Math383 Test 1 Instructions:

Read the following instructions carefully:

• Write each problem on a separate page.

• Write the number of the problem, and your name, in the top right corner of your answer sheet.

• You may NOT use any calculators on this exam.

• It is important to show your work for each problem. Credit will NOT be given for correct answers without justification. Also, partial credit will be given for incorrect answers if some of the work is correct.

• Clearly mark out (cross out) any work that you are not including in your answer and you do not want graded.

• Be sure to staple your exam at the end and hand it in.

• Sign the Honor Pledge after you finish the exam.

• Good Luck.

Honor Pledge

I have neither given nor received any unauthorized help on this test, and I have conducted myself within the guidelines of the university honor code.

Pledge: __________________________________________________________
1. (15 points total) Give a short answer to each of the following questions.

(a) (2 points) Is the following equation linear or nonlinear?

\[ \sin^2(t) \frac{d^3y}{dt^3} + (t^2 + y) \frac{d^2y}{dt^2} + t^2 \frac{dy}{dt} = \cos(t) \]  

(b) (2 points) What is the order of the following equation?

\[ \sin^2(t) \frac{d^3y}{dt^3} + (t^2 + y) \left( \frac{d^2y}{dt^2} \right)^4 + t^2 \frac{dy}{dt} = \cos(t) \]  

(c) (3 points) Is \( y(t) = \sin(t) \) a solution to the following equation?

\[ \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + e^t y = (e^t - 1) \sin(t) + 2 \cos(t) \]  

(d) (2 points) What are the two components which compose an initial value problem?

(e) (2 points) What is the general form of a linear first order differential equation?

(f) (2 points) What is the general form of a first order differential equation?

(g) (2 points) What is the general form of an autonomous first order differential equation?

2. (22 points total) A tank in the lab has capacity of 100 gallons. Initially, the tank has 100 gallons of water with 10 lbs of salt mixed in solution. Water containing 3 lbs of salt per gallon flows into the tank at a rate of 5t gallons per minute. The mixture in the tank flows out at the same rate (after being well mixed).

(a) (9 points) Give an initial value problem for the rate of change of salt in the tank.

(b) (9 points) Solve for the amount of salt in the tank at any time \( t \).

(c) (4 points) Find the limiting amount of salt in the tank as \( t \to \infty \).

3. (20 points total) Given the initial value problem

\[ t \frac{dy}{dt} + 2y = e^{-2t} \]  

\[ y \left( \frac{1}{2} \right) = 0. \]  

(a) (15 points) Solve the initial value problem using the method of integrating factors. You must show how the integrating factor is derived and NOT just use the formula. Also, do NOT just use a formula for the solution but show how you solved the equation by using the integrating factor.

(b) (5 points) On what interval are we guaranteed to have a unique solution? Make sure to justify your answer.
4. (23 points total) Given the differential equation

\[ \frac{dy}{dt} = y^2 - 5y + 6. \]  

(a) (4 points) Find the critical (equilibrium) points.

(b) (7 points) Determine the stability of these equilibrium points and draw the phase line.

(c) (8 points) Determine where the graph of \( y \) versus \( t \) is concave up and where it is concave down.

(d) (4 points) Sketch the graph of \( y \) versus \( t \).

5. (20 points total) Given the differential equation (the same as in problem (4))

\[ \frac{dy}{dt} = y^2 - 5y + 6. \]  

(a) (14 points) Find the general solution. The solution should be given explicitly \( y(t) = \ldots \).

(b) (6 points) Discuss how the solution agrees with the plot you drew in 4(d) as \( t \to \infty \).

Bonus

(1 point) What is something you do for fun?

(1 point) What is a band that you enjoy listening to?

(1 point) What do you want to be doing in 10 years?