1. **Chapter 22 - Problem 37.**
   A diffraction grating having 500 lines/mm diffracts visible light at 30°. What is the light’s wavelength?

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**Solution**

Here we use the diffraction equation that says $d \sin \theta_m = m \lambda$, where $m$ is the diffraction order. Now, solving for wavelength gives

$$
\lambda = \frac{d \sin \theta_m}{m}.
$$

The spacing is $d = 1/500$ lines/mm, or $d = 2 \times 10^{-6}$ m. So, $\lambda = \frac{2 \times 10^{-6} \sin 30^\circ}{m} = 10^4$ nm. Now, we know that *visible* light has a range of wavelengths 400 - 700 nm, and so only $m = 2$ works. Thus, $\lambda = 500$ nm.
2. Chapter 22 - Problem 40.
Light emitted by Element X passes through a diffraction grating having 1200 lines/mm. The diffraction pattern is observed on a screen 75.0 cm behind the grating. Bright fringes are seen on the screen at distances of 56.2 cm, 65.9 cm, and 93.5 cm from the central maximum. No other fringes are seen.

(a) What is the value of \( m \) for each of these diffracted wavelengths? Explain why only one value is possible.

(b) What are the wavelengths of light emitted by Element X?

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Solution

(a) Because we are told that we see the fringes, this means that the fringes are in the visible range of wavelengths, from 400 - 700 nm. Once again, \( d \sin \theta_m = m \lambda \), where \( d = 1/1200 = 8.3 \times 10^{-7} \text{m} \) or 830 nm. Thus, \( \sin \theta_m = \frac{m \lambda}{d} = \frac{m \lambda}{830} \).

Now, since \( 400 \leq \lambda \leq 700 \) nm or so, and since \( \sin \theta_m \leq 1 \), only \( m = 1 \) works for most of the wavelengths. However, \( m = 2 \) does work for wavelengths in the range 400 - 415 nm, but these wavelengths are pushed out to angles approaching 90°, and so don’t fall in the range of fringes seen above. So, \( m = 1 \).

(b) The positions of the fringes are given by \( y = L \tan \theta \). But, \( \sin \theta = \frac{m \lambda}{d} \Rightarrow \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) \). This means that \( y = L \tan \left[ \sin^{-1} \left( \frac{m \lambda}{d} \right) \right] \). Now, \( \tan \left( \sin^{-1} \theta \right) = \frac{\theta}{\sqrt{1-\theta^2}} \), and so \( y = \frac{L \left( \frac{m \lambda}{d} \right)}{\sqrt{1-\left( \frac{m \lambda}{d} \right)^2}} \). Solving for \( \lambda \) gives, after some algebra

\[
\lambda = \frac{d \ y}{m \sqrt{y^2 + L^2}} = \frac{830 \ y}{\sqrt{y^2 + L^2}},
\]

after plugging in \( d = 830 \text{ nm} \), and \( m = 1 \).

Let’s make a table of the wavelengths, taking \( L = 75 \text{ cm} \).

<table>
<thead>
<tr>
<th>( y ) (cm)</th>
<th>( \lambda ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>56.2</td>
<td>500</td>
</tr>
<tr>
<td>65.9</td>
<td>550</td>
</tr>
<tr>
<td>93.5</td>
<td>600</td>
</tr>
</tbody>
</table>
3. Chapter 22 - Problem 48.

The wings of some beetles have closely spaced parallel lines of melanin, causing the wing to act as a reflection grating. Suppose sunlight shines straight onto a beetle wing. If the melanin lines on the wing are spaced 2.0 \( \mu \text{m} \) apart, what is the first-order diffraction angle for green light \( (\lambda = 550 \text{ nm}) \)?

