Lab: Electric Potential & Electric Field I

INTRODUCTION
In this lab, you will determine the electric potential produced by a set of electrodes held at a fixed voltage. The working surface of the experiment will be a two-dimensional sheet of paper. Rather than measure the potential at every single point, you will use the equipotential contour lines to visualize the potential. You will then use these lines to draw and calculate the corresponding electric field lines.

Objectives for the lab:
- Learn how to visualize electric fields and potentials using field lines and contour lines
- Plot contour lines using a digital voltmeter and the CENCO Overbeck Electric Field Apparatus
- Utilize the mathematical relationship between electric fields and electric potential.
- Learn how to “measure” the electric field using only voltage measurements.

BACKGROUND [1]
I. Electric potential and electric field
   a. Any arrangement of static charges produces an electric field in its vicinity. Based on Coulomb’s Law and the principle of superposition, it is possible to (theoretically) calculate the field produced by a set of charges. In practice, though, it is mathematically very difficult to do so for all but the simplest charge distributions. And even in the simple configurations, the task is difficult because of the vector nature of the electric field.
   b. Electric potential, on the other hand, is a scalar, which makes it much easier to work with. And the best part is, the electric potential contains all of the same information as the electric field—if you know the potential, you can calculate the field, and vice versa. The term voltage is often used as a synonym for “potential difference.”
      i. If you know \( V(x, y, z) \) then the electric field is the vector
         \[
         \vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}
         \]
      ii. If you know the electric field, then you can calculate \( V \) anywhere by taking the line integral of the \( E \) field. For any two points A and B,
         \[
         V(B) - V(A) = -\int_A^B \vec{E} \cdot d\vec{s}.
         \]
      iii. You can arbitrarily choose any one point to set \( V = 0 \), but then the line integral determines \( V \) everywhere else:
         \[
         V(\text{any point } P) = -\int_{\text{the place where } V=0}^P \vec{E} \cdot d\vec{s}.
         \]
II. Visualizing the electric field and electric potential
   a. It can be difficult to develop a useful intuitive understanding of the electric field, which is a vector quantity that is a function of position, i.e. it can be different at every point in space. One way to visualize the electric field near an arrangement of charges is to draw zillions of tiny arrows, filling the page, each of which points in the direction of the electric field in its vicinity. In Figure 1, the strength of the electric field is indicated by the brightness, which is difficult to reproduce in black and white. In the 19th century, Michael Faraday came up with
a better plan: draw long, continuous arrows, which point along the direction of the electric field (curving, if necessary, as the $\mathbf{E}$ field changes direction). The strength of the $\mathbf{E}$ field is indicated by the how close the electric field lines are to each other; see Figure 2 [2].

The situation gets more complicated, of course, if there are multiple charges. Below are examples [3].

b. Visualizing the electric potential is a little bit easier, since we only need to think about one number at every point instead of a whole vector. For example, here is the same charge distribution as Figure 4, with the potential indicated by color: “warm” colors indicated high potential and “cool” colors indicate low potential [2].

c. The above image won’t look very good if you print it out in black and white. Also, it’s very difficult to make such an image without a computer. However, there is an easier way to
represent the potential: by drawing **equipotential surfaces** (in 3 dimensions) or **equipotential lines** (in 2 dimensions) [2].

Equipotential lines, or contour lines, are lines of constant electric potential. So for instance, you might have one equipotential which includes all locations where \( V = 0 \), and then another equipotential line that includes all places where \( V = +1 \text{ Volt} \). The image above could be rendered with equipotential lines drawn as shown in Figure 6. (Note: the contour lines very close to the point charges themselves have been suppressed. Otherwise, there would be infinitely many of them in the small region around each charge.

**d.** Contour lines are *equally spaced in terms of potential difference*, but that is definitely not the same thing as saying that they are equally spaced on the page. In particular, regions where the contour lines are very close together (look at the region directly between the two charges in Figure 6 are regions where the potential is changing rapidly with position. But mathematically, saying that the potential is changing rapidly with position is equivalent to simply saying that the **magnitude of the electric field is large**. Indeed, the \( \mathbf{E} \) field between two opposite charges is strongest directly between them, and weaker further away or off-axis—and in the figure above, the contour lines are closely bunched between the charges, and spread further apart off-axis, far away, or “behind” one charge or the other.

