CSE 135: Introduction to Theory of Computation
Turing Machines

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    - Come up with a model that describes all "conceivable" computation
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- In this course: Turing Machines

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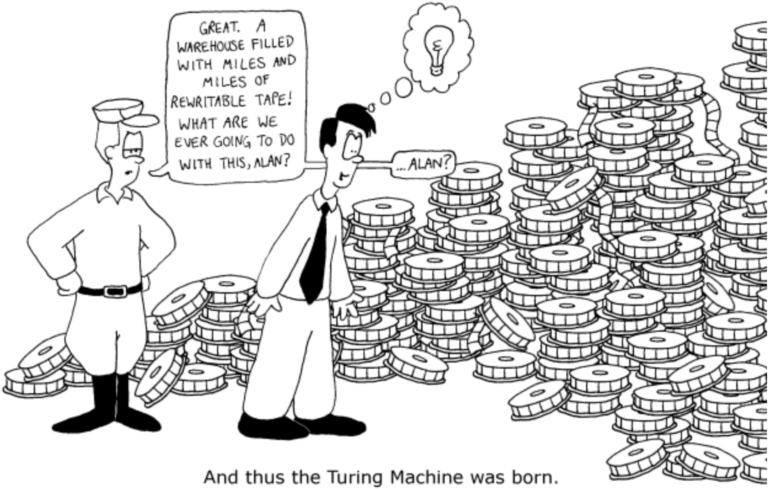
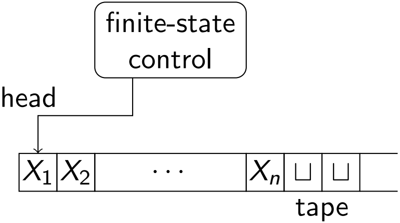


And thus the Turing Machine was born.

Turing Machines



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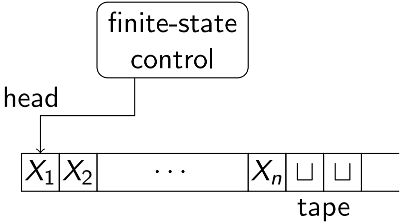
Turing Machines



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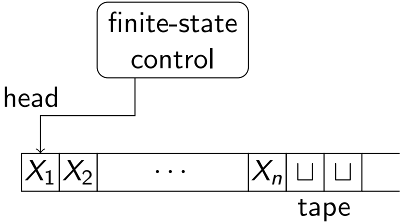

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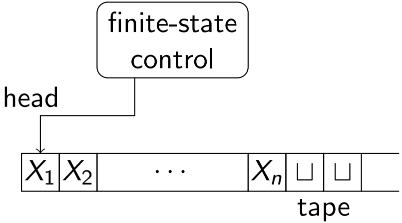

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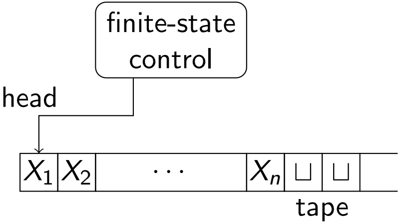

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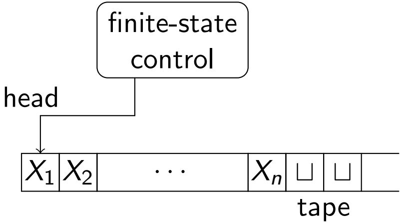

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- Initially, tape has input and the machine is reading (i.e., tape head is on) the leftmost input symbol.
- Transition (based on current state and symbol under head):
  - Change control state
  - Overwrite a new symbol on the tape cell under the head
  - Move the head left, or right.

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Formal Definition

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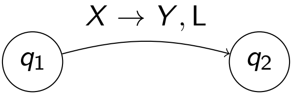
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function. Given the current state and symbol being read, the transition function describes the next state, symbol to be written and direction (left or right) in which to move the tape head.

