Physics 18 Spring 2011  
Homework 8 - Solutions  
Wednesday March 9, 2011

Make sure your name is on your homework, and please box your final answer. Because we will be giving partial credit, be sure to attempt all the problems, even if you don’t finish them. The homework is due at the beginning of class on Wednesday, March 16th. Because the solutions will be posted immediately after class, no late homeworks can be accepted! You are welcome to ask questions during the discussion session or during office hours.

1. Two points are on a disk that is turning about a fixed axis perpendicular to the disk and through its center at increasing angular velocity. One point is on the rim and the other point is halfway between the rim and the center.

(a) Which point moves the greater distance in a given time?
(b) Which point turns through the greater angle?
(c) Which point has the greater speed?
(d) Which point has the greater angular speed?
(e) Which point has the greater tangential acceleration?
(f) Which point has the greater angular acceleration?
(g) Which point has the greater centripetal acceleration?

Solution

(a) The point on the rim travels a greater distance in the same amount of time, since it has to make a bigger “orbit.”
(b) Both points move through the same angle.
(c) The speed is \( v = r\omega = r\dot{\theta} \). Since point particles change their angle by the same amount in the same time, the point with the bigger radius has the bigger velocity. So, the point on the rim moves with the greater speed.
(d) Since the angular speed is \( \omega = \frac{d\theta}{dt} \), and both points travel through the same angle in the same time, they both have the same angular speed.
(e) The problem states that the disk is turning at increasing angular velocity, and so \( \omega \) is increasing. Thus, \( v = r\omega \) is increasing. The tangential acceleration is \( a_{tang} = \frac{dv}{dt} \), and so \( a_{tang} = r\alpha \). So, the particle with the bigger radius has the bigger tangential acceleration; i.e., the point on the rim has the bigger tangential acceleration.
(f) The angular acceleration is \( \alpha \), and is the same for both points.
(g) The centripetal acceleration is \( a_{cent} = v^2/r = (r\omega)^2/r = r\omega^2 \). Since \( \omega \) is the same for both points, the point on the rim has the bigger centripetal acceleration, since it has the bigger value of \( r \).
2. The methane molecule (CH$_4$) has four hydrogen atoms located at the vertices of a regular tetrahedron of edge length 0.18 nm, with the carbon atom at the center of the tetrahedron. Find the moment of inertia of this molecule for rotation about an axis that passes through the centers of the carbon atom and one of the hydrogen atoms.

Solution

Because the axis of rotation is through the carbon and one of the hydrogens, neither of these atoms contribute. The moment of inertia is then due only to the remaining three hydrogens. The moment of inertia is

\[
I = \sum_{i=1}^{3} m_i r_i^2 = m_H \sum_{i=1}^{3} r_i^2,
\]

since all of the masses are identical. Now the radii are also all equal is is just the distance from the central hydrogen to the atoms on the vertex. if the distance between the hydrogens is $a$, then the radius is

\[
r = \frac{a}{2} \cos 30^\circ = \frac{a}{2} \frac{2}{\sqrt{3}} = \frac{a}{\sqrt{3}}.
\]

So, we find that

\[
I = 3m_H \left( \frac{a}{\sqrt{3}} \right)^2
\]

\[
= m_H a^2
\]

\[
= (1.67 \times 10^{-27}) (0.18 \times 10^{-9})^2
\]

\[
= 5.41 \times 10^{-47} \text{ kg m}^2.
\]
3. A pendulum consisting of a string of length \( L \) attached to a bob of mass \( m \) swings in a vertical plane. When the string is at an angle \( \theta \) to the vertical,

(a) calculate the tangential acceleration of the bob using \( \sum F_t = ma_t \).

(b) What is the torque exerted about the pivot point?

(c) Show that \( \sum \tau = I\alpha \) with \( a_t = L\alpha \) gives the same tangential acceleration as found in Part (a).

