An electric dipole is formed from ±1.0 nC charges spread 2.0 mm apart. The dipole is at the origin, oriented along the $y$–axis. What is the electric field at the points (a) $(x, y) = (10 \text{ cm}, 0 \text{ cm})$ and (b) $(x, y) = (0 \text{ cm}, 10 \text{ cm})$?

---

**Solution**

(a) The dipole is oriented along the $y$–axis, and so the point $(x, y) = (10, 0)$ is *perpendicular* to the dipole, for which the electric field is given by

$$E_{\text{dip}} = -\frac{1}{4\pi\varepsilon_0} \frac{p}{r^3},$$

where $p = qs$ is the dipole moment. Plugging in the numbers gives

$$E_{\text{dip}} = -\frac{1}{4\pi\varepsilon_0} \frac{9 \times 10^{-9}}{0.002} \frac{10^{-9} \times 0.002}{0.13} = -18 \text{ N/C}.$$ 

(b) Now the point $(x, y) = (0, 10)$ is *along* the axis of the dipole, with an electric field

$$E_{\text{dip}} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}.$$ 

The only difference between this expression and our answer to part (a) is factor of $-2$. Thus, we can immediately write down the electric field as $E_{\text{dip}} = 36 \text{ N/C}$.  

---

1
2. **Chapter 27 - Exercise 7.**  
The electric field strength 5.0 cm from a very long charged wire is 2000 N/C. What is the electric field strength 10.0 cm from the wire?

---

**Solution**

Recall that the electric field of a very long wire is

\[ E = \frac{\lambda}{2\pi \epsilon_0 r}, \]

which falls off *linearly* with distance. This means that if we go twice as far away from the wire, then the field is *half* as strong. See that, as \( r \to 2r \), and if the field starts at \( E_0 = \lambda / (2\pi \epsilon_0) \), then

\[ E \to \frac{\lambda}{2\pi \epsilon_0 (2r)} = \frac{1}{2} \frac{\lambda}{2\pi \epsilon_0 r} = \frac{1}{2} E_0. \]

So, if the electric field is 2000 N/C at 5.0 cm, then at 10 cm, then the electric field must be \( 2000/2 = 1000 \) N/C. Thus, \( E(10 \text{ cm}) = 1000 \) N/C.
3. **Chapter 27 - Exercise 20.**
A 0.10 g plastic bead is charged by the addition of $1.0 \times 10^{10}$ excess electrons. What electric field $\vec{E}$ (strength and direction) will cause the bead to hang suspended in the air?

---

**Solution**

The bead carries a charge $Q = 10^{10} \times (-1.602 \times 10^{-19}) = -1.602 \times 10^{-9}$ C, due to the excess electrons. We want to support this bead in space, against the pull of gravity. Gravity pulls the bead down, and so the electric force must push the bead up. Since the bead is negatively charged, in order for the force to point up, the electric field must point *down*. So, to balance the forces we need $Q E = mg$, and so $E = \frac{mg}{Q}$. Plugging in the values gives

$$E = \frac{mg}{Q} = \frac{10^{-3} \times 9.8}{-1.602 \times 10^{-9}} = -6.1 \times 10^{5} \text{ N/C}.$$  

Expressed as a vector, $\vec{E} = -6.1 \times 10^{5} \hat{j} \text{ N/C}$.  

---

3
4. Chapter 27 - Problem 34.
Derive Equation 27.12 for the field \( \vec{E}_{dipole} \) in the plane that bisects an electric dipole.

Solution

Equation 27.12 in the text gives the result as

\[
\vec{E}_{dip} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3},
\]

where \( r \) is the distance from the center of the dipole. Let’s start by looking at the picture to the right. The electric field from the positive charge points down and to the right, while that from the negative charge points down and to the left. The net field points straight down.

The magnitude of the net field is \( E = -2E_{pos}\sin\theta \), where \( E_{pos} \) is the field from a single charge, and the factor of 2 comes from the contributions of the two charges. The sine factor comes from the vertical components of the fields, while the horizontal components cancel. Now,

\[
E_{pos} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + (s/2)^2},
\]

while \( \sin\theta = \frac{s/2}{\sqrt{r^2 + (s/2)^2}} \). So, adding in the unit vector, \( \hat{j} \), we have

\[
\vec{E} = -2 \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + (s/2)^2} \frac{s/2}{\sqrt{r^2 + (s/2)^2}} \hat{j} = -\frac{1}{4\pi\epsilon_0} \frac{q s \hat{j}}{(r^2 + (s/2)^2)^{3/2}}.
\]

But, the dipole moment \( \vec{p} = qs\hat{j} \). Finally, for distances \( r \gg s/2 \), then \( r^2 + (s/2)^2 \approx r^2 \).

