CSE 135: Introduction to Theory of Computation
Turing Machine’s variants and Church-Turing Thesis

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Multi-Tape Turing Machine

- Initially all heads scanning cell 1, and tapes 2 to $k$ blank.
- In one step: Read symbols under each of the $k$-heads, and depending on the current control state, write new symbols on the tapes, move the each tape head (possibly in different directions), and change state.
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Formal Definition

A $k$-tape Turing Machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ where

- $Q$ is a finite set of control states
- $\Sigma$ is a finite set of input symbols
- $\Gamma \supseteq \Sigma$ is a finite set of tape symbols. Also, a blank symbol $\bot \in \Gamma \setminus \Sigma$
- $q_0 \in Q$ is the initial state
- $q_{\text{acc}} \in Q$ is the accept state
- $q_{\text{rej}} \in Q$ is the reject state, where $q_{\text{rej}} \neq q_{\text{acc}}$
- $\delta : (Q \setminus \{q_{\text{acc}}, q_{\text{rej}}\}) \times \Gamma^k \to Q \times (\Gamma \times \{L, R\}^k)$ is the transition function.
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Computation, Acceptance and Language

- A configuration of a multi-tape TM must describe the state, contents of all $k$-tapes, and positions of all $k$-heads. Thus, $c \in Q \times (\Gamma^*\{\ast}\Gamma\Gamma^*)^k$, where $\ast$ denotes the head position.
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Accepting configuration is one where the state is $q_{\text{acc}}$, and starting configuration on input $w$ is $(q_0, \ast w, \ast \sqcup, \ldots, \ast \sqcup)$.
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$L(M) = \{ w \mid w \text{ accepted by } M \}$
Expressive Power of multi-tape TM

Theorem

For any $k$-tape Turing Machine $M$, there is a single tape TM $\text{single}(M)$ such that $L(\text{single}(M)) = L(M)$.

Challenges

▶ How do we store $k$-tapes in one?
▶ How do we simulate the movement of $k$ independent heads?
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1-tape equivalent single($M$)

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Simulating One Step

**Challenge 1:** Head of 1-Tape TM is pointing to one cell. How do we find out all the $k$ symbols that are being read by the $k$ heads, which maybe in different cells?

▶ Read the tape from left to right, storing the contents of the cells being scanned in the state, as we encounter them.

**Challenge 2:** After this scan, 1-tape TM knows the next step of $k$-tape TM. How do we change the contents and move the heads?

▶ Once again, scan the tape, change all relevant contents, “move” heads (i.e., move ∗s), and change state.
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Overall Algorithm

On input $w$, the 1-tape TM will work as follows.

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   - Read from left-to-right, changing symbols, and moving those heads that need to be moved right.
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   - Read from left-to-right, changing symbols, and moving those heads that need to be moved right.
   - Scan back from right-to-left moving the heads that need to be moved left.
Nondeterministic Turing Machine

**Deterministic TM:** At each step, there is one possible next state, symbols to be written and direction to move the head, or the TM may halt.

**Nondeterministic TM:** At each step, there are finitely many possibilities. So formally, 

\[ M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej}) \]

where

- \( Q \), \( \Sigma \), \( \Gamma \), \( q_0 \), \( q_{acc} \), \( q_{rej} \) are as before for 1-tape machine
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- A single step $\vdash$ is defined similarly. $X_1X_2\cdots X_{i-1}qX_i\cdots X_n \vdash X_1X_2\cdots pX_{i-1}Y\cdots X_n$, if $(p, Y, L) \in \delta(q, X_i)$
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Theorem

For any nondeterministic Turing Machine $M$, there is a (deterministic) TM $\text{det}(M)$ such that $L(\text{det}(M)) = L(M)$.

Proof Idea

$\text{det}(M)$ will simulate $M$ on the input.

▶ Idea 1: $\text{det}(M)$ tries to keep track of all possible "configurations" that $M$ could possibly be after each step. Works for DFA simulation of NFA but not convenient here.

▶ Idea 2: $\text{det}(M)$ will simulate $M$ on each possible sequence of computation steps that $M$ may try in each step.
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Nondeterministic Computation

- If \( r = \max_{q,X} |\delta(q, X)| \) then the runs of \( M \) can be organized as an \( r \)-branching tree.

\( c_{i_1} c_{i_2} \cdots c_{i_n} \) is the configuration of \( M \) after \( n \)-steps, where choice \( i_1 \) is taken in step 1, \( i_2 \) in step 2, and so on.

