

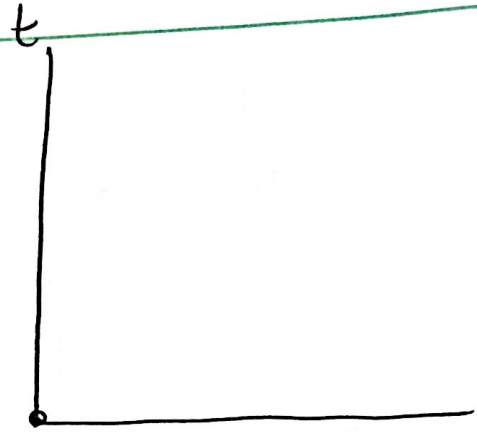
Chap 4 #5

$$u_{tt} = 4u_{xx} \quad x > 0, t > 0$$

$$u(0, t) = 0 \quad t > 0$$

$$u(x, 0) = \phi(x) \quad x > 0$$

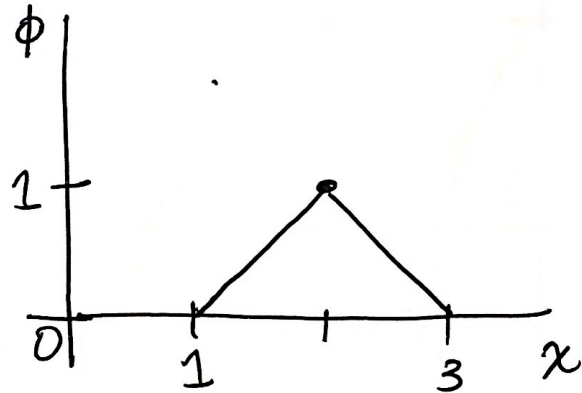
$$u_t(x, 0) = \psi(x) \quad x > 0$$



~~Read the solution in this case~~

$$\psi(x) = 0 \text{ and } \phi(x) \Rightarrow$$

$$\phi(x) = \begin{cases} x-1 & 1 \leq x \leq 2 \\ -x+3 & 2 \leq x \leq 3 \end{cases}$$



The soln can be written in this case as

$$u(x, t) = \begin{cases} \frac{1}{2} [\phi(x+2t) + \phi(x-2t)] & x > 2t \\ \frac{1}{2} [\phi(x+2t) - \phi(2t-x)] & 0 < x < 2t \end{cases}$$

~~to be~~

~~u(x,t) = ...~~

Here, I will compute in detail for one value of  $t$  and then you can look at the matlab file.

$$t = \frac{1}{4}$$

$$u(x, \frac{1}{4}) = \begin{cases} \frac{1}{2} [\phi(x + \frac{1}{2}) + \phi(x - \frac{1}{2})] & x > \frac{1}{2} \\ \frac{1}{2} [\phi(x + \frac{1}{2}) + \phi(\frac{1}{2} - x)] & 0 < x < \frac{1}{2} \end{cases}$$

$$= \begin{cases} \frac{1}{2} [(0) - 0] & 0 < x < \frac{1}{2} \\ \frac{1}{2} [x + \frac{1}{2} - 1 + 0] & \frac{1}{2} < x < \frac{3}{2} \\ \frac{1}{2} [-(x + \frac{1}{2}) + 3 + (x - \frac{1}{2}) - 1] & \frac{3}{2} < x < \frac{5}{2} \\ \frac{1}{2} [-0 + -(x - \frac{1}{2}) + 3] & \frac{5}{2} < x < \frac{7}{2} \\ 0 & x > \frac{7}{2} \end{cases}$$

$$= \begin{cases} 0 & 0 < x < \frac{1}{2} \\ \frac{1}{2}(x - \frac{1}{2}) & \frac{1}{2} < x < \frac{3}{2} \\ \frac{1}{2}(1) & \frac{3}{2} < x < \frac{5}{2} \\ \frac{1}{2}(\frac{7}{2} - x) & \frac{5}{2} < x < \frac{7}{2} \\ 0 & x > \frac{7}{2} \end{cases}$$