**Solution**

Yet again, we take \( d \sin \theta = m\lambda \). Taking \( m = 1 \) for the first-order gives

\[
\theta = \sin^{-1} \left( \frac{m\lambda}{d} \right) = \sin^{-1} \left( \frac{550}{2 \times 10^3} \right) = 16^\circ.
\]

So, the first-order diffraction angle is 16°.
Scientists use laser range-finding to measure the distance to the moon with great accuracy. A brief laser pulse is fired at the moon, then the time interval is measured until the “echo” is seen by a telescope. A laser beam spreads out as it travels because it diffracts through a circular exit as it leaves the laser. In order for the reflected light to be bright enough to detect, the laser spot on the moon must be no more than 1.0 km in diameter. Staying within this diameter is accomplished by using a special large-diameter laser. If $\lambda = 532 \text{ nm}$, what is the minimum diameter of the circular opening from which the laser beam emerges? The earth-moon distance is 384,000 km.

**Solution**

Since the aperture is circular, the beam spreads out to a width $w = \frac{2.44\lambda}{D}L$, where $\lambda$ is the wavelength, $D$ is the diameter of the aperture, and $L$ is the distance away. Solving for $D$ gives simply,

$$D = \frac{2.44\lambda}{w}L.$$

Plugging in for the variables gives

$$D = \frac{2.44\lambda}{w}L = \frac{2.44 \times 532 \times 10^{-9} \times 384 \times 10^6}{10^3} = 0.498 \text{ m}.$$

So, the diameter of the aperture needs to be $D \geq 50 \text{ cm}$. 

4. Chapter 22 - Problem 60.
5. Chapter 22 - Problem 71.
Light consisting of two nearly equal wavelengths $\lambda + \Delta \lambda$ and $\lambda$, where $\Delta \lambda \ll \lambda$, is incident on a diffraction grating. The slit separation of the grating is $d$.

(a) Show that the angular separation of these two wavelengths in the $m$th order is

$$\Delta \theta = \frac{\Delta \lambda}{\sqrt{(d/m)^2 - \lambda^2}}.$$ 

(b) Sodium atoms emit light at 589.0 nm and 589.6 nm. What are the first-order and second-order angular separations (in degrees) of these two wavelengths for a 600 line/mm grating?

Solution

(a) We begin with the diffraction equation, solving for $\sin \theta = \frac{m \lambda}{d}$. Now, taking the derivatives with respect to $\lambda$ gives

$$\cos \theta \frac{d\theta}{d\lambda} = \frac{m}{d} \Rightarrow \frac{d\theta}{d\lambda} = \frac{1}{\cos \theta} \frac{m}{d}.$$ 

Now, $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - (m \lambda / d)^2}$, from the diffraction equation. Thus, $\frac{d\theta}{d\lambda} = \frac{1}{d} \frac{1}{\sqrt{1 - (m \lambda / d)^2}} = \frac{1}{d} \frac{1}{\sqrt{(d/m)^2 - \lambda^2}}$. Now, letting the infinitesimal differentials become finite changes, $d\theta \rightarrow \Delta \theta$, and $d\lambda \rightarrow \Delta \lambda$, we find, after separating,

$$\Delta \theta = \frac{\Delta \lambda}{\sqrt{(d/m)^2 - \lambda^2}}.$$ 

(b) Here, $d = 1/600 = 1.67 \times 10^{-6}$ m or 1670 nm, while $\Delta \lambda = 589.6 - 589.0 = 0.6$ nm. So, for $m = 1$,

$$\Delta \theta = \frac{\Delta \lambda}{\sqrt{(d/m)^2 - \lambda^2}} = \frac{0.6}{\sqrt{(1670/1)^2 - 589^2}} = 0.022^\circ,$$

while for $m = 2$,

$$\Delta \theta = \frac{\Delta \lambda}{\sqrt{(d/m)^2 - \lambda^2}} = \frac{0.6}{\sqrt{(1670/2)^2 - 589^2}} = 0.058^\circ.$$
6. **Chapter 23 - Problem 51.**

It’s nighttime, and you’ve dropped your goggles into a 3.0-m-deep swimming pool. If you hold a laser pointer 1.0 m above the edge of the pool, you can illuminate the goggles if the laser beam enters the water 2.0 m from the edge. How far are the goggles from the edge of the pool?