So, we find

\[
\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3},
\]

which is precisely Eq. 27.12.
5. Chapter 27 - Problem 45.

(a) Show that the maximum electric field strength on the axis of a ring of charge occurs at \( z = R/\sqrt{2} \).

(b) What is the electric field strength at this point?

---

### Solution

(a) The electric field along the axis of a ring of charge \( Q \) is given by \( E(z) = \frac{1}{4\pi \epsilon_0} \frac{z^2 Q}{(z^2 + R^2)^{3/2}} \), where \( z \) is the distance along the axis, and \( R \) is the radius of the ring. So, the field depends on \( z \). It has a maximum when \( \frac{dE}{dz} = 0 \). Taking the derivative gives

\[
\frac{dE}{dz} = \frac{Q}{4\pi \epsilon_0} \frac{d}{dz} \left( \frac{z}{(z^2 + R^2)^{3/2}} \right) = \frac{Q}{4\pi \epsilon_0} \left( \frac{1}{(z^2 + R^2)^{3/2}} - \frac{3}{2} \frac{2z^2}{(z^2 + R^2)^{5/2}} \right) = 0
\]

This is zero when the term inside the parenthesis vanishes. This gives \( z^2 + R^2 = 3z^2 \), or \( z = \frac{R}{\sqrt{2}} \), as claimed.

(b) When \( z = \frac{R}{\sqrt{2}} \), then

\[
E \left( \frac{R}{\sqrt{2}} \right) = \frac{1}{4\pi \epsilon_0} \frac{(R/\sqrt{2}) Q}{\left( (R/\sqrt{2})^2 + R^2 \right)^{3/2}} = \frac{1}{4\pi \epsilon_0} \frac{RQ}{\sqrt{2} \left( 3R^2/2 \right)^{3/2}} = \frac{2}{3\sqrt{3}} \frac{1}{4\pi \epsilon_0} \frac{Q}{R^2}
\]

Thus, the maximum value of the electric field is \( E = \frac{2}{3\sqrt{3}} \frac{1}{4\pi \epsilon_0} \frac{Q}{R^2} \).
6. **Chapter 27 - Problem 54.**
A problem of practical interest is to make a beam of electrons turn a 90° corner. This can be done with the parallel-plate capacitor shown in the figure. An electron with kinetic energy $3.0 \times 10^{-17}$ J enters through a small hole in the bottom plate of the capacitor.

(a) Should the bottom plate be charged positive or negative relative to the top plate if you want the electron to turn to the right? Explain.

(b) What strength electric field is needed if the electron is to emerge from an exit hole 1.0 cm away from the entrance hole, traveling at right angles to its original direction? **Hint:** The difficulty of this problem depends on how you choose your coordinate system.

(c) What minimum separation $d_{\text{min}}$ must the capacitor plates have?

---

**Solution**

Because the field between the plates is constant, this problem is *exactly* like a gravitational projectile problem, only with a different acceleration! So, for a given acceleration, we want to figure out what the electric field needs to be to give the electron a range of 1 cm. We can see the angles from the diagram to the right, orienting our coordinates as shown.

(a) We want the bottom plate to *attract* the electron, and so it should be *positively* charged.

(b) Recall the projectile equations

$$
\begin{align*}
x(t) &= x_0 + v_{0x}t + \frac{1}{2}a_xt^2 = v_{0x}t \\
y(t) &= y_0 + v_{0y}t + \frac{1}{2}a_yt^2 = v_{0y}t - \frac{1}{2}a_yt^2,
\end{align*}
$$

where we have set $x_0 = y_0 = 0$, and note that $a_x = 0$. Solving the first equation for time gives $t = x/v_{0x}$, which, upon substituting into the second equation gives
\[
y(x) = \frac{v_{0y}}{v_{0x}} x - \frac{a_y}{2v_{0x}^2} x^2.
\]
Now, when \( x \) hits its maximum range, \( R \), then \( y = 0 \). So,

\[
y(R) = 0 = \frac{v_{0y}}{v_{0x}} R - \frac{a_y}{2v_{0x}^2} R^2 \Rightarrow a_y = \frac{2v_{0x}v_{0y}}{R}.
\]

This is our acceleration. The electric field needed to give this acceleration is

\[
E = \frac{F}{q} = \frac{ma_y}{q} = \frac{2mv_{0x}v_{0y}}{qR}.
\]

