Physics 18 Spring 2011
Homework 14 - Solutions
Wednesday April 27, 2011

Make sure your name is on your homework, and please box your final answer. Because we will be giving partial credit, be sure to attempt all the problems, even if you don’t finish them. The homework is due at the beginning of class on Wednesday, May 4th. Because the solutions will be posted immediately after class, no late homeworks can be accepted! You are welcome to ask questions during the discussion session or during office hours.

1. A scuba diver is 40 m below the surface of a lake, where the temperature is 5.0°C. He releases an air bubble that has a volume of 15 cm³. The bubble rises to the surface, where the temperature is 25°C. Assume that the air in the bubble is always in thermal equilibrium with the surrounding water, and assume that there is no exchange of molecules between the bubble and the surrounding water. What is the volume of the water right before it breaks the surface? Hint: Remember that the pressure also changes.

Solution

From the ideal gas law, $PV = nRT$, since the number of moles of gas doesn’t change we can write $P_iV_i/T_i = nR = P_fV_f/T_f$. So, we can solve for the final volume

$$V_f = \frac{P_iT_f}{P_fT_i}V_i.$$

Remember that the pressure at a depth $h$ is $P = P_{atm} + \rho gh$. So,

$$V_f = \frac{(P_{atm} + \rho gh)T_f}{P_{atm}T_i}V_i.$$

Thus,

$$V_f = \frac{(P_{atm} + \rho gh)T_f}{P_{atm}T_i}V_i = \frac{(1.01 \times 10^5 + 10^3(9.8)(40))298}{(1.01 \times 10^5)278}15 = \frac{78 \text{ cm}^3}{78 \text{ cm}^3}. $$


2. A 50.0 g piece of aluminum at 20\(^\circ\) C is cooled to \(-196\)^\(^\circ\) C by placing it in a large container of liquid nitrogen at that temperature. How much nitrogen is vaporized? (Assume that the specific heat of aluminum is constant over this temperature range.)

**Solution**

The heat lost by the aluminum is gained by the liquid nitrogen. Since the liquid nitrogen is already at its boiling point all the heat energy goes into vaporizing it. Thus, \(-Q_{Al} = -m_{Al}c\Delta T = +Q_{N} = m_{N}L_{f}\). So, the mass of nitrogen vaporized is \(m_{N} = -\frac{m_{Al}c\Delta T}{L_{f}}\). Thus,

\[
m_{N} = -\frac{m_{Al}c\Delta T}{L_{f}}
\]
\[
= -\frac{(0.05) \times (900) \times (-196-20)}{199 \times 10^{3}}
\]
\[
= 48.8 \text{ grams.}
\]
3. A lead bullet initially at 30° C just melts upon striking a target. Assuming that all of the initial kinetic energy of the bullet goes into the internal energy of the bullet, calculate the impact speed of the bullet.

Solution

The initial energy of the bullet is just kinetic energy, \( \frac{1}{2}mv^2 \), which is changed into thermal energy, which goes into both heating the bullet and also melting it, \( Q = mc\Delta T + mL_f \). Setting the two values equal and noting that the masses cancel we find

\[ v = \sqrt{2 \left( c\Delta T + L_f \right)} \]

Now, the final temperature of the lead is the melting point, 600 Kelvins, while \( c = 128 \text{ J/kg K} \), and \( L_f = 24.7 \times 10^3 \text{ J/kg} \). So, we find

\[
\begin{align*}
v &= \sqrt{2 \left( 128 \times (600 - 303) + 24.7 \times 10^3 \right)} \\
&= 354 \text{ m/s}.
\end{align*}
\]
4. The initial state of 1.00 mol of a dilute gas is $P_1 = 3.00$ atm, $V_1 = 1.00$ L, and $E_{\text{int}_1} = 456$ J, and its final state is $P_2 = 2.00$ atm, $V_2 = 3.00$ L, and $E_{\text{int}_2} = 912$ J. The gas is allowed to expand isothermally until it reaches its final volume and its pressure is 1.00 atm. It is then heated at constant volume until it reaches its final pressure.

(a) Illustrate this process on a PV diagram and calculate the work done by the gas.
(b) Find the heat absorbed by the gas during this process.

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Solution

(a) The PV diagram is seen in the figure to the right. Because there is no work done on the constant volume part, the net work done is just from the isothermal part which is

$$W = nRT_i \ln \left( \frac{V_f}{V_i} \right),$$

where $T_i$ is the initial temperature.

Plugging in the values, and replacing $nRT_i = P_iV_i$, using the ideal gas law, gives

$$W = P_iV_i \ln \left( \frac{V_f}{V_i} \right) = (3 \times 1.01 \times 10^5)(1 \times 10^{-3}) \ln(3) = 334 \text{ Joules},$$

where we have recalled that 1 atm is 101.3 kPa, and one liter is 1/1000 cubic meter.

(b) Remember the first law of thermodynamics, $\Delta E_{\text{in}} = Q + W$, and so $Q = \Delta E - W = (912 - 456) + 334 = 790$ Joules.
5. As part of a laboratory experiment, you test the calorie content of various foods. Assume that when you eat these foods, 100% of the energy released by the foods is absorbed by your body. Suppose you burn a 2.50 g potato chip, and the resulting flame warms a small aluminum can of water. After burning the potato chip, you measure its mass to be 2.20 g. The mass of the can is 25.0 g, and the volume of water contained in the can is 15.0 mL. If the temperature increase in the water is 12.5° C, how many kilocalories (1 kcal = 1 dietary calorie) per 150 g serving of these potato chips would you estimate there are? Assume the can of water captures 50.0 percent of the heat released during the burning of the potato chip. Note: Although the joule is the SI unit of choice in most thermodynamic situations, the food industry in the United States currently expresses the energy released during metabolism in terms of the “dietary calorie,” which is our kilocalorie.

Solution

The heat that’s released in the burning of the chip goes in to heating both the can and also the water. We can determine this heat by looking at the change in temperature of the can and water, 

\[ Q_{\text{total}} = Q_{\text{can}} + Q_{\text{water}} = m_{\text{can}}c_{\text{can}}\Delta T + m_{\text{water}}c_{\text{water}}\Delta T, \]

where the change in temperature is the same for both (the can heats up first, then heats the water). Plugging in the numbers we can figure out how much energy is absorbed by the can and water,

\[
Q_{\text{total}} = (m_{\text{can}}c_{\text{can}} + m_{\text{water}}c_{\text{water}})\Delta T = (0.025 \times 900 + 0.0150 \times 4184) \times 12.5 = 1066 \text{ Joules.}
\]

This is the energy absorbed by the system, which is only half of the energy lost by the chip. So, the energy lost by burning 0.30 grams of a single 2.20 gram chip is \(1066 \times 2 = 2132\) Joules. Finally, this is for only a single 2.20 gram chip, the entire serving contains (assuming that the entire amount of energy is absorbed by the body)

\[
Q_{\text{chips}} = 2132 \times \left( \frac{150}{0.30} \right) = 1.06 \times 10^6 \text{ Joules} = 106 \times 10^3 \text{ kJ.}
\]

Now, one kilocalorie (Calorie) is 4184 Joules, and so \(Q = 106 \times 10^3 /4184 = 255\) Calories.