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**Solution**

The light bends, of course, because of Snell’s law, \( n_{\text{air}} \sin \theta_{\text{air}} = n_{\text{water}} \sin \theta_{\text{water}} \). We can determine the angle of incidence, \( \theta_{\text{air}} \), by finding \( \phi \) in the diagram to the right. From the geometry, we see that \( \phi = \tan^{-1} \frac{1}{2} = 26.57^\circ \), which means that \( \theta_{\text{air}} = 90^\circ - \phi = 63.43^\circ \).

Now, using Snell’s law, we solve for the angle in water, remembering that \( n_{\text{air}} = 1 \), and \( n_{\text{water}} = 4/3 \),

\[
\theta_{\text{water}} = \sin^{-1} \left( \frac{\sin 63.43^\circ}{4/3} \right) = 42.12^\circ.
\]

Now, we’re almost done. We just need to figure out the distance \( x \). From the geometry, we can write \( \tan \theta_{\text{water}} = \frac{x}{3.0} \Rightarrow x = 3 \tan \theta_{\text{water}} = 2.71 \text{ m} \). So, the total distance that the goggles are from the edge of the pool is \( D = x + 2 = 4.71 \text{ m} \).
7. Chapter 23 - Problem 58.
There’s one angle of incidence $\beta$ onto a prism for which the light inside an isosceles prism travels parallel to the base and emerges at an angle $\beta$.

(a) Find an expression for $\beta$ in terms of the prism’s apex angle $\alpha$ and index of refraction $n$.

(b) A laboratory measurement finds that $\beta = 52.2^\circ$ for a prism shaped like an equilateral triangle. What is the prism’s index of refraction?

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**Solution**

(a) Snell’s law says that, taking $n_{\text{air}} = 1$,

$$\sin \beta = n \sin \phi,$$

where $\phi$ is the angle in the prism, the angle between the light path and the normal to the prism, denoted by the dotted line. Since the light path is parallel to the base, if we call $\theta$ the angle of the two isosceles angles, then the light path makes the same angle $\theta$ with respect to the wall of the prism. This means that $\theta + \phi = 90^\circ$. Furthermore, since the triangle is isosceles, $2\theta + \alpha = 180^\circ$. Plugging in $\theta = 90^\circ - \phi$ gives $\phi = \frac{\alpha}{2}$. Furthermore, from Snell’s law, we find that $\beta = \sin^{-1} \left( n \sin \frac{\alpha}{2} \right)$.

(b) If the triangle is an equilateral triangle, then all the angles are $60^\circ$. Taking $\beta = 52.2^\circ$, and solving for $n$ gives

$$n = \frac{\sin \beta}{\sin (\alpha/2)} = \frac{\sin 52.2^\circ}{\sin 30^\circ} = 1.58,$$

which sounds about right for glass or plastic.
Chapter 23 - Problem 60.
Parallel light rays enter a transparent sphere along a line passing through the center of the sphere. The rays come to a focus on the far surface of the sphere. What is the sphere’s index of refraction?

Solution

The ball is a spherical refracting surface, as seen in the figure. Because the rays start off parallel, the object distance \( s = \infty \). The rays converge to the back surface of the sphere, and so \( s' = 2R \), where \( R \) is the radius of the sphere. Now, recall that for image formation due to refraction at a spherical surface,

\[
\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R},
\]

where \( n_1 \) is the index of refraction outside the sphere, and \( n_2 \) is that inside the sphere.

Taking \( s = \infty \), and \( s = 2R \), then \( \frac{n_2}{2R} = \frac{n_1 - n_2}{R} \), which, upon solving for \( n_2 \) gives \( n_2 = 2n_1 \), independent of \( R \)!

So, if the outside environment is air, with \( n_1 = 1.00 \), then \( n_2 = 2.00 \).

(a) Estimate the diameter of your eyeball.