**e.** If you’ve ever used a trail map while hiking, you’ve seen contour lines before. In topographical maps, instead of representing places of equal electric potential, the contour lines represent places of equal elevation. If you walk along a contour line you stay at constant elevation. If you walk perpendicular to a contour line, you are going either straight uphill or straight downhill. Below is a topographical map showing the hike to Half Dome in Yosemite National Park [4].
f. Play with the following online applet to get a feeling for what the electric field lines and contour lines look like and how they are related to each other
   
   i. [http://www.falstad.com/emstatic](http://www.falstad.com/emstatic)
   
   ii. The “Show E lines” is particularly useful for seeing the electric field lines; the arrows give the directions of the electric field and the color of the arrows indicates the electric field strength (brighter = stronger). ‘rho’ is the charge density within a conductor. The most useful configurations at this stage are those that do not include dielectrics or current. Make sure to play around with the double charge, dipole charge, and the conducting planes.

**REFERENCES**


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**PRE-LAB**
1. Read through the entire lab handout.
2. Answer the following questions based on the applicable sections of your textbook and your observations from the online applet referenced in the Background section of this handout.
   a. What are three properties of electric field lines?
   b. In what way are equipotential lines oriented with respect to the electric field lines?
   c. Why must the \( E \) field be perpendicular to the surface of an ideal conductor?
   
   ![Plate and Circular Charge](image)

   d. Based on equipotential contour lines shown in the textbook, sketch your prediction of the equipotential contour lines for the following charge configuration. Assume the plate is at a higher potential than the circular charge.
e. If the electric potential is uniform throughout a region of space, what can be said about the electric field in that region?

f. In what direction can you move relative to an electric field so that the electric potential does not change?

g. A uniform electric field is in the \(-x\) direction. Points \(a\) and \(b\) are on the \(x\)-axis, with \(a\) at \(x = 2.00\, \text{m}\) and \(b\) at \(x = 6.00\, \text{m}\)
   
   i. Is the potential difference \(V_b - V_a\) positive or negative? Explain.

   ii. If \(|V_b - V_a|\) is 100 kV, what is the magnitude of the electric field? Show your work.
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EQUIPMENT SETUP
1. The power supply and the multimeter should be off at the beginning of the lab.
2. You will be mapping equipotential contour lines for the three different charge configurations shown below.

![Diagram of charge configurations]

3. Attach a blank sheet of paper to the top of the CENCO Overbeck Electric Field Mapping apparatus. By pressing down on the board, you can secure the paper under the rubber mounts near the corners of the board. Do not reposition the paper once you’ve started mapping equipotentials.
4. Position the plastic template for the configuration at your station over the blank paper using the alignment pegs at the top of the apparatus board. Trace the appropriate outline, then place the template to the side.
5. WITH THE POWER SUPPLY OFF, connect a black lead from the negative (−) output from the power supply to the electric binding post on the left-hand side of the apparatus, and connect a red lead from the positive (+) output from the power supply to the electric binding post on the right-hand side of the apparatus. Connect the black lead from the common (COM) plug on the multimeter to the long
U-shaped probe. Connect the red lead from the Voltage plug on the multimeter lead to E1 jack at the top of the board.

6. Position the U-probe with one arm passing under the apparatus to make an electrical contact with the resistance board and the multimeter and the other arm passing on top; the silver knob and the hole for a pencil to pass through to mark the paper should be on top. **Note: do not squeeze the U-probe to squish the apparatus; this damages the resistance board underneath. Always move the probe using the silver knob on top.**

**PROCEDURE**

7. Turn the power supply on and set the voltage to approximately 5 Volts. The power supply maintains that potential difference between the two electrical binding posts, which are connected to the various conducting areas on the apparatus boards.

