The concept of work is central to physics, and leads to the idea of kinetic energy, which will be discussed here, as well as potential energy. The concept of conservation of energy is among the most important in all of physics, encompassing every branch. Not only does it make a practical difference, allowing us to solve a number of problems very easily, but it also helps us to discover new laws. Regardless of whether we understand a new phenomenon (like dark matter, for example), we expect that that phenomenon will still obey laws like conservation of energy. Energy is an even more important concept when we venture outside classical physics, where we learn that an object can have energy, not only from how it’s moving and where it is, but also due to it’s sheer existence! Furthermore, a system can’t have any energy it likes, but only certain amounts. Such ideas are the foundations of more advanced topics like quantum mechanics and quantum field theory. This worksheet introduces you to some of the consequences of such new ideas.

1 Conceptual Questions

1. Give two examples in which a force is exerted on an object without doing any work on that object.

2. Can kinetic energy be negative? If so, give an example. If not, explain what would be required, and comment on the realism of those requirements.

3. If only one external force acts on a particle, does it necessarily change the particle’s kinetic energy? What about it’s velocity?

4. Does the kinetic energy of an object depend on the frame of reference in which its motion is measured? Provide an example to prove this point.

5. Which method does less work against gravity: carrying a 10 kg weight up 50 meters by taking the stairs, or taking the elevator? What about carrying the same object 50 meters horizontally, or holding it stationary for the same amount of time?

6. If you slide a block along a frictional surface, you do work against friction. Suppose you smack the block so that it slides along for a while, but comes to a stop. The block started at rest and ends up at rest, but you’ve done work. Does the work-energy theorem fail, here? If so, why? If not, where did the energy go?

7. Suppose that you are riding a bike. In what sense is that bike solar-powered?
8. One person drops a ball from the top of a building while you watch from the ground. Will you both necessarily agree on the gravitational potential energy of the ball? What about the change in that potential energy as it falls? The kinetic energy of the ball?

9. Why are we free to set the zero of our potential energy wherever we want? Why can’t we do the same with kinetic energy?

10. Give two examples of a system that has a negative total energy.

11. If the potential energy of an object is zero at a point, does that mean that the force acting on that object must be zero, too? Explain your answer.

12. Typically, you turn off light switches when you aren’t in a room saying that you “don’t want to waste energy.” However, you know that energy is conserved. In what sense are you “wasting energy,” in this case?

13. What does \( E = mc^2 \) actually mean?

14. What does it mean that energy is quantized? Can you give any examples? Why don’t we notice this in everyday life?

15. One of the Heisenberg uncertainty principles of quantum mechanics states that you can’t know the energy of a system exactly - there are always fluctuations. What does this imply about the energy of empty space? Explain your answer.
2 Work

1. To assess a patient who is suspected of having heart disease, the physician must examine the cardiac function when (a) the patient is at rest with a heart beating at a normal pace, and (b) when under stress, for example, after exercise. To simulate the conditions of stress, the patient exercises by walking a treadmill to increase heart beat and sustain high levels of cardiac stress.

Suppose a 50 kg patient exercises on the treadmill, angled at $30^\circ$ from the ground, exerting a constant force of 500 N up the slope of the treadmill while running a constant velocity of 4 m/s along the treadmill for 5 minutes. The coefficient of friction of the treadmill is $\mu_s = 0.45$. Determine the work done by the patient.

3 Conservation of Energy

1. Suppose you eat a Snickers bar that contains 280 Calories.
   
   (a) If one Calorie is 4187 joules, then how much energy is contained in the Snickers bar?

   (b) Digestion requires some energy, and so only about 85% of the food energy is directly available for other uses (like living). How much energy is this?

   (c) Suppose you want to work off the candy by climbing stairs. Because the efficiency of the muscles is relatively low, only about 20% of the food energy is available for conversion to mechanical energy. If you have a mass of 80 kg, and a single step is 15 centimeters tall, how many steps would you be able to climb from the energy in part (b) available to you?