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**Solution**

(a) The pendulum is seen in the figure to the right. The only tangential force is the component of gravity along the rotation path,

\[
F_t = mg \sin \theta.
\]

Since \( F_t = ma_t \), then we just solve for the acceleration,

\[
a_t = \frac{F_t}{m} = g \sin \theta.
\]

(b) The torque is just \( \tau = rF_t \), where \( r = L \) is the distance from the pivot point. So,

\[
\tau = mgL \sin \theta.
\]

(c) The torque is \( \tau = I\alpha \). Since the string is massless and all the mass is in the bob, then \( I = mr^2 = mL^2 \). So, \( \tau = mgL \sin \theta = mL^2\alpha \). Furthermore, since \( \alpha = \frac{\tau}{I} = \frac{a_t}{L} \), then \( a_t = \frac{\tau L}{I} \), or

\[
a_t = \frac{\tau L}{I} = \frac{mgL^2 \sin \theta}{mL^2} = g \sin \theta,
\]

which is the same result we found in part (a).
4. A uniform solid sphere of mass $M$ and radius $R$ is free to rotate about a horizontal axis through its center. A string is wrapped around the sphere and is attached to an object of mass $m$. Assume that the string does not slip on the sphere. Find

(a) the acceleration of the object and

(b) the tension in the string.

Solution

(a) We can draw the force diagram for the system as seen in to the right. The weight of the block creates a torque on the wheel, causing it to rotate. We can write down the net forces and torques acting on the system. For the sphere,

$$
\sum \tau = TR = I\alpha.
$$

While for the hanging mass, calling the vertical direction $x$,

$$
\sum F_x = T - mg = -ma.
$$

Now, since the string isn’t slipping around the wheel, the angular acceleration is just $\alpha = a/R$. Furthermore, the moment of inertia for a sphere is $I = \frac{2}{5}MR^2$. So, we find the following system of equations

$$
T = \frac{2}{5}Ma,
$$

$$
T = m(g - a).
$$

Setting these two expressions equal to each other and solving for the acceleration gives

$$
a = \frac{g}{1 + \frac{2M}{5m}}.
$$

(b) Now we just substitute this result back in to the expression $T = \frac{2}{5}Ma$ to find

$$
T = \frac{2}{5} \frac{Mg}{1 + \frac{2M}{5m}} = \frac{2Mmg}{5m + 2M}.
$$
5. A basketball rolls without slipping down an incline of angle $\theta$. The coefficient of static friction is $\mu_s$. Model the ball as a thin spherical shell. Find

(a) the acceleration of the center of mass of the ball,
(b) the frictional force acting on the ball, and
(c) the maximum angle of the incline for which the ball will roll without slipping.

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**Solution**

(a) We can begin by writing down Newton’s laws for the ball, including the torques:

\[
\sum F_x = -mg \sin \theta + F_f = -ma \\
\sum F_y = -mg \cos \theta + F_n = 0 \\
\sum \tau_i = F_f R = I \alpha.
\]

Furthermore, since the ball is rolling without slipping, we have that $a = \alpha R$. So, we have enough to solve this system of equations. Plugging in $F_f = I \alpha / R = I \alpha / R^2$ to the first equation and solving for $a$ gives

\[
a = \frac{mg \sin \theta}{m + I / R^2}.
\]

The moment of inertia of the spherical shell is just $I = \frac{2}{3} m R^2$, and so

\[
a = \frac{mg \sin \theta}{m + 2m / 3R^2} = \frac{3}{5} g \sin \theta.
\]

(b) Since $F_f = I \frac{\alpha}{R^2} a = \frac{2}{3} m R^2 a = \frac{2}{3} ma$, we have

\[
F_f = \frac{2}{5} mg \sin \theta.
\]

(c) Now, we know that the frictional force is $F_f = \mu_s F_N = \mu_s mg \cos \theta$. Setting this equal to our result from part (b) we find

\[
\frac{2}{5} mg \sin \theta = \mu_s mg \cos \theta \Rightarrow \tan \theta = \frac{5}{2} \mu_s.
\]

So, the maximum angle of the incline that the ball will roll without slipping is

\[
\theta = \tan^{-1} \left( \frac{5}{2} \mu_s \right).
\]

The rolling of the ball increases the angle from the sliding case.