EECS 279: Approximation Algorithms | Release: 09-23-2014
Homework 2-B | Due: 3:00pm, Thursday, 10-02-2014

Make your solutions concise and formal. Your goal is to convince me that you know the solutions. You are highly encouraged to typeset your solutions in Latex. See the course website for further homework policies. Please write down your collaborators names and references if any.

1. In the k-center problem, we are given a complete graph $G = (V, E)$ where the distances on the edges satisfy the metric property ($d_{i,j} + d_{j,k} \geq d_{i,k}$ for all $i, j, k \in V$), $d_{i,i} = 0$ for all $i \in V$ and $d_{i,j} = d_{j,i}$ for all $i, j \in V$. Let $d(S, i) = \min_{j \in S} d_{i,j}$. The goal is to choose a set S of k points such that $\max_{i \in V} d(S, i)$ is minimized. Consider the following algorithm that is a variation of the algorithm we considered in class. Set $S = \emptyset$. The algorithm first adds an arbitrary vertex to S. Then it iteratively adds verticies to S until S consists of k vertices. In each step, it adds *any* vertex i such that $d(S, i) \geq \frac{1}{2} \max_{j \in V} d(S, j)$. Note that the algorithm in class adds a vertex i such that $d(S, i) = \max_{j \in V} d(S, j)$. Show that this new algorithm is a 4-approximation for k-center.

2. In the multiway cut problem, we are given a graph $G = (V, E)$ where each edge $e \in$ has a nonnegative weight w_e. We are also given a set $S \subseteq V$ of terminals. The goal is to find a subset of edges of the graph E' which disconnects every vertex in S from each other and $\sum_{e \in E'} w_e$ is minimized. In class we saw two approximation algorithms for this problem. Consider the following variant of the algorithm shown in class. The algorithm chooses an *arbitrary* vertex $v \in S$ and finds the minimum weight set of edges E' in G that disconnects v from all other terminals in S. The algorithm then recurses on $G' = (V, E \setminus E')$ and $S \setminus \{v\}$. Show the best possible approximation ratio for this algorithm.

3. Consider the following variant of the bin packing problem. There is a set U of n items where each item has a size s_i and a weight w_i. Unlike the bin packing problem discussed in class, now an item can be rejected. If an item i is rejected the algorithm has to pay a cost of w_i. All items not rejected must be packed into bins of size 1. The goal is to find a set of items S and pack them into m bins to minimize $m + \sum_{i \in U \setminus S} w_i$. Show a constant factor approximation for this problem.