Finally, note that \( v_{0x} = v_0 \cos \theta \), while \( v_{0y} = v_0 \sin \theta \), and so \( 2v_{0x}v_{0y} = 2v_0^2 \sin \theta \cos \theta = v_0^2 \sin 2\theta \). So, \( E = \frac{mv_0^2}{qR} \sin 2\theta \). Finally, recall that the kinetic energy of the electron is \( KE = \frac{1}{2}mv_0^2 \), and so

\[
E = \frac{2(KE)}{qR} \sin 2\theta.
\]

Now, we plug in the numbers. The launch angle is \( \theta = 45^\circ \), and so

\[
E = \frac{2(KE)}{qR} \sin 2\theta = \frac{2 \times 3.0 \times 10^7}{1.602 \times 10^{-19} \times 0.01} \sin 90^\circ \approx 37,500 \text{ N/C}.
\]

(c) Now, we want the minimum separation. If the plates are too close, then the electron will collide with the top plate, and won’t continue on its full parabolic arc. The electron reaches its maximum height when \( x = R/2 \), i.e., at the halfway point. So, when \( x = R/2 \), then

\[
y(R/2) = \frac{v_{0y}}{v_{0x}} \frac{R}{2} - \frac{a_y}{2v_{0x}^2} \left( \frac{R}{2} \right)^2
\]

\[
= \frac{v_{0y}}{v_{0x}} \frac{R}{2} - \frac{1}{2v_{0x}^2} \left( \frac{2v_{0x}v_{0y}}{R} \right) \left( \frac{R}{2} \right)^2
\]

\[
= \frac{R}{4} \tan \theta.
\]

This is the minimum height. Now, \( \theta = 45^\circ \), and \( \tan \theta = 1 \), so \( h_{\text{min}} = R/4 \). Since \( R = 1 \text{ cm} \), then \( h_{\text{min}} = 2.5 \text{ mm} \).
7. Chapter 27 - Problem 58.
In a classical model of the hydrogen atom, the electron orbits the proton in a circular orbit of radius 0.053 nm. What is the orbital frequency? The proton is so much more massive than the electron that you can assume the proton is at rest.

Solution

The electric force holds the electron to the proton. Since it’s moving in a circle, there is also a centripetal force. The two forces are equal in magnitude, keeping the circular orbit stable. So, \( F_{\text{elec}} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r} = F_{\text{cent}} \). Now, we can write \( v = r\omega \), where \( \omega \) is the angular frequency. The ordinary frequency is \( f = \omega/2\pi \). Thus, we can solve for the frequency to find

\[
\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \frac{mv^2}{r} = \frac{m}{r} (r\omega)^2 = mr\omega^2.
\]

Thus, \( \omega^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{mr^3} \), or since \( \omega = 2\pi f \),

\[
f = \frac{1}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mr^3}}.
\]

Plugging in the numbers gives

\[
f = \frac{1}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mr^3}} = \frac{1}{2\pi} \sqrt{(9 \times 10^9) \frac{(1.602 \times 10^{-19})^2}{9.11 \times 10^{-31} (0.053 \times 10^{-9})}} = 6.65 \times 10^{15} \text{ rev/sec}.
\]
8. Chapter 27 - Problem 61.
Show that an infinite line of charge with linear charge density \( \lambda \) exerts an attractive force on an electric dipole with magnitude \( F = \frac{2\lambda p}{4\pi \epsilon_0 r^2} \). Assume that \( r \) is much larger than the charge separation in the dipole.

Solution

Suppose that we hold the dipole a distance \( r \) away, oriented such that the negative side of the dipole points towards the line of charge, while the positive charge points away. The electric field of an infinite line charge is given by \( E = \frac{\lambda}{2\pi \epsilon_0 d} \), where \( d \) is the distance away.

Now, if the distance from the line to the center of the dipole is \( r \), then the distance to the negative charge is \( r - s/2 \), while it’s \( r + s/2 \) to the positive charge. So, the net force is \( F_{\text{net}} = -qE_{\text{neg}} + qE_{\text{pos}} \), or

\[
F = -q \frac{\lambda}{2\pi \epsilon_0 (r-s/2)} + q \frac{\lambda}{2\pi \epsilon_0 (r+s/2)}
= -q \frac{\lambda}{2\pi \epsilon_0} \left( \frac{1}{r-s/2} - \frac{1}{r+s/2} \right)
= -q \frac{\lambda}{2\pi \epsilon_0} \left( \frac{r+s/2-r+s/2}{(r-s/2)(r+s/2)} \right)
= -q \frac{\lambda}{2\pi \epsilon_0} \frac{1}{r^2-(s/2)^2}.
\]

Now, the dipole moment is defined as \( p = qs \). Furthermore, for \( r \gg s/2 \), then \( r^2 - (s/2)^2 \approx r^2 \), and so we find

\[
F \approx -\frac{\lambda p}{2\pi \epsilon_0 r^2},
\]

which is precisely what we wanted, recalling that the negative sign means that the force is attractive.
A positron is an elementary particle identical to an electron except that its charge is $+e$. An electron and a positron can rotate about their center of mass as if they were a dumbbell connected by a massless rod. What is the orbital frequency for an electron and a positron 1.0 nm apart?