(b) Bring this page up to the closest distance at which the text is sharp – not the closest at which you can still read it, but the closest at which the letters remain sharp. If you wear glasses or contact lenses, leave them on. This distance is called the near point of your (possibly corrected) eye. Measure it.

(c) Estimate the effective focal length of you eye. The effective focal length includes the focusing due to the lens, the curvature of the cornea, and any corrections you wear. Ignore the effects of the fluid in your eye.

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Solution

Here are the values that I estimated, actual milage may vary:

(a) I estimate that the diameter of my eyeball is about 4.5 cm.

(b) I estimate the neat point to be about 10 cm.

(c) The focal length is given by the the thin-lens formula, \( \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \), which, upon solving for \( f \) gives

\[
 f = \frac{ss'}{s + s'}.
\]

Since the image forms on the retina at the back of the eyeball, \( s' = 4.5 \) cm, and the object is \( s = 10 \) cm, then

\[
 f = \frac{ss'}{s + s'} = \frac{45}{14.5} \approx 3.1 \text{ cm}.
\]

So, a very rough estimation of the focal length of my eyeball gives about 3.1 cm.
The figure shows a light ray that travels from point A to point B. The ray crosses the boundary at position $x$, making angles $\theta_1$ and $\theta_2$ in the two media. Suppose that you did not know Snell’s law.

(a) Write an expression for the time $t$ it takes the light ray to travel from A to B. Your expression should be in terms of the distances $a$, $b$, and $w$; the variable $x$; and the indices of refraction $n_1$ and $n_2$.

(b) The time depends on $x$. There’s one value of $x$ for which the light travels from A to B in the shortest possible time. We’ll call it $x_{\text{min}}$. Write an expression (but don’t try to solve it!) from which $x_{\text{min}}$ could be found.

(c) Now, by using the geometry of the figure, derive Snell’s law from your answers to part (b).

You’ve proven that Snell’s law is equivalent to the statement that “light traveling between two points follows the path that requires the shortest time.” This interesting way of thinking about refraction is called Fermat’s principle.

**Solution**

(a) The time to travel from A to B is just the time required to go from A to the boundary, plus the time required to go from the boundary to B, $t = t_1 + t_2$. Since the light travels at speed $v_1 = c/n_1$, and travels a distance $d_1$, then $t_1 = d_1/v_1 = n_1d_1/c$. Similarly, $t_2 = n_2d_2/c$. So, the net time is $t = n_1d_1/c + n_2d_2/c$. Or, since $d_1 = \sqrt{a^2 + x^2}$, and $d_2 = \sqrt{b^2 + (w - x)^2}$, then

$$t = \frac{n_1}{c} \sqrt{a^2 + x^2} + \frac{n_2}{c} \sqrt{b^2 + (w - x)^2}.$$

(b) There is one value of $x$ for which the time is minimized, and that occurs when the derivative $\frac{dt}{dx} = 0$. So, taking the derivative and setting the result equal to zero gives the expression

$$\frac{n_1x}{\sqrt{a^2 + x^2}} = \frac{n_2(w - x)}{\sqrt{b^2 + (w - x)^2}}.$$
You could try to solve this expression, but you would in general get a fourth-order polynomial, which is very difficult to solve. Instead, we’ll leave the expression and move on to the next part.

(c) From the geometry of the figure, we can see that \( \sin \theta_1 = \frac{x}{d_1} = \frac{x}{\sqrt{x^2 + a^2}} \), while \( \sin \theta_2 = \frac{w-x}{d_2} = \frac{(w-x)}{\sqrt{(w-x)^2 + b^2}} \). So, replacing these values in our expression above we find

\[
\frac{n_1 x}{\sqrt{a^2 + x^2}} = \frac{n_2 (w-x)}{\sqrt{b^2 + (w-x)^2}} \Rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2,
\]

which is precisely Snell’s law! Incidentally, the law of reflection can be derived in the same way. It’s really quite remarkable that Snell’s law can be derived in this way, and is a very deep idea of physics. The principle of least time is a special case of a more general idea called the principle of least action, which you will study if you go on to further study in physics.