Physics 9 Fall 2010
Final Solutions

For the midterm, you may use one sheet of notes with whatever you want to put on it, front and back. Please sit every other seat, and please don’t cheat! If something isn’t clear, please ask. You may use calculators. All problems are weighted equally. PLEASE BOX YOUR FINAL ANSWERS! You have the full length of the class. If you attach any additional scratch work, then make sure that your name is on every sheet of your work. Good luck!

1. Light in air is incident on the surface of a transparent substance at an angle of 58° with the normal. The reflected and refracted rays are observed to be mutually perpendicular.

   (a) What is the index of refraction of the transparent substance? Hint: Note that 
   \[ \sin (90° - \theta) = \cos \theta. \]
   (b) What is the critical angle for the total internal reflection in this substance?

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Solution

(a) Recall Snell’s law, taking \( n_{\text{air}} = 1 \), and the index of refraction in the substance \( n \),
   \[ \sin \theta_1 = n \sin \theta_2. \]
   We’re told that the reflected and refracted rays are mutually perpendicular, such that \( \theta_1 + \theta_2 = 90° \). So, solving for \( \theta_2 = 90° - \theta_1 \), such that \( \sin \theta_2 = \sin (90° - \theta_1) = \cos \theta_1 \), then we have
   \[ n = \frac{\sin \theta_1}{\cos \theta_1} = \tan \theta_1 = \tan (58°) = 1.6. \]

(b) The condition for total internal reflection is, again taking \( n_{\text{air}} = 1 \),
   \[ n_2 \sin \theta_c = \sin (90°) = 1 \Rightarrow \theta_c = \sin^{-1} \left( \frac{1}{n_2} \right). \]
   In part (a) we found \( n_2 = 1.6 \), and so
   \[ \theta_c = \sin^{-1} \left( \frac{1}{1.6} \right) = 39°. \]
2. A stationary naval destroyer is equipped with sonar that sends out 40 MHz pulses of sound. The destroyer receives reflected pulses back from a submarine directly below with a time delay of 80 ms at a frequency of 39.959 MHz.

(a) What is the depth of the submarine?

(b) Based on the frequency shift, is the submarine ascending or descending?

(c) If the speed of sound in seawater is 1.54 km/s, then what is the vertical speed of the submarine (remember that, since the pulse is reflected, it’s going to be Doppler shifted \textit{twice}!).

Solution

(a) The sound pulse takes 80 ms to travel down to the submarine and back up to the destroyer. If the sound travels at speed \(v\), then \(2D = v\Delta t\), so

\[
D = \frac{1}{2}v\Delta t = \frac{1}{2} (1540) (0.080) = 62 \text{ m}.
\]

(b) The reflected frequency is smaller than the original frequency which tells us that the submarine is moving away from the destroyer, and so it is descending.

(c) The destroyer sends out the \(f_0 = 40\) MHz pulse, which is Doppler shifted once as it is traveling towards the submarine. If the sound travels at speed \(v\), then the submarine receives the pulse with a frequency

\[
f_{\text{Sub}} = \frac{v - v_{\text{sub}}}{v} f_0.
\]

Now, the submarine reflects this same frequency back to the destroyer, but it is Doppler shifted again, since the submarine is still descending. So, the frequency received by the destroyer is

\[
f_{\text{destroyer}} = \frac{v}{v + v_{\text{sub}}} f_{\text{sub}} = \left(\frac{v - v_{\text{sub}}}{v + v_{\text{sub}}}\right) f_0.
\]

So we just have to now solve for \(v_{\text{sub}}\). Doing so gives

\[
v_{\text{sub}} = \left(\frac{f_0 - f_{\text{destroyer}}}{f_0 + f_{\text{destroyer}}}\right) v = \left(\frac{40 - 39.959}{40 + 39.959}\right) \times 1540 = 0.81 \text{ m/s}.
\]
When electrons transition from one orbit to lower one, they emit a photon of a specific frequency (and so a specific color). In one of these cases the electron jumps from the third to second orbit, emitting a photon of frequency \( f = 4.57 \times 10^{14} \) Hz, which is in the red region. In another case the electron jumps from the fifth orbit down to the second, and emits a photon of frequency \( 6.91 \times 10^{14} \) Hz, which is in the blue region of the spectrum. This light is then sent through a diffraction grating with 4500 lines/cm.

