 \frac{N_e}{\Delta t} = \frac{1}{2} \left(9.11 \times 10^{-31}\right) (4.0 \times 10^7)^2 (3.1 \times 10^{14}) = 0.23 \) W.
9. **Chapter 31 - Problem 50.**
You need to design a 1.0 A fuse that “blows” if the current exceeds 1.0 A. The fuse material in your stockroom melts at a current density of 500 A/cm². What diameter wire of this material will do the job?

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**Solution**

The current density is \( J = I/A \). For a wire \( A = \pi r^2 \), or since \( r = D/2 \), where \( D \) is the diameter of the wire, \( A = \pi D^2/4 \). So, plugging in for \( A \) and solving for \( D \) gives \( D = 2 \sqrt{\frac{I}{\pi J}} \). Now, with \( I = 1 \) A, and \( J = 500 \) A/cm², we find

\[
D = 2 \sqrt{\frac{1}{500\pi}} = 0.50 \text{ mm}.
\]

So, a wire with a diameter of 0.50 mm will work.
10. **Chapter 31 - Problem 72.**
A 5.0-mm-diameter proton beam carries a total current of 1.5 mA. The current density in the proton beam, which increases with distance from the center, is given by \( J = J_{\text{edge}} \left( \frac{r}{R} \right) \), where \( R \) is the radius of the beam and \( J_{\text{edge}} \) is the current density at the edge.

(a) How many protons per second are delivered by this proton beam?

(b) Determine the value of \( J_{\text{edge}} \).

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**Solution**

(a) The number of protons per second is just the total current, divided by the charge,
\[
N_p = \frac{I}{e} = \frac{1.5 \times 10^{-3}}{1.602 \times 10^{-19}} = 9.4 \times 10^{15} \text{ protons/second}
\]

(b) When the current density is not constant, then \( I = \int J(r) \, dA \). For a disk, \( dA = 2\pi r \, dr \), as we’ve seen before. So, \( I = 2\pi \int r J(r) \, dr \). Plugging in for the current density we find
\[
I = 2\pi \frac{J_{\text{edge}}}{R} \int_0^R r \, dr = \frac{2\pi}{3} J_{\text{edge}} R^2.
\]

Solving for \( J_{\text{edge}} \) gives
\[
J_{\text{edge}} = \frac{3}{2\pi R^2} I.
\]

Plugging in the numbers gives
\[
J_{\text{edge}} = \frac{3}{2\pi \left(2.5 \times 10^{-3}\right)^2} \left(1.5 \times 10^{-3}\right) = 115 \text{ A/m}^2.
\]