Quiz 7 - Solutions

Make sure your name is on your quiz, and please box your final answer. Because we will be giving partial credit, be sure to attempt all the problems, even if you don’t finish them!

1. It is believed that there is a “supermassive” black hole at the center of our galaxy. One datum that leads to this conclusion is the important recent observation of stellar motion in the vicinity of the galactic center. Suppose one such star moves in a circular orbit with a period of 15.2 years and has a radius of 5.5 light days (the distance light travels in 5.5 days).

   (a) Using Kepler’s laws, what is the mass around which the star moves in its orbit? If the mass of our sun is $2 \times 10^{30}$ kg, how many solar masses is this mass? Note: Newton’s gravitational constant, $G_N = 6.672 \times 10^{-11}$ N kg$^2$/m$^2$, and the speed of light is $c = 3.00 \times 10^8$ m/s.

   (b) What is the Schwarzschild radius of the black hole?

Solution

1. (a) Kepler’s law says that the square of the period, $T$, of an orbit is proportional to the cube of it’s semimajor axis, $a$ (which is just the radius for a circular orbit), so

   \[ T^2 = \frac{4\pi^2}{G_N M} a^3. \]

   So, all we need to do is to solve for the mass,

   \[ M = \frac{4\pi^2}{G_N T^2} a^3. \]

   Now, we can just plug in numbers. One year has about $\pi \times 10^7$ seconds, while one day has $3600 \times 24 = 86400$ seconds. So, one light day is $3 \times 10^8 \times 86400 = 2.6 \times 10^{13}$ meters. So,

   \[ M = \frac{4\pi^2}{G_N T^2} a^3 = \frac{4\pi^2}{6.7 \times 10^{-11} (15.2 \times \pi \times 10^7)^2} (5.5 \times 2.6 \times 10^{13})^3 = 7.5 \times 10^{36} \text{ kg.} \]

   Our sun has a mass of about $2 \times 10^{30}$ kg, and so this star has a mass of nearly four million suns! This is clearly a black hole.

   (b) The Schwarzschild radius of a black hole is

   \[ R_S = \frac{2G_N M}{c^2} = \frac{2 (6.67 \times 10^{-11}) (7.5 \times 10^{36})}{(3 \times 10^8)^2} = 11 \times 10^9 \text{ m,} \]

   meaning that four million suns are crammed into a space only about ten times larger than our Solar System!