Transition Function



$\delta(q_1, X) = (q_2, Y, \mathsf{L})$: Read transition as "the machine when in state q_1, and reading symbol X under the tape head, will move to state q_2, overwrite X with Y, and move its tape head to the left"

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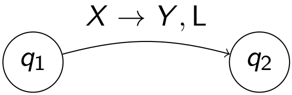

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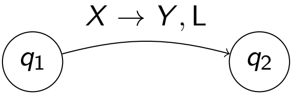

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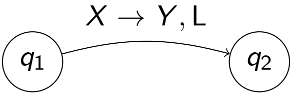

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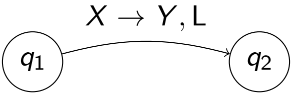

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- Transitions are deterministic
- Convention: if $\delta(q, X)$ is not explicitly specified, it is taken as leading to q_{rej}, i.e., say $\delta(q, X) = (q_{\mathsf{rej}}, \sqcup, \mathsf{R})$

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Single Step

Definition

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Acceptance and Recognition

Definition

A Turing machine M accepts w iff $q_0 w \vdash^* \alpha_1 q_{\text{acc}} \alpha_2$, where α_1, α_2 are some strings. In other words, the machine M when started in its intial state and with w as input, reaches the accept state.

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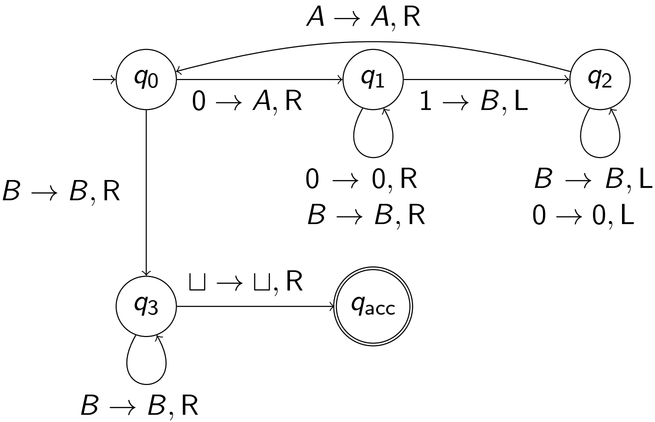
High level description

```
On input string w
    while there are unmarked 0s, do
        Mark the left most 0
        Scan right till the leftmost unmarked 1;
            if there is no such 1 then crash
        Mark the leftmost 1
    done
    Check to see that there are no unmarked 1s;
        if there are then crash
    accept
```

Example 1: TM for $\{0^n1^n \mid n > 0\}$



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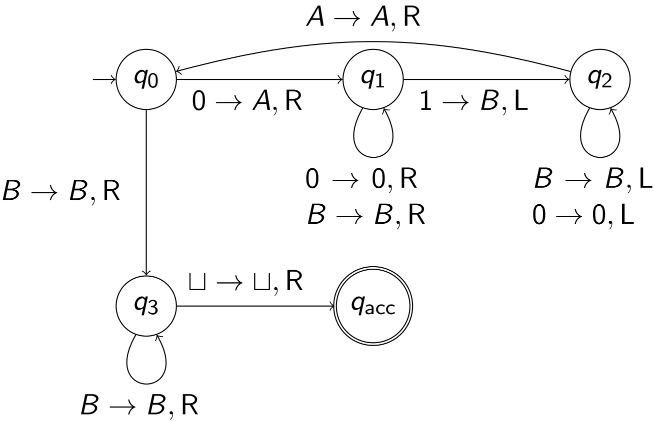

- Accepts input 0011: $q_0 0011 \vdash$

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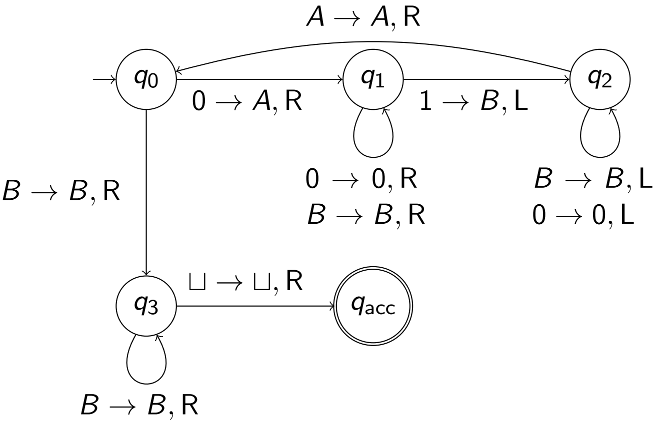