Input \( w \) is accepted iff \( \exists \) accepting configuration in tree.
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\[
\begin{align*}
C_\epsilon &= q_0w \\
&= C_1 \cdots C_i \cdots C_{ij} \cdots C_{r1} \cdots C_{rr} \\
&= \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots
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- The configuration at any vertex can be obtained by simulating $M$ on the appropriate sequence of nondeterministic choices

Why not a DFS?

Observe that $\text{det}(M)$ may not terminate if $w$ is not accepted.
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The machine $\text{det}(M)$ will search for an accepting configuration in computation tree

- The configuration at any vertex can be obtained by simulating $M$ on the appropriate sequence of nondeterministic choices
- $\text{det}(M)$ will perform a BFS on the tree.
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The machine det($M$) will search for an accepting configuration in computation tree

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- Tape 1, called input tape, will always hold input $w$
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det(\(M\)) will use 3 tapes to simulate \(M\) (note, multitape TMs are equivalent to 1-tape TMs)

- Tape 1, called **input tape**, will always hold input \(w\)
- Tape 2, called **simulation tape**, will be used as \(M\)’s tape when simulating \(M\) on a sequence of nondeterministic choices
- Tape 3, called **choice tape**, will store the current sequence of nondeterministic choices
Execution of \text{det}(M)

1. Initially: Input tape contains \( w \), simulation tape and choice tape are blank
2. Copy contents of input tape onto simulation tape
3. Simulate \( M \) using simulation tape as its (only) tape
   3.1 At the next step of \( M \), if state is \( q \), simulation tape head reads \( X \), and choice tape head reads \( i \), then simulate the \( i \)th possibility in \( \delta(q, X) \); if \( i \) is not a valid choice, then goto step 4
   3.2 After changing state, simulation tape contents, and head position on simulation tape, move choice tape’s head to the right. If Tape 3 is now scanning \( \square \), then goto step 4
   3.3 If \( M \) accepts then accept and halt, else goto step 3(1) to simulate the next step of \( M \).
4. Write the lexicographically next choice sequence on choice tape, erase everything on simulation tape and goto step 2.
Deterministic Simulation

In a nutshell

- $\text{det}(M)$ simulates $M$ over and over again, for different sequences, and for different number of steps.

- If $M$ accepts $w$ then there is a sequence of choices that will lead to acceptance. $\text{det}(M)$ will eventually have this sequence on choice tape, and then its simulation $M$ will accept.

- If $M$ does not accept $w$ then no sequence of choices leads to acceptance. $\text{det}(M)$ will therefore never halt!
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Random Access Machines

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- Initially, the program instructions are stored in a contiguous block of memory locations starting at location 1. All registers and memory locations, other than those storing the program, are set to 0.
Instruction Set

- **add X, Y**: Add the contents of registers X and Y and store the result in X.
- **loadc X, I**: Place the constant I in register X.
- **load X, M**: Load the contents of memory location M into register X.
- **loadI X, M**: Load the contents of the location “pointed to” by the contents of M into register X.
- **store X, M**: Store the contents of register X in memory location M.
- **jmp M**: The next instruction to be executed is in location M.
- **jmz X, M**: If register X is 0, then jump to instruction M.
- **halt**: Halt execution.
Expressive Power of RAMs
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Theorem

Anything computed on a RAM can be computed on a Turing machine.
Expressive Power of RAMs

Theorem

Anything computed on a RAM can be computed on a Turing machine.
Various efforts to capture mechanical computation have the same expressive power.
Robustness of the Class of TM Languages

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- Non-Turing Machine models: random access machines, $\lambda$-calculus, type 0 grammars, first-order reasoning, $\pi$-calculus, ...
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- Restricted Turing Machine models: queue machines, 2-stack machines, 2-counter machines, ...
Church-Turing Thesis

“Anything solvable via a mechanical procedure can be solved on a Turing Machine.”
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- Not a mathematical statement that can be proved or disproved!
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- Not a mathematical statement that can be proved or disproved!
- Strong evidence based on the fact that many attempts to define computation yield the same expressive power
In the course, we will use an informal pseudo-code to argue that a problem/language can be solved on Turing machines.
Consequences

- In the course, we will use an informal pseudo-code to argue that a problem/language can be solved on Turing machines.
- To show that something can be solved on Turing machines, you can use any programming language of choice, unless the problem specifically asks you to design a Turing Machine.