   (d) How many steps would you need to climb in order to work off all the Calories from the candy?

2. A 75 kilogram base runner has an initial speed of 8 m/s (almost 18 miles per hour), and he then slides to a stop over 4 m. What is the coefficient of sliding friction between the runner and ground during the slide?

3. A mass $m$ is dropped from height $h$ above the top of a spring of constant $k$ mounted vertically on the floor. Show that the spring’s maximum compression is given by

$$\Delta y = \frac{mg}{k} \left[ 1 + \sqrt{1 + \frac{2kh}{mg}} \right].$$

Hint: Recall that the quadratic equation, $ax^2 + bx + c = 0$, is solved by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

and explain why we need to take the positive sign (Super-hint: Consider what happens when $h \to 0$; how much does the spring compress in that case?).
4. Let’s look the maximum safe heights that a person could jump from in two different landing cases.

(a) First, suppose a person of mass $m$ jumps from a height $h$, and lands with straight legs such that they stop in a small distance $d$, cushioned only by the padding on the bottom of their feet. Using conservation of energy, show that the force acting back up on them from the ground is $F = \frac{mgh}{d}$.

(b) Upon compression bones typical break when subjected to a force per area above about $1.7 \times 10^8 \text{ N/m}^2$. If the tibia near the ankle has a radius of about 1 cm, estimate the maximum height from which a 70 kg person could just land without breaking any bones if the padding on the soles of their feet is about 1 cm.

(c) Now, suppose the person lands at the end of the jump using the muscles in his knees to help cushion the collision. Now his center of mass falls a height $h$ before his feet make contact with the ground, but then he bends down, dropping his center of mass a further distance $s$ (we can ignore the padding on his feet now, since it’s small compared to how much he bends). Show that the required force is now

$$F = mg \left(1 + \frac{h}{s}\right).$$

(d) If he bends down 0.5 meters, what is the maximum height that he could just land from without breaking any bones? (Kids - don’t try this at home!)

5. A 190 gram block is launched by compressing a spring of constant $k = 200 \text{ N/m}$ by 15 cm. The spring is mounted horizontally, and the surface directly under it is frictionless. But, beyond the equilibrium position of the spring end, the surface has frictional coefficient $\mu_k = 0.27$. This frictional surface extends 85 cm, followed by a frictionless curved rise, as shown in the figure to the right.

(a) If there was no frictional part, how high up the ramp would the block slide?

(b) If we now include the frictional part, how high up the ramp does the block slide?

(c) After the block reaches the top of the ramp, it slides back down, over the frictional surface, compressing the spring. The spring then pushes the block back to do the same trip over, again. However, each during each trip, the block slows down when passing over the rough patch. How many times does the block slide over the frictional surface? (Note: you’ll get a decimal, meaning that it doesn’t make it all the way. Just give the whole number actual completed trips.)

(d) How far along the rough patch (measured from the left-hand-side) does the block ultimately stop?
6. If a black hole and a “normal” star orbit each other, gases from the normal star falling into the black hole can have their temperatures increased by millions of degrees due to frictional heating. When the gases are heated that much, they begin to radiate light in the X-ray region of the electromagnetic spectrum (high-energy light photons). Cygnus X-1, the second strongest known X-ray source in the sky, is thought to be one such binary system; it radiates at an estimated power of $4 \times 10^{31}$ W. If we assume that 1.0 percent of the in-falling mass escapes as X ray energy, at what rate is the black hole gaining mass?

7. A large nuclear power plant produces 1000 MW of electrical power by nuclear fission.

   (a) By how many kilograms does the mass of the nuclear fuel decrease in one year? (Assume an efficiency of 33 percent for a nuclear power plant.)

   (b) In a coal-burning power plant, each kilogram of coal releases 31 MJ of thermal energy when burned. How many kilograms of coal are needed each year for a 1000 MW coal-burning power plant? (Assume an efficiency of 38 percent for a coal-burning power plant.)