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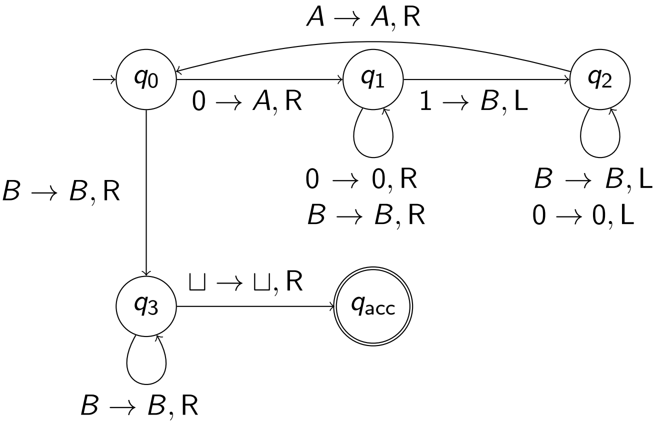

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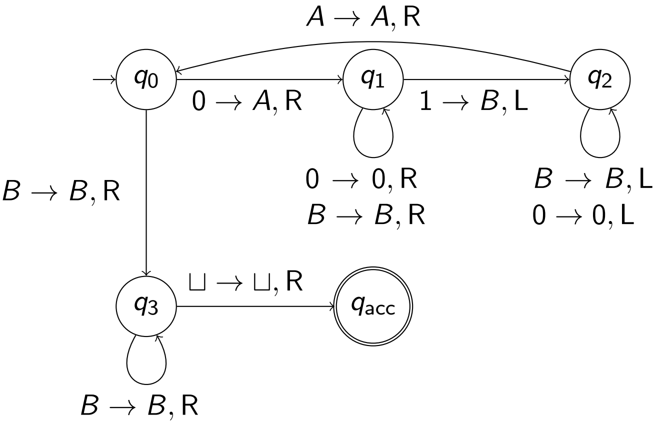

- Accepts input 0011: $q_0 0011 \vdash A q_1 011 \vdash A0 q_1 11 \vdash A q_2 0B1 \vdash$

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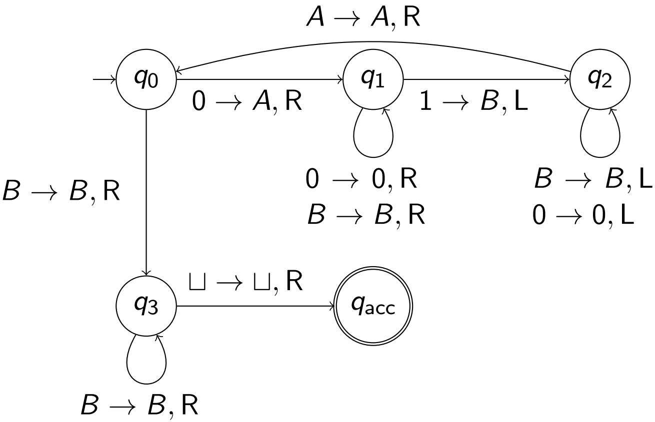

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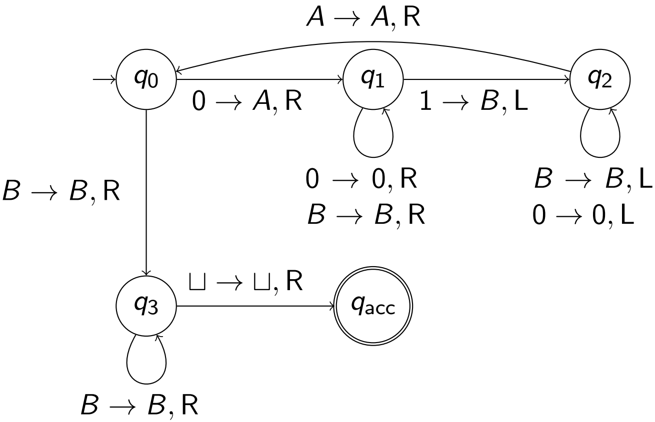