(a) What are the wavelengths of the two spectral lines?
(b) What is the distance between the slits in the grating?
(c) What is the angular separation, \( \Delta \theta \), (in degrees) between the two \( m = 1 \) spectral lines?
(d) What about for the \( m = 2 \) spectral lines?

Solution

(a) Recall that \( \lambda f = c \) for light. So, the wavelength is \( \lambda = c/f \), and so

\[
\begin{align*}
\lambda_{\text{red}} &= \frac{2.998 \times 10^8}{4.57 \times 10^{14}} = 656 \text{ nm}, \\
\lambda_{\text{blue}} &= \frac{2.998 \times 10^8}{6.91 \times 10^{14}} = 434 \text{ nm}.
\end{align*}
\]

(b) Since the grating has \( N = 450000 \) lines per meter, then the spacing is \( d = 1/N = 2000 \) nm.

(c) Each color will be diffracted through a different angle. Remember the diffraction equation for \( m = 1 \), \( d \sin \theta = \lambda \). Then we’ll have two different angles, \( \theta_{\text{blue}} \), and \( \theta_{\text{red}} \), each determined by the wavelengths. So, \( \theta_{\text{blue}} = \sin^{-1}(\lambda_{\text{blue}}/d) \), and \( \theta_{\text{red}} = \sin^{-1}(\lambda_{\text{red}}/d) \). Thus, \( \Delta \theta = \theta_{\text{red}} - \theta_{\text{blue}} \), or

\[
\Delta \theta = \sin^{-1} \left( \frac{\lambda_{\text{red}}}{d} \right) - \sin^{-1} \left( \frac{\lambda_{\text{blue}}}{d} \right).
\]

Now we just have to plug in the two values of wavelength to find

\[
\Delta \theta = \sin^{-1} \left( \frac{656}{2000} \right) - \sin^{-1} \left( \frac{434}{2000} \right) = 6.61^\circ.
\]

(d) Here we just repeat the analysis of part (c), but let \( m = 2 \), instead. This gives

\[
\Delta \theta = \sin^{-1} \left( \frac{2 \lambda_{\text{red}}}{d} \right) - \sin^{-1} \left( \frac{2 \lambda_{\text{blue}}}{d} \right) = 19.15^\circ - 12.53^\circ = 6.61^\circ.
\]
4. Suppose you have two infinite straight line charges, of charge per unit length \( \lambda \), a distance \( d \) apart, moving along at a constant speed \( v \). The moving charge densities constitute a current \( I = \lambda v \), and so the charges behave like current-carrying wires. Because the wires carry the same charge density, \( \lambda \), they repel each other electrically. But, because they are current-carrying wires, with currents moving in the same direction, they attract each other magnetically. How great would \( v \) have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number... Is this a reasonable sort of speed? (Hint - use Gauss’s law to get the electric field, and Ampere’s law to get the magnetic field of the wire. Then determine the force per unit length from each field, \( F_{\text{elec}}/L = \lambda E \), and \( F_{\text{mag}}/L = IB \).)

Solution

From Gauss’s law, the electric field from a line charge is \( E = \frac{1}{2\pi \epsilon_0} \frac{\lambda}{r} \). The force per unit length on the other line charge at a distance \( d \) is \( F/L = \frac{1}{2\pi \epsilon_0} \frac{\lambda^2}{d} \). From Ampere’s law, the magnetic field from a current-carrying wire is \( B = \frac{\mu_0 I}{2\pi r} \). The force per unit length on the other wire at a distance \( d \) is \( F/L = \frac{\mu_0 I^2}{2\pi d} \). When these forces balance, we have

\[
\frac{1}{2\pi \epsilon_0} \frac{\lambda^2}{d} = \frac{\mu_0 I^2}{2\pi d} \Rightarrow \frac{1}{\epsilon_0} \lambda^2 = \mu_0 I^2.
\]

