

# Homework 3

Due before class February 28, 2017

1. Read Pinchover and Rubenstein Section 2.7 in detail.
2. Work through Shearer and Levy Chapter 13 Example 3 in detail.
3. Shearer and Levy Chapter 13: 5 and 7
4. Show that

$$u(x, t) = \begin{cases} u_L & x < st \\ u_R & x > st \end{cases} \quad (1)$$

where  $s = (u_L + u_R)/2$  is a weak solution to Burgers' equation (Riemann problem) by showing that

$$\int_0^\infty \int_{-\infty}^\infty \left[ \phi_t u + \phi_x \left( \frac{1}{2} u^2 \right) \right] dx dt = - \int_{-\infty}^\infty \phi(x, 0) u(x, 0) dx \quad (2)$$

for all  $\phi \in C_0^1$ . Consider both  $u_L > u_R$  and  $u_L < u_R$ . (from LeVeque)

5. Show that

$$u(x, t) = \begin{cases} u_L & x < s_m t \\ u_m & s_m t \leq x \leq u_m t \\ x/t & u_m t \leq x \leq u_r t \\ u_R & x > u_r t \end{cases} \quad (3)$$

is a weak solution of Burgers' equation (Riemann problem) for any  $u_m$  with  $u_L \leq u_m \leq u_r$  and  $s_m = (u_L + u_m)/2$  where  $u_R > u_L$ .

6. Pinchover and Rubenstein Chapter 3: 3.1, 3.2, 3.5, 3.7, 3.9, 3.10
7. Levy and Shearer Chapter 2: 1

Note that this is the minimum number of problems you should be solving. Other problems in Pinchover and Rubenstein Chapter 3, and in Shearer and Levy Chapters 2 and 13 are good practice problems.