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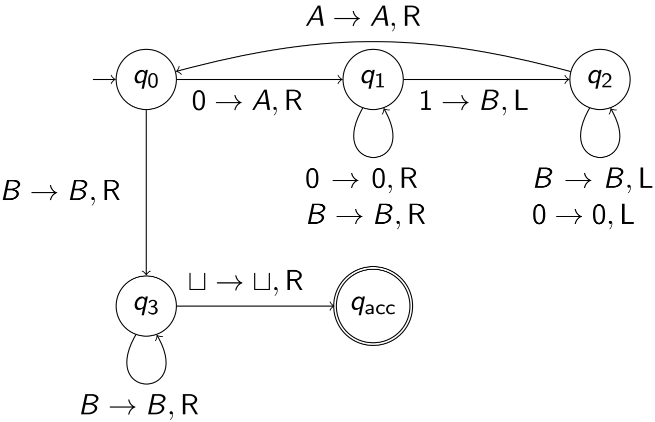

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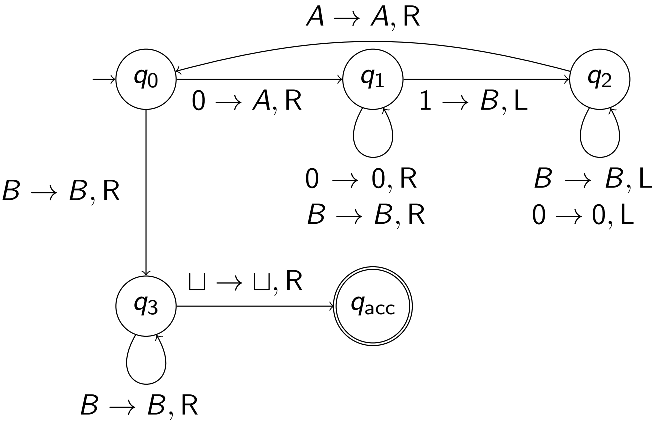

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash AABq_1 1 \vdash$

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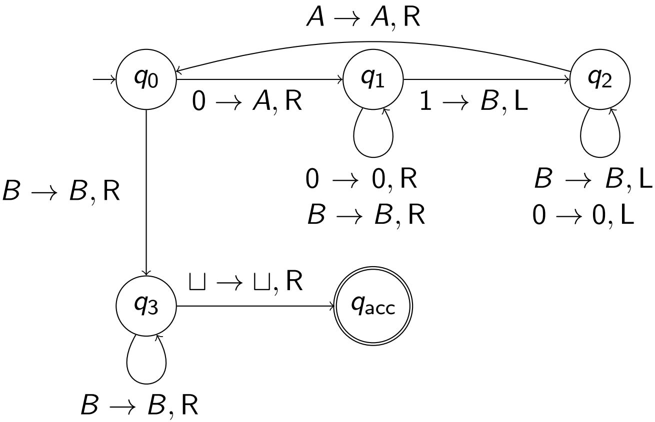

- Accepts input 0011: $q_00011 \vdash Aq_1011 \vdash A0q_111 \vdash Aq_20B1 \vdash q_2A0B1 \vdash Aq_00B1 \vdash AAq_1B1 \vdash AABq_11 \vdash AAq_2BB \vdash$

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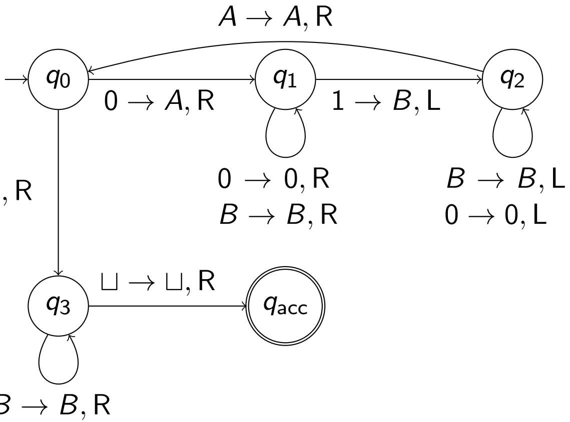