**Solution**

This problem is very similar to the hydrogen problem that we did earlier, and we can solve it in the same way. However, this time the electron doesn’t go around the positron - both go around each other! So, the distance between the two particles is twice the orbital radius. So, the force between the charges is

$$F_E = \frac{1}{4\pi\epsilon_0} \frac{e^2}{(2r)^2},$$

and since they are going around in a circle,

$$F_{\text{cent}} = \frac{mv^2}{r}.$$

Again, replacing $v = r\omega = 2\pi rf$, we find that

$$F_{\text{cent}} = F_E \Rightarrow \frac{mv^2}{r} = mr(2\pi f)^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{4r^2},$$

which gives

$$f = \frac{1}{2\pi} \sqrt{\frac{e^2}{16\pi\epsilon_0 mr^3}}.$$

Now, $r = 0.5$ nm is half the distance between the positron and electron. So, we find

$$f = \frac{1}{2\pi} \sqrt{\frac{(9 \times 10^9) \times (1.602 \times 10^{-19})^2}{4 (9.11 \times 10^{-31}) (.5 \times 10^{-9})^3}} = 1.13 \times 10^{14} \text{ Hz.}$$
10. **Chapter 27 - Problem 74.**

You have a summer intern position with a company that designs and builds nanomachines. An engineer with the company is designing a microscopic oscillator to help keep time, and you’ve been assigned to help him analyze the design. He wants to place a negative charge at the center of a very small, positively charged metal loop. His claim is that the negative charge will undergo simple harmonic motion at a frequency determined by the amount of charge on the loop.

(a) Consider a negative charge near the center of a positively charged ring. Show that there is a restoring force on the charge if it moves along the \( z \)-axis but stays close to the center. That is, show there’s a force that tries to keep the charge at \( z = 0 \).

(b) Show that for small oscillations, with amplitude \( \ll R \), a particle of mass \( m \) with charge \(-q\) undergoes simple harmonic motion with frequency

\[
f = \frac{1}{2\pi} \sqrt{\frac{qQ}{4\pi \varepsilon_0 m R^3}}.
\]

\( R \) and \( Q \) are the radius and charge of the ring.

(c) Evaluate the oscillation frequency for an electron at the center of a 2.0 \( \mu \)m-diameter ring charged to \( 1.0 \times 10^{-13} \) C.

---

**Solution**

(a) Recall that, along the axis the electric field of a ring of charge is

\[
E_z = \frac{1}{4\pi \varepsilon_0} \frac{Qz}{(z^2 + R^2)^{3/2}}.
\]

The force on a negative charge is \( F = qE \), or

\[
F = -\frac{1}{4\pi \varepsilon_0} \frac{qQz}{(z^2 + R^2)^{3/2}}.
\]

When \( z < 0 \) (i.e., when the charge is to the right of the ring), the force is to the left. When \( z < 0 \), when the charge is on the left, then the force is positive, and to the right. So, there is always a restoring force.

(b) Suppose that we always stay close to the ring, such that \( z \ll R \). In this case, \( z^2 + R^2 \approx R^2 \), and so the force becomes

\[
F = -\frac{1}{4\pi \varepsilon_0} \frac{qQz}{(z^2 + R^2)^{3/2}} \approx -\left( \frac{qQ}{4\pi \varepsilon_0 R^3} \right) z.
\]

So, we have a force of the form \( F = -kz \), where \( k \) is the term in parenthesis. So, the force is proportional to the distance. This is just Hooke’s law, and so the system behaves like a mass on a spring!
The natural frequency of a spring with constant \( k \) is \( f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \), and so we find

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{qQ}{4\pi\varepsilon_0 m R^3}}.
\]

(c) Now, we have an electron of mass \( m_e = 9.11 \times 10^{-31} \) kg, and charge \( q = e = 1.602 \times 10^{-19} \) C, at the center of a ring of radius \( R = 10^{-6} \) m, carrying a charge \( Q = 10^{-13} \) C. With these numbers,

\[
f = \frac{1}{2\pi} \sqrt{\frac{qQ}{4\pi\varepsilon_0 m R^3}} = \frac{1}{2\pi} \sqrt{(9 \times 10^9) \frac{1.602 \times 10^{-31} \times 10^{-13}}{9.11 \times 10^{-31} \times 10^{-6}}} = 2.0 \times 10^{12} \text{ Hz}.
\]