Motion in more than one direction is most easily described in terms of vectors. A vector is an object that has a magnitude and a direction. For example, displacement is a vector since you have to say how far you went *plus* in what direction you moved. Other vectors include velocity, acceleration, and forces. Objects that can be specified in terms of a magnitude only are called scalars. Examples of scalars include mass, energy, and temperature. In this worksheet you will practice manipulating vectors including adding and subtracting them graphically and by using the components, as well as differentiating them. Once you have gotten more skill with vectors, we will apply them to two-dimensional motion.

## 1 Manipulating Vectors Graphically

1. On the graph to the left, draw a coordinate \((x, y)\) axis, and then draw the following vectors:
   
   (a) \(4\, \hat{i}\)
   
   (b) \(6\, \hat{j}\)
   
   (c) \(4\, \hat{i} + 6\, \hat{j}\)
   
   (d) \(-4\, \hat{i} - 6\, \hat{j}\).

What are the angles from the \(x\) axis of each of the vectors above?
2. On the graph to the left, draw a coordinate \((x, y)\) axis, and then add the following vectors, \(\vec{A} = 3\hat{i} - 2\hat{j}\) and \(\vec{B} = \hat{i} + 3\hat{j}\) graphically. What is resultant vector from the graph? What is the angle that the resultant vector makes with the \(x\) axis?

3. On the graph to the left show that the following vectors are perpendicular to each other:

\[
\begin{align*}
\vec{A} &= 2\hat{i} + 4\hat{j} \\
\vec{B} &= 2\hat{i} - \hat{j}.
\end{align*}
\]
4. *This one is tougher.* Suppose you don’t know the vectors \( \vec{A} \) and \( \vec{B} \), but you do know that:

\[
\vec{A} + \vec{B} = 5\hat{i} + 3\hat{j} \\
\vec{A} - \vec{B} = -\hat{i} + 5\hat{j}.
\]

Show how you can *graphically* (i.e., work it out using arrows, not components) determine \( \vec{A} \) and \( \vec{B} \), and then draw them on the graph.

## 2 Manipulating Vectors by Components

1. Consider the following vectors:

\[
\vec{A} = 2\hat{i} + 4\hat{j} + 3\hat{k} \\
\vec{B} = 3\hat{i} + 6\hat{j} + 9\hat{k} \\
\vec{C} = -2\hat{i} + 7\hat{j} - 3\hat{k}.
\]

Find the following vectors in terms of their components:

(a) \( \vec{A} + \vec{B} \)  
(b) \( \vec{A} - \vec{B} \)  
(c) \( 2\vec{A} + 5\vec{B} - 2\vec{C} \)  
(d) \( 3\vec{A} + 3\vec{C} \)  
(e) \( \vec{C} + \vec{B} \)

2. Given the vectors above,

(a) find the unit vectors,

\[
\hat{A} \equiv \frac{\vec{A}}{|\vec{A}|}, \quad \hat{B} \equiv \frac{\vec{B}}{|\vec{B}|}, \quad \hat{C} \equiv \frac{\vec{C}}{|\vec{C}|}.
\]

(b) Does \( |\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}| \)?
3. **This one is tough!** Consider the vector $\vec{A}$ in the figure to the right, making an angle $\theta$ with respect to the $x$ and $y$ axes. Show that, if the length of the vector is $|\vec{A}| = A$, the components in this coordinate system are given by

$$
A_x = A \cos \theta \\
A_y = A \sin \theta.
$$

Now, suppose you rotate the coordinate system by an angle $\phi$, so that the vector now makes an angle $\alpha$ with respect to the $x'$ and $y'$ axes. Show that the new components can be written in terms of the old ones as

$$
A_{x'} = A_x \cos \phi + A_y \sin \phi \\
A_{y'} = -A_x \sin \phi + A_y \cos \phi.
$$

Finally, show that the length of the vector $|\vec{A}'| = |\vec{A}|$. That is, the length of the arrow doesn’t change if we turn our heads and look at it sideways.

4. The vector describing a particular particle is given by

$$
\vec{r}(t) = R \cos (\omega t) \hat{i} + R \sin (\omega t) \hat{j},
$$

where $R$ and $\omega$ are constants.

(a) What is the magnitude of the position, $|\vec{r}|$? What is its unit vector, $\hat{r}$?

(b) What is the velocity, $\vec{v}$, of this particle? What is it’s magnitude, $|\vec{v}|$? What is its unit vector, $\hat{v}$?

(c) What is the acceleration, $\vec{a}$, of this particle? What is it’s magnitude, $|\vec{a}|$? What is its unit vector, $\hat{a}$?

(d) Show that the acceleration points opposite to the position, $\vec{r}$ (i.e., $\hat{a} \propto -\hat{r}$), and determine the proportionality constant.

5. Integrate the vector $\vec{A}(t) = 3t^2 \hat{i} + 5t \hat{j} + 7 \hat{k}$ from $t = 0$ to $t = 5$. What is the magnitude of the resulting vector?
3 Two-Dimensional Motion

1. In a chase scene, a movie stuntman runs horizontally off the roof of one building and lands on another roof 1.9 m lower. If the gap between the buildings is 4.5 m wide, how fast must he run to cross the gap?

2. Wile E. Coyote (*Carnivorus hungribilus*) is chasing the Roadrunner (*Speedibus cantcatchmi*) yet again. While running down the road, they come to a deep gorge, 15.0 m straight across and 100 m deep. The Roadrunner launches himself across the gorge at a launch angle of 15° above the horizontal, and lands with 1.5 m to spare.

   (a) What was the Roadrunner’s launch speed?

   (b) Wile E. Coyote launches himself across the gorge with the same initial speed, but at a different launch angle. To his horror, he is short of the other lip by 0.50 m. What was his launch angle? (Assume that it was less than 15°.)

3. You are watching an archery tournament when you start wondering how fast an arrow is shot from the bow. Remembering your physics, you ask one of the archers to shoot an arrow parallel to the ground. You find the arrow stuck in the ground 60 meters away, making an angle of 3° with the ground.

   (a) How fast was the arrow shot?

   (b) From what height was the arrow shot?

   *Hint: imagine the arrow traveling backwards, from the ground to the bow.*