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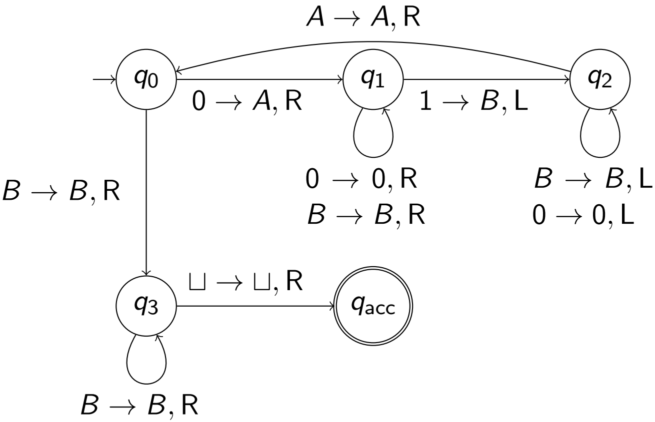

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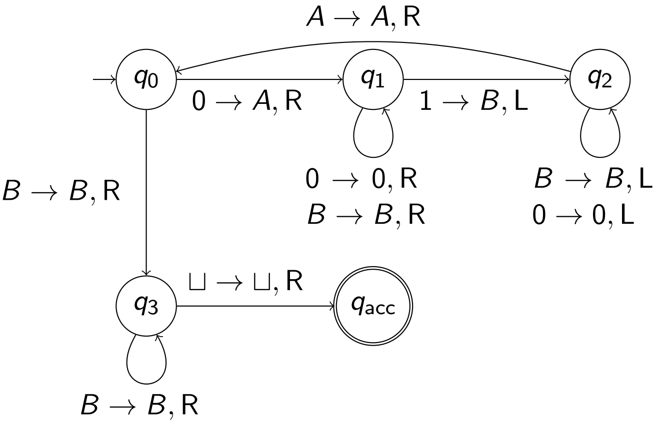

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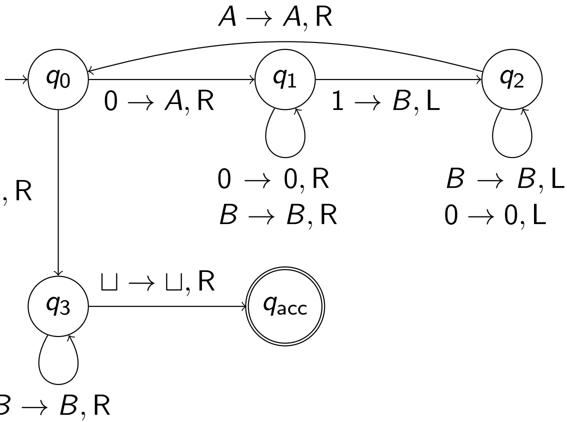

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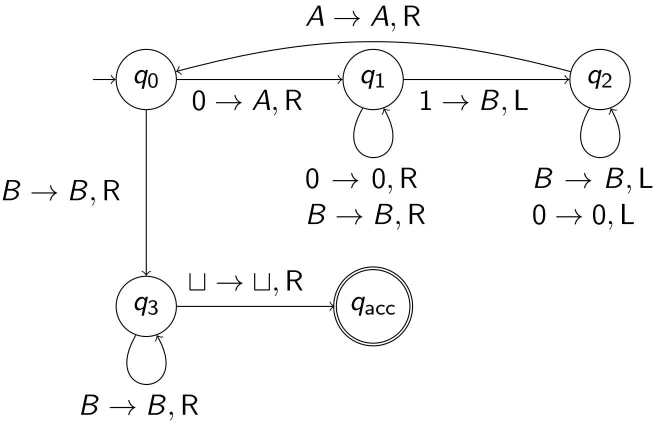

