Proof by Induction

Step 1: Show for \( n = 1 \) or \( n = 2 \)
Step 2: Assume true for \( n = k \)
Step 3: Show for \( n = k + 1 \)

Example:
The sum of the first \( n \) integers is \( \frac{n(n+1)}{2} \).

\[
1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}
\]

Proof: By induction on \( n \)
Step 1: \( n = 1 \)

\[
1 = \frac{1(2)}{2} = 1
\]

Step 2: Assume true for \( n = k \)

\[
1 + 2 + 3 + \ldots + k = \frac{k(k + 1)}{2}
\]

Step 3: \( n = k + 1 \)

\[
1 + 2 + 3 + \ldots + k + (k + 1) = \frac{k(k + 1)}{2} + (k + 1)
\]

\[
= \frac{k(k + 1) + 2(k + 1)}{2}
\]

\[
= \frac{k(k + 1) + 2(k + 1)}{2}
\]

\[
= \frac{(k + 2)(k + 1)}{2}
\]

The first line here uses the assumption made in Step 2. The last line gives us that

\[
1 + 2 + 3 + \ldots + k + (k + 1) = \frac{(k + 1)(k + 2)}{2}
\]

which is what the statement we are trying to prove says for \( n = k + 1 \).