

CSE 135: Introduction to Theory of Computation

Sungjin Im

University of California, Merced

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Nondeterminism

Michael Rabin and Dana Scott (1959)



Michael Rabin



Dana Scott

Nondeterminism

Given a current state of the machine and input symbol to be read, the next state is not uniquely determined.

Comparison to DFAs

Nondeterministic Finite Automata (NFA)

NFAs have 3 features when compared with DFAs.

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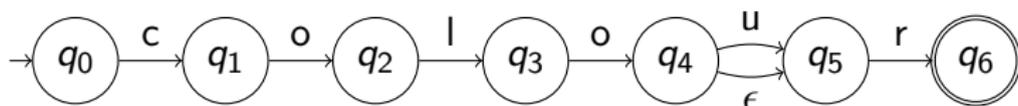
1. Ability to take a step without reading any input symbol
2. A state may have no transition on a particular symbol
3. Ability to transition to more than one state on a given symbol

ϵ -Transitions

Transitions without reading input symbols

Example

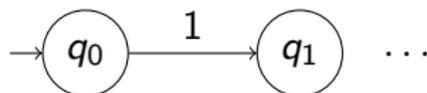
The British spelling of “color” is “colour”. In a web search application, you may want to recognize both variants.



NFA with ϵ -transitions

No transitions

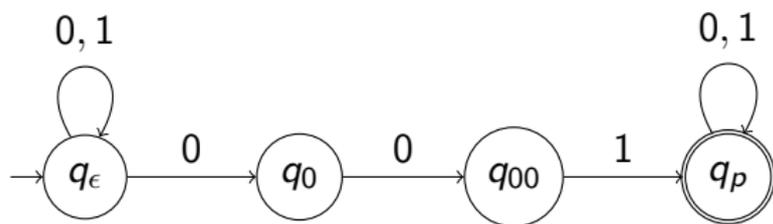
Example



No 0-transition out of initial state

In the above automaton, if the string starts with a 0 then the string has no computation (i.e., rejected).

Multiple Transitions



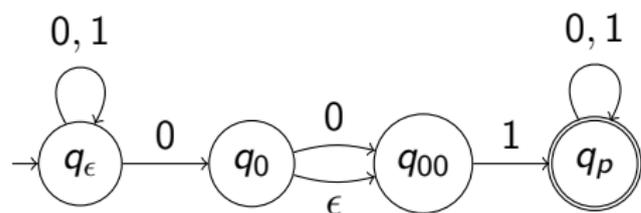
q_ϵ has two 0-transitions

Parallel Computation View

At each step, the machine “forks” a thread corresponding to one of the possible next states.

- ▶ If a state has an ϵ -transition, then you fork a new process for each of the possible ϵ -transitions, without reading any input symbol
- ▶ If the state has multiple transitions on the current input symbol read, then fork a process for each possibility
- ▶ If from current state of a thread, there is no transition on the current input symbol then the thread dies

Parallel Computation View: An Example



Example NFA

Nondeterministic Acceptance

Parallel Computation View

Input is **accepted** if after reading all the symbols, one of the live threads of the automaton is in a final/accepting state.

Nondeterministic Acceptance

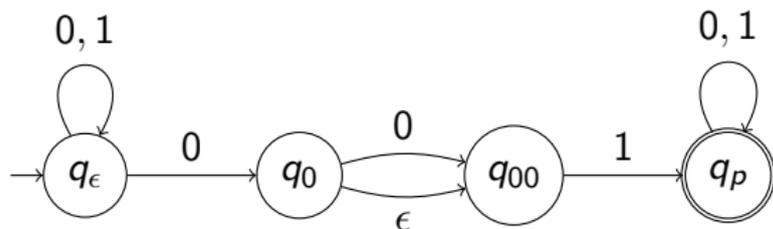
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0100 is accepted because one thread of computation is

$$q_\epsilon \xrightarrow{0} q_0 \xrightarrow{\epsilon} q_{00} \xrightarrow{1} q_p \xrightarrow{0} q_p \xrightarrow{0} q_p$$

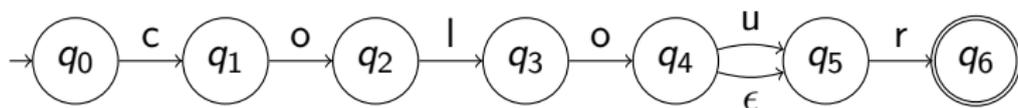
Computation: Guessing View

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Example



NFA M_{color}

After seeing “colo” the automaton guesses if it will see the british or the american spelling. If it guesses american then it moves without reading the next input symbol.

Observations: Guessing View

- ▶ If there is a sequence of choices that will lead to the automaton (not “dying” and) ending up in an accept state, then those choices will be magically guessed

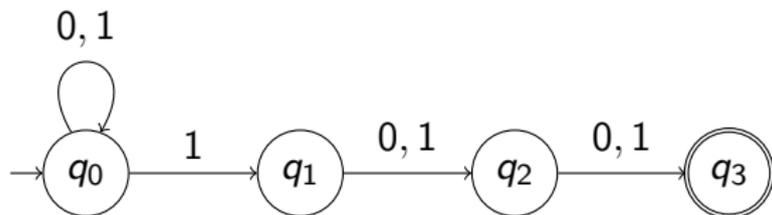
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- ▶ If there is a sequence of choices that will lead to the automaton (not “dying” and) ending up in an accept state, then those choices will be magically guessed
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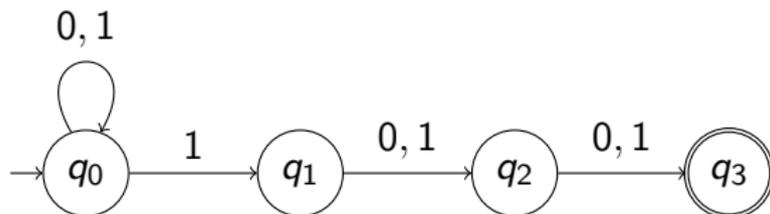
Observations: Guessing View

- ▶ If there is a sequence of choices that will lead to the automaton (not “dying” and) ending up in an accept state, then those choices will be magically guessed
- ▶ On the other hand, if the input will not be accepted then no guess will lead the to automaton being in an accept state
 - ▶ On the input “colobr”, whether automaton M_{color} guesses british or american, it will not proceed when it reads ‘b’.

Example 1



Example 1



Automaton accepts strings having a 1 two positions from end of input

The automaton “guesses” at some point that the 1 it is seeing is 2 positions from end of input.