8. With the red multimeter lead connected to the E1 jack, turn on the multimeter to the 200 V setting.

9. Measuring and marking equipotential contour lines.
   a. **Measure the equipotential value:** Using the probe and the multimeter, measure the potential difference between E1 and one of the charged regions you traced with the template. Make a note of the potential difference, $\Delta V$, shown on the multimeter and remember which charged region you reference.
   
   b. **Marking the equipotential contour lines:** Move the probe (using the silver knob) until you find a position where the multimeter reading is zero and mark that position on the paper (there is a hole on the top of the U-probe for this purpose). Find 4 – 5 other positions, separated by 1 – 2 centimeters, where the multimeter reading is also zero. Connect these dots by a smooth, dashed line and write down the potential difference measured in part (a) near the end of the line. You have mapped your first equipotential for this charge configuration.

   c. Repeat steps (a) and (b) for the remaining jacks E2 through E7, making sure to reference the same charged region when measuring the potential difference for that jack.

   d. Turn off the power supply and the multimeter after taking your measurements.

10. Follow steps 1 through 9 for the other two charge configurations.

**DATA AND CALCULATIONS**

11. Electric Field Lines:
   a. Using solid lines draw electric field lines for each charge configuration, including arrows on the field lines to show the direction of the electric field. Explain here how you know how to draw the fields.

   b. Mark locations A – L on your configurations, as shown below, on the electric field lines you sketched in part (a), making sure the marks are between two equipotential lines. Using your data and a ruler, **estimate** the magnitude of the electric field at each. Hint: $E_x = -\frac{\Delta V}{\Delta x}$. Units of $V/m$ are especially convenient for the electric field in this case. Show your work and fill in
the values in the table below. Split up the work evenly, indicating who calculated that point in the table.

| Configuration               | Location | $|E|$ | Initials for calculation |
|-----------------------------|----------|-----|--------------------------|
| Paired circular charges     | A        |     |                          |
|                             | B        |     |                          |
|                             | C        |     |                          |
| Parallel plate capacitor    | D        |     |                          |
|                             | E        |     |                          |
|                             | F        |     |                          |
|                             | G        |     |                          |
|                             | H        |     |                          |
| Plate and circular charge   | I        |     |                          |
|                             | J        |     |                          |
|                             | K        |     |                          |
|                             | L        |     |                          |
ANALYSIS

12. How did your prediction of the equipotential contour lines of the plate and circular charge compare to your measurement? Explain any differences.

13. Why is it important to obtain a zero (or null) reading on the multimeter in order to establish an equipotential point on the paper?

14. What feature(s) of the equipotential map indicates where the electric field is the strongest? Why? Is the electric field strongest where you expect it to be?

15. According to theory, the electric field for the parallel plate configuration should be a uniform field. Is this consistent with your experimental result? Clearly state the evidence for your answer.
16. What do you consider the most helpful guide when attempting to determine the shape of the electric fields for the different charge configurations?

17. Is it possible for electric field lines to cross? Explain.

FINISHING UP
Clean up your area. Return all materials to their appropriate places, and make sure the power supplies and the multimeters are turned off.
**POST-LAB QUESTIONS**

18. Are the electric field representation and the equipotential contour representation equivalent in terms of how much information they contain about the electric field?

19. In figure 8, at which of these two points, X or Y, is the electric field stronger? How do you know?

20.
   a. In question 18, you compared the electric field at two points on the same equipotential line. Now consider two points on different equipotential lines. In figure 9, where is the field stronger—point W or point Z? How do you know?
b. Sketch on figure 9 the direction in which a positive charge placed at point $Z$ would move. Sketch the direction in which a negative charge placed at point $W$ would move.

21. The gray charge distribution shown generates an electric field corresponding to the following equipotential surfaces. The potentials at points $A$ and $B$ are $V_A = 3.0 \text{ V}$ and $V_B = 1.0 \text{ V}$.

   a. On this diagram, sketch some of the electric field lines resulting from the charge distribution. Is the charge distribution positive or negative? (Yes, you have enough information to tell.)

   b. Where is the electric field strongest? Explain.

   c. How much work would it take to move a $Q = 0.50 \text{ C}$ point charge along a straight line from $B$ to $A$?

   d. Now consider a semicircular path from $B$ to $A$. To move the $Q = 0.50 \text{ C}$ charge along this path, would it take more work, less work, or the same work, as compared to part (c)? Explain.

   e. Which takes more work: Moving charge $Q$ from point $C$ to point $A$, or moving it from point $B$ to point $A$? Justify your answer.
22. Challenge Puzzle: Based on the electric field shown, what would be the charge distribution?
   a.
   b.