Compressive Coded Aperture Superresolution Image Reconstruction

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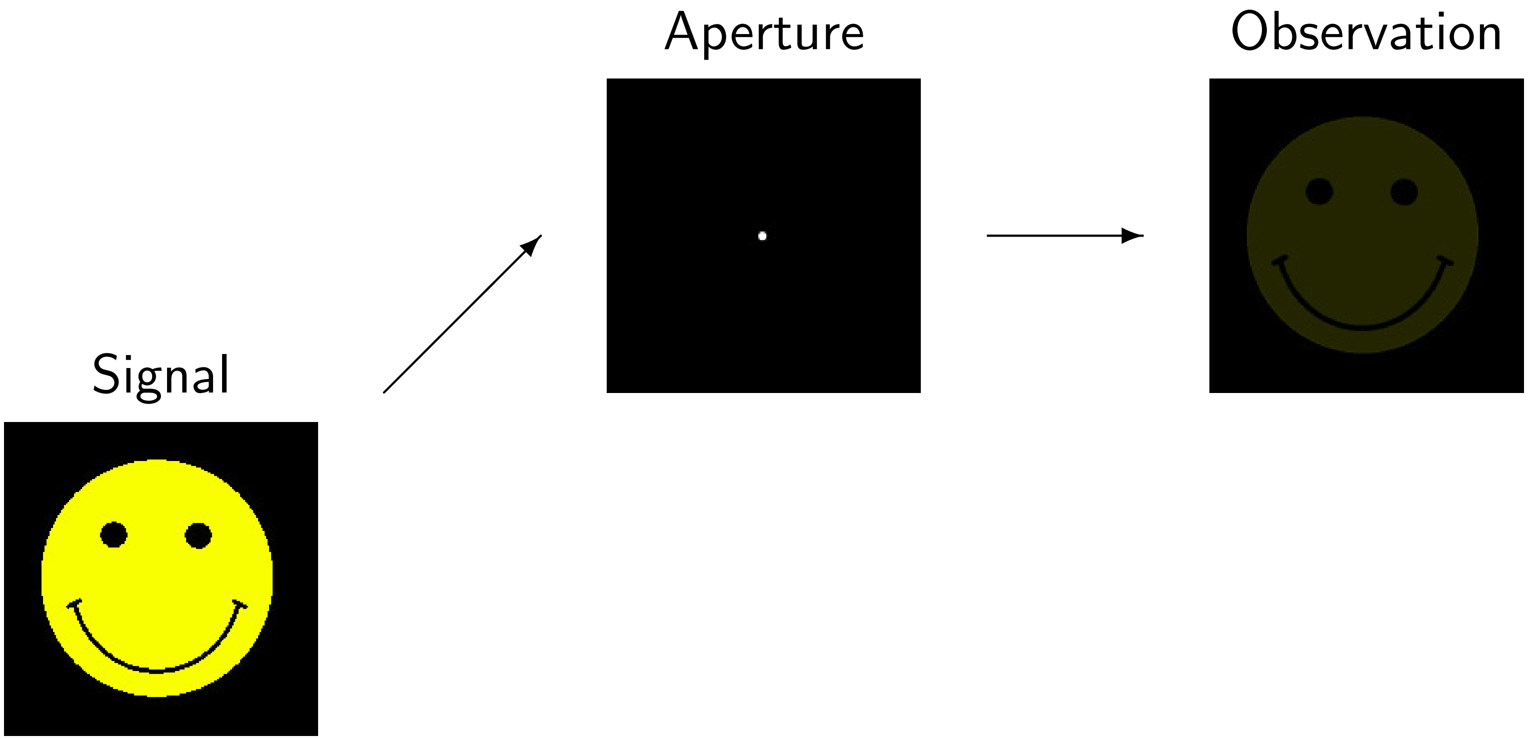
Aperture imaging

Signal

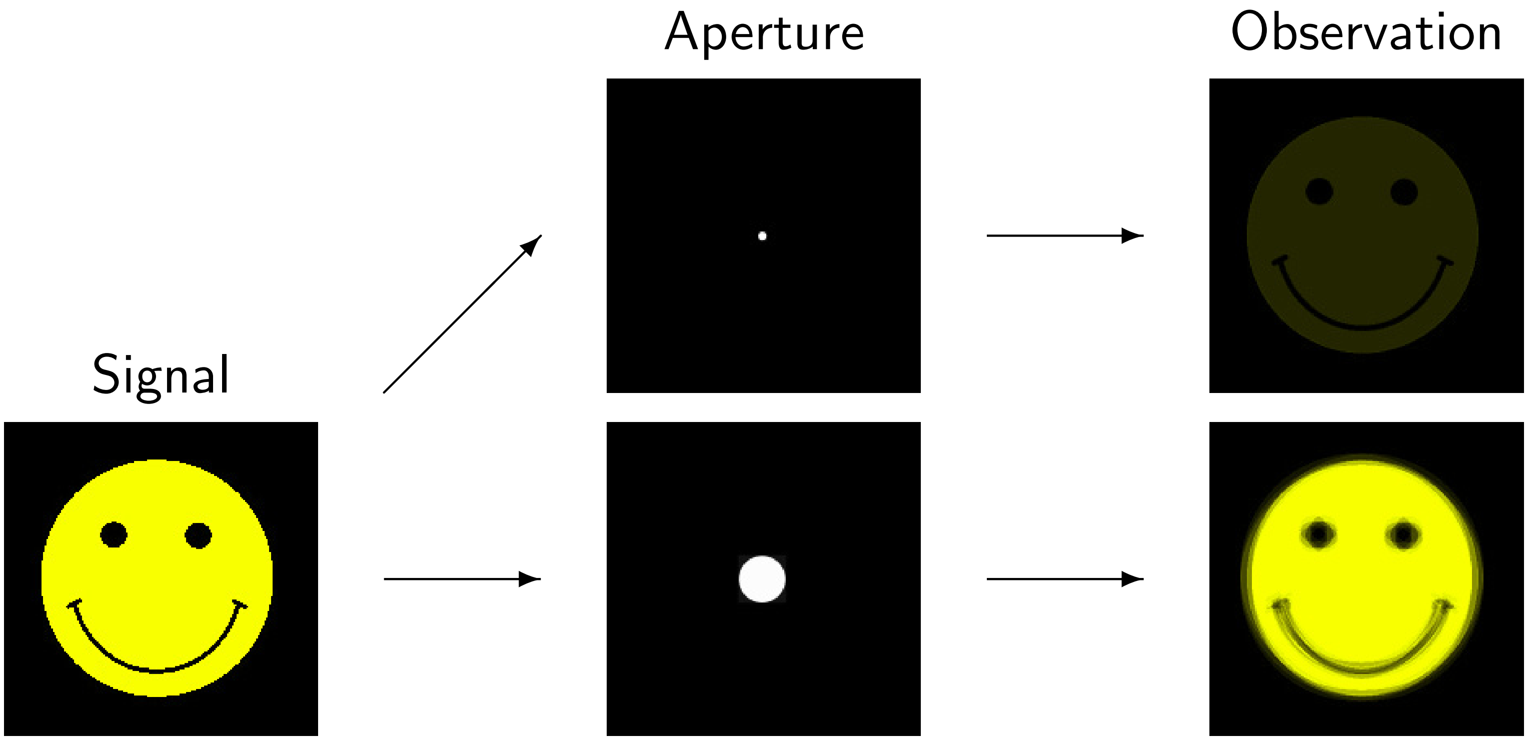
Images can be taken using pinhole cameras, which have infinite depth of field and do not suffer from chromatic aberration.

Aperture imaging