But, \( I = \lambda v \), and so \( \frac{1}{\epsilon_0} \lambda^2 = \mu_0 \lambda^2 v^2 \), or

\[
v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.
\]

Plugging in the values gives \( v \approx 2.99 \times 10^8 \text{ m/s} = c \), which is the speed of light! So, the wires would need to be moving at the speed of light for the forces to exactly balance out! So, this means that the two wires will always repel each other electrically.
5. The galactic magnetic field in some region of interstellar space has a magnitude of $1.00 \times 10^{-9}$ T. A particle of dust has mass $10.0 \, \mu g$ and a total charge of $0.300 \, nC$. How many years does it take for the particle to complete a revolution of the circular orbit caused by its interaction with the magnetic field? You can assume that the orbit is perpendicular to the magnetic field.

Solution

If the dust has a velocity $v$ and makes a circular orbit of radius $r$, then it takes a time

$$T = \frac{2\pi r}{v}$$

to do so. Now, the magnetic force makes it go in a circle and so, since the orbit is perpendicular to the field,

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB},$$

where $q$ is the charge. Plugging this back into the time equation gives

$$T = \frac{2\pi m}{qB}.$$  

Plugging in the numbers gives

$$T = \frac{2\pi m}{qB} = \frac{2\pi \times 10 \times 10^{-6}}{0.300 \times 10^{-9} \times 1.00 \times 10^{-9}} = 2.1 \times 10^{11} \text{ seconds},$$

which is about 6,640 years! Notice that the velocity and radius of the dust doesn’t make any difference.
6. Inchy, an inchworm, is inching along a cotton clothesline. The 5 meter long clothesline has a mass of 0.2 kilograms, and is pulled tight under 100 N of tension. Vivian is hanging up her swimsuit 0 meters from one end when she sees Inchy 2.54 cm (one inch) from the opposite end. She plucks the clothesline sending a terrifying 3 centimeter high wave pulse toward Inchy. If Inchy crawls at 1 inch per second, will he get to the end of the clothesline before the pulse reaches him? If so, how much time does he have to spare? If not, how far does he make it before the pulse kicks poor Inchy off the line? *(Note: don’t forget that Inchy is running away from the pulse, so the time for it to reach him is not just his distance divided by the wave velocity!)*

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**Solution**

It would take Inchy one second to get to the end of the clothesline. Will the pulse get to him before then? The pulse travels at a speed

\[ v = \sqrt{\frac{T}{\mu}}, \]

where \( T \) is the tension, and \( \mu = \frac{M}{L} \) is the linear mass density of the line. Plugging in the values gives a speed

\[ v = \sqrt{\frac{TL}{M}} = \sqrt{\frac{100 \times 5}{0.2}} = 50 \text{ m/s}. \]

Since Inchy starts only 5 meters away, the pulse will reach him before he can get to the end of the clothesline. Suppose it takes a time \( t \) for the pulse to reach him. In that time he can travel a distance \( d = v_I t \), where \( v_I = 2.54 \text{ cm/s} \) is Inchy’s velocity. During that time, the pulse travels the initial distance \( D = (5 - 0.0254) \) meters, plus the extra distance \( d \) that Inchy went. So, it travels a distance \( D + d = vt \), where \( v = 50 \text{ m/s} \) is the speed of the wave. The time is the same in both cases, so solving for the time in each and setting the results equal gives, upon solving for \( d \)

\[ \frac{d}{v_I} = \frac{D + d}{v} \Rightarrow d = \frac{v_I}{v - v_I}D. \]

Plugging in the numbers gives

\[ d = \frac{v_I}{v - v_I}D = \frac{0.0254}{50 - 0.0254} \times (5 - 0.0254) = 2.5 \text{ millimeters}, \]

which is about a *tenth* of the distance that he needed to go!
Extra Credit Question!!

The following is worth 15 extra credit points!

The figure to the right shows the actual production of the antimatter partner to the electron, the positron, in a bubble chamber immersed in a uniform magnetic field. The positron has exactly the same mass as an electron, but opposite charge. The different tracks show the trajectories of the different particles. In the reaction at the top, a high-energy gamma ray (just a high-frequency light wave) is absorbed by an electron, which scatters away to the right, producing an electron-positron pair through the reaction

\[ \gamma + e^- \rightarrow e^- + e^+ + e^- , \]

where \( \gamma \) is the incident gamma ray. A similar effect occurs in the bottom reaction, but the original electron very likely had little final velocity. Let’s see what we can tell about this reaction.