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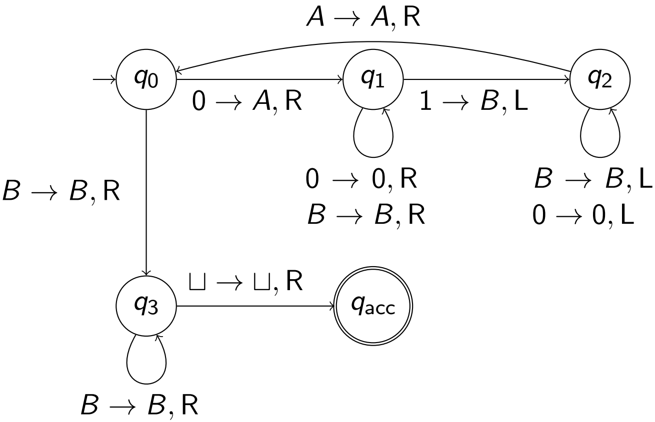

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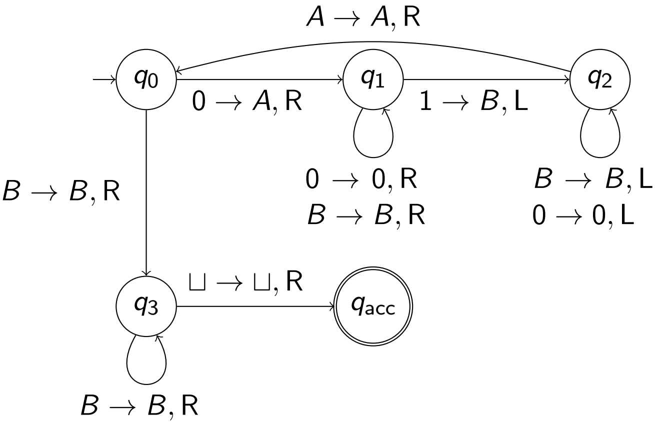

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Formal Definition

The machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ where

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- $\Sigma = \{0, 1\}$, and $\Gamma = \{0, 1, A, B, \sqcup\}$
- δ is given as follows

$$
\begin{array}{ll}
\delta(q_0, 0) = (q_1, A, R) & \delta(q_0, B) = (q_3, B, R) \\
\delta(q_1, 0) = (q_1, 0, R) & \delta(q_1, B) = (q_1, B, R) \\
\delta(q_1, 1) = (q_2, B, L) & \delta(q_2, B) = (q_2, B, L) \\
\delta(q_2, 0) = (q_2, 0, L) & \delta(q_2, A) = (q_0, A, R) \\
\delta(q_3, B) = (q_3, B, R) & \delta(q_3, \sqcup) = (q_{acc}, \sqcup, R)
\end{array}
$$

In all other cases, $\delta(q, X) = (q_{rej}, \sqcup, R)$.

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The machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ where

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\end{array}
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In all other cases, $\delta(q, X) = (q_{\text{rej}}, \sqcup, R)$. So for example, $\delta(q_0, 1) = (q_{\text{rej}}, \sqcup, R)$.

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Design a TM to accept the language $L = \{0^n1^n2^n \mid n > 0\}$

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High level description

```
On input string w
    while there are unmarked 0s, do
        Mark the left most 0
        Scan right to reach the leftmost unmarked 1;
            if there is no such 1 then crash
        Mark the leftmost 1
        Scan right to reach the leftmost unmarked 2;
            if there is no such 2 then crash
        Mark the leftmost 2
    done
    Check to see that there are no unmarked 1s or 2s;
        if there are then crash
    accept
```

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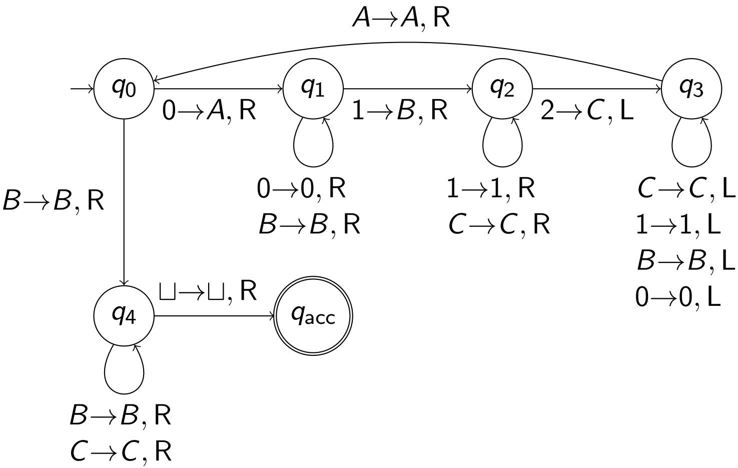

