EECS 279: Approximation Algorithms | Release: 09-03-2014
Homework 1-A | Due: 3:00pm, Thursday, 9-11-2014

Make your solutions concise and formal. Your goal is to convince me that you know the solutions. You are highly encouraged to typeset your solutions in Latex. See the course website to further homework policies.

1. In the class, we learned that the algorithm MST-TSP is a 2-approximation for Metric-TSP. Prove that MST-TSP is in fact a $2(1-1/n)$-approximation where n is the number of vertices.

2. Show that there is no "additive" 100-approximation algorithm A for TSP unless P = NP. We say that an algorithms is a α additive approximation if the solution produced by the algorithm has cost at most $\mathrm{OPT}(I) + \alpha$ for every instance I. (E.g. algorithm A finds a Hamiltonian cycle with cost at most $\mathrm{OPT}(I) + 100$ for all instances I).

3. The Partition problem is defined as follows.

   Partition Problem
   Input: positive integers $a_1, a_2, ..., a_n$ such that $\sum_{i=1}^{n} a_i$ is even.
   Goal: To decide if there is a partition of $\{1, 2, ..., n\}$ into S and T such that $\sum_{i \in S} a_i = \sum_{i \in T} a_i$.

   Show that Partition is *weakly* NP-complete by showing the following:

   (a) Give an (exact) algorithm with running time $O(n \sum_{i=1}^{n} a_i)$ for Partition.
   (b) Show that Partition is in NP.
   (c) Show that Partition is NP-complete via a reduction from Subset-Sum. The input to the Subset-Sum consists of n positive integers $b_1, b_2, ..., b_n$, and a target integer B. The goal is to decide if there is a subset S of $\{1, 2, ..., n\}$ such that $\sum_{i \in S} b_i = B$. Subset-sum is known to be NP-hard.

4. Show that the following problem is NP-complete: A *double-Hamiltonian* circuit in an undirected graph G is a closed walk that visits every vertex in G exactly twice. Given a graph G, does G have a *double-Hamiltonian* circuit?

   You can use the fact that the Hamiltonian Path (or Cycle) problem is NP-hard where the goal is to decide if a given graph has a Hamiltonian path (or cycle) or not; we say a path is Hamiltonian if it visits every vertex exactly once.