(a) Is charged conserved in this reaction? Explain.

(b) Why do the electrons and positrons curve in different directions?

(c) In what direction (into or out of the page) is the magnetic field? How can you tell?

(d) How does the velocity of the spiraling electron compare to that of the spiraling positron? How can you tell?

(e) Accelerating charges radiate energy. Explain why this leads to the spiraling motion for the particles.

(f) Positrons and electrons can come together and annihilate, releasing two photons in the process

\[ e^- + e^+ \rightarrow \gamma + \gamma . \]

Explain why we get two photons, and not one. Hint: consider the two particles initially at rest right next to each other.

Solution
A more colorful picture is seen to the right, where the positrons are shown in red, and the electrons in green.

(a) Initially we have an electron and a photon. Since the photon isn’t charged, the initial charge is just that on the electron, $-e$. After the reaction, we’ve produced an electron and positron pair. The net charge of the pair is $-e + e = 0$, and since we still have an electron the net charge after the reaction is $-e$, and so charge is conserved.

(b) The electrons and positrons are oppositely charged, and so experience opposite forces in the magnetic field. This leads to motion in different directions.

(c) The electron is curving in the counterclockwise direction, while the positron is curving in the clockwise direction. From the magnetic force law, $\vec{F} = q\vec{v} \times \vec{B}$, for the positively charged positron the velocity is down, and the force is initially to the left, and so the direction of the magnetic field is out of the page.

(d) The faster the speed of the particle, the bigger the radius of the curve. Since the positron has a bigger spiral, it has a bigger speed.

(e) As the charges radiate, they lose energy. The only place that they can lose it from is their kinetic energy, and so, they slow down. The slower they move, the smaller the radius of their curvature. So, the curve tightens as they slow down, leading to the spiral.

(f) Suppose we had an electron and a positron at rest and just brought them together. In this case the initial momentum is zero, but the photon carries momentum. So, a single photon would violate momentum conservation. Two photons, emitted back-to-back, would conserve momentum. This means we need two photons.
Some Possibly Useful Information

Some Useful Constants.

Coulomb’s Law constant \( k \equiv \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \frac{Nm^2}{C^2} \).

The magnetic permeability constant \( \mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2} \).

Speed of Light \( c = 2.99 \times 10^8 \text{ m/s} \).

Newton’s Gravitational Constant \( G = 6.672 \times 10^{-11} \frac{Nm^2}{kg^2} \).

The charge on the proton \( e = 1.602 \times 10^{-19} \text{ C} \).

The mass of the electron, \( m_e = 9.11 \times 10^{-31} \text{ kg} \).

The mass of the proton, \( m_p = 1.673 \times 10^{-27} \text{ kg} \).

Boltzmann’s constant, \( k_B = 1.381 \times 10^{-23} \text{ J/K} \).

\( 1 \text{ eV} = 1.602 \times 10^{-19} \text{ Joules} \Rightarrow 1 \text{ MeV} = 10^6 \text{ eV} \).

\( 1 \text{ Å} = 10^{-10} \text{ meters} \).

Planck’s constant, \( h = 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s} \).

The reduced Planck’s constant, \( \hbar \equiv \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J s} = 6.58 \times 10^{-16} \text{ eV s} \).

Some Useful Mathematical Ideas.

\[
\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} & n \neq -1, \\ \ln(x) & n = -1. \end{cases}
\]

\[
\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left( x + \sqrt{a^2 + x^2} \right).
\]

\[
\int \frac{x dx}{\sqrt{a^2 + x^2}} = \sqrt{a^2 + x^2}.
\]

Other Useful Stuff.

The force on an object moving in a circle is \( F = \frac{mv^2}{r} \).

Kinetmatic equations \( x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2, \ y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \).

The binomial expansion, \((1 + x)^n \approx 1 + nx\), if \( x \ll 1 \).