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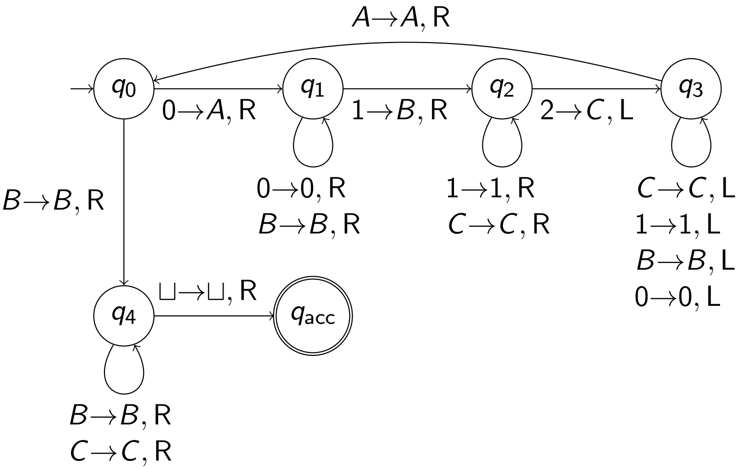

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2$

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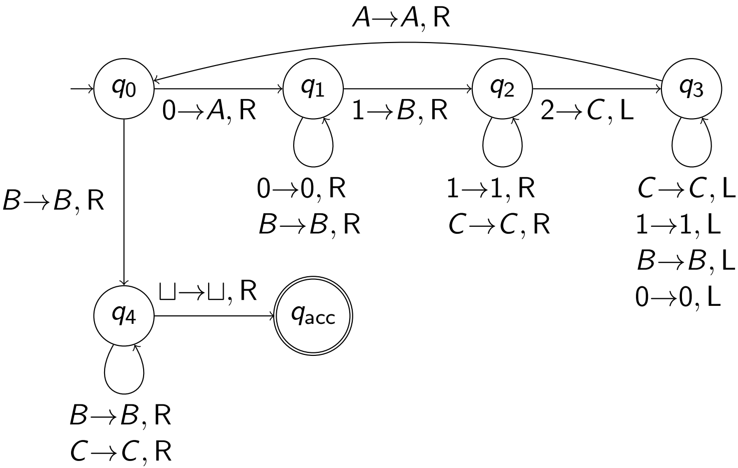

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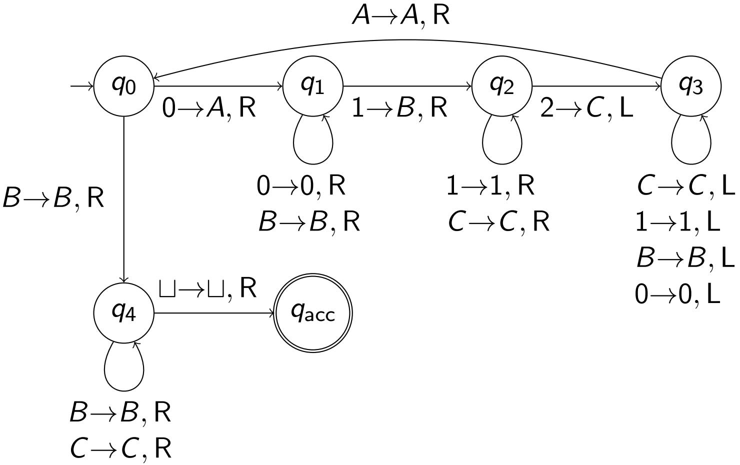

