Momentum conservation is another extremely useful concept, because (as with energy) it allows us to completely ignore the detailed interactions of particles, and simply focus on the initial and final conditions. Momentum conservation has a number of practical applications, for example investigating car crashes and designing automotive safety equipment. Momentum conservation is also extremely important in other areas of physics, for example particle physics. In fact, the neutrino was discovered theoretically by Pauli before it was discovered experimentally, simply by looking at the energy and momentum of a radioactive decay. Momentum calculations can be algebraically rough, however, and this worksheet will give you some practice in those calculations.

1 Conceptual Questions

1. Under what conditions is the total momentum conserved in a collision? Under what conditions is the total energy conserved? The total kinetic energy?

2. Why are cars designed so that their front ends crumple during an accident?

3. You are standing perfectly still and then take a step forward. Before the step, your total momentum was zero, but afterwards you have some momentum. Does this violate conservation of momentum? Explain.

4. In numerous action movies the bad guy is shot with a bullet from a handgun and goes flying backwards due to the impact. Is this realistic? Explain why or why not.

5. A sharpshooter fires a rifle while standing with the butt of the rifle against her shoulder. If the forward momentum of the bullet is the same as the backward momentum of the gun, why isn’t it as dangerous to be hit by the gun as by the bullet?

6. Explain how a rocket can fly through space.

7. What do we mean by reference frame?

8. What quantities are conserved when a heavy nucleus undergoes fission and splits up into lighter ones? List as many as you can (there are lots).

9. You are designing a nuclear reactor and know that you need to work out a way to slow down neutrons to mediate the nuclear reactions. Should you use a material that is made up of heavy atoms, like lead, or lighter atoms, like carbon? Explain your decision.
2 Conservation of Momentum

1. A 300 gram bird flying along at 6.0 m/s sees a 10 g insect heading straight toward it with a speed of 30 m/s. The bird opens its mouth wide and enjoys a nice lunch. What is the bird’s speed immediately after swallowing?

2. The light isotope, $^5$Li, of lithium is unstable and breaks up spontaneously into a proton and an $\alpha$ particle. In this process, $3.15 \times 10^{-13}$ J of energy are released, appearing as the kinetic energy of the two decay products. Determine the velocities of the proton and the $\alpha$ particle that arise from the decay of a $^5$Li nucleus at rest. (*Note:* The masses of the proton and the alpha particle are $m_p = 1.67 \times 10^{-27}$ kg and $m_\alpha = 4m_p = 6.64 \times 10^{-27}$ kg.)

3. A block of mass $m_1 = 2.0$ kg slides along a frictionless table with a speed of 10 m/s. Directly in front of it, and moving in the same direction with a speed of 3.0 m/s, is a block of mass $m_2 = 5.0$ kg. A massless spring that has a force constant $k = 1120$ N/m is attached to the second block, as in the figure.

   (a) What is the velocity of the center of mass of the system?
   
   (b) During the collision, the spring is compressed by a maximum amount $\Delta x$. What is the value of $\Delta x$?
   
   (c) The blocks will eventually separate again. What are the velocities of the two blocks measured in the reference frame of the table, after they separate?

4. A bomb explodes into three fragments. Immediately after the explosion, the first fragment of mass $m_1 = 2$ kg travels leftward at speed $v_1 = 100$ m/s. The second fragment of mass $m_2 = 3$ kg moves at a 45$^\circ$ angle from the horizontal at $v_2 = 80$ m/s. The third fragment of mass $m_3 = 4$ kg moves at an angle of 30$^\circ$ below the horizontal with velocity $v_3 = 50$ m/s. Was the bomb moving before it exploded, and if so, what was its velocity vector, and also it’s speed and direction?
5. In the “slingshot effect,” the transfer of energy in an elastic collision is used to boost the energy of a space probe so that it can escape from the solar system. All speeds are relative to an inertial frame in which the center of the Sun remains at rest. The figure shows a space probe moving at 10.4 km/s toward Saturn, which is moving at 9.6 km/s toward the probe. Because of the gravitational attraction between Saturn and the probe, the probe swings around Saturn and heads back in the opposite direction with speed \( v_f \).

(a) Assuming this collision to be a one-dimensional elastic collision with the mass of Saturn much much greater than that of the probe, find \( v_f \).

(b) By what factor is the kinetic energy of the probe increased? Where does this energy come from?

6. A neutron of mass \( m \) makes an elastic head-on collision with a stationary nucleus of mass \( M \).

(a) Show that the kinetic energy of the nucleus after the collision is given by \( K_{\text{nucleus}} = [4mM/(m+M)^2] K_n \), where \( K_n \) is the initial kinetic energy of the neutron.

(b) Show that the fractional change in the kinetic energy of the neutron is given by

\[
\frac{\Delta K_n}{K_n} = -\frac{4 (m/M)}{(1 + [m/M])^2}.
\]

(c) Show that this expression gives plausible results both if \( m \ll M \) and if \( m = M \). What is the best stationary nucleus for the neutron to collide head-on with if the objective is to produce a maximum loss in kinetic energy of the neutron?