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$

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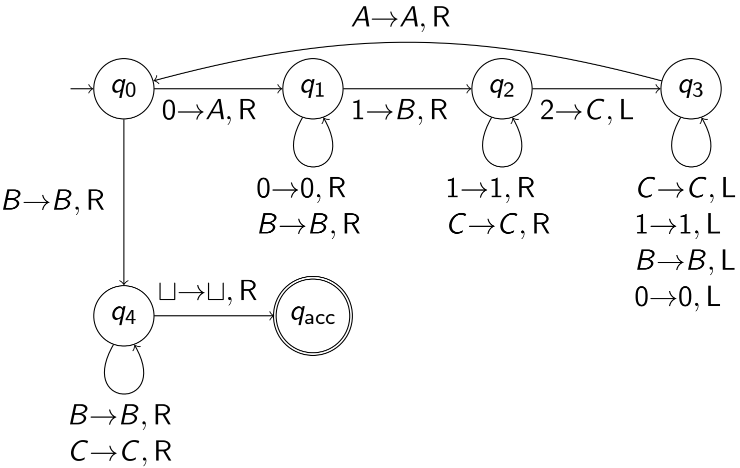

e.g.: $q_0001122 \vdash^* A0Bq_31C2 \vdash^* q_3A0B1C2 \vdash Aq_00B1C2$
$\vdash^* AAq_0BBCC$

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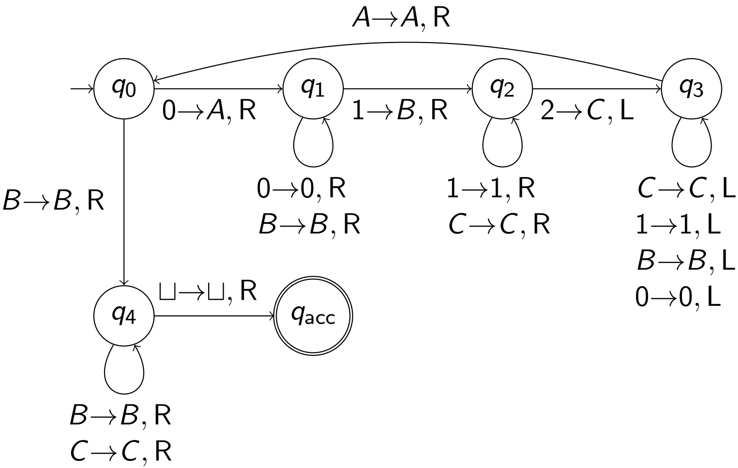

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC \vdash^* AABBCCq_4 \sqcup$

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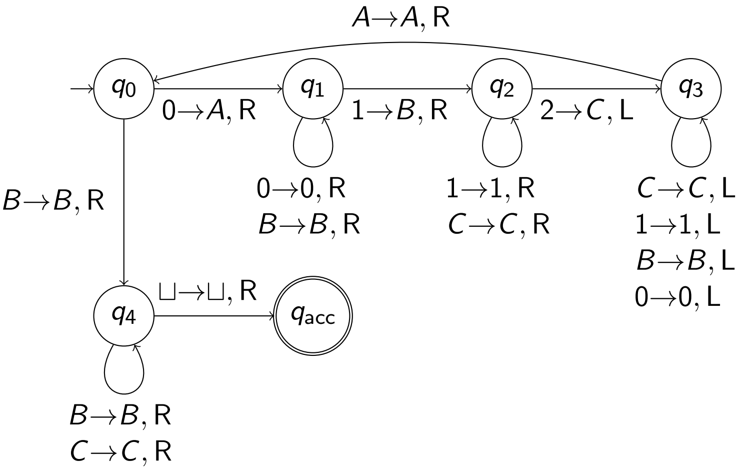

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC \vdash^* AABBCCq_4 \sqcup \vdash AABBCC\sqcup q_{acc} \sqcup$

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Recognizing/Deciding a Language

- Only halting configurations are those with state q_{acc} or q_{rej}
- A Turing machine may keep running forever on some input
- Then the machine does not accept that input
- So two ways to not accept: reject or never halt (loop) Three possible outcomes: acept, reject, or loop

Recognizing a Language

Definition

A language L is said to be Turing-recognizable if there is some Turing machine M that recognizes it.

That is, for every $w \in L$, M must accept w. Also for every $w \in L$, M must not accpet w (either reject it or does not halt).

Deciding a Language

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A Turing machine M is said to decide a language L if $L = L(M)$ and M halts on every input

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A language L is said to be Turing-decidable (or simply decidable) if there is some Turing machine M that decides it.

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Deciding a language is more than recognizing it!

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Deciding a language is more than recognizing it! There are languages which are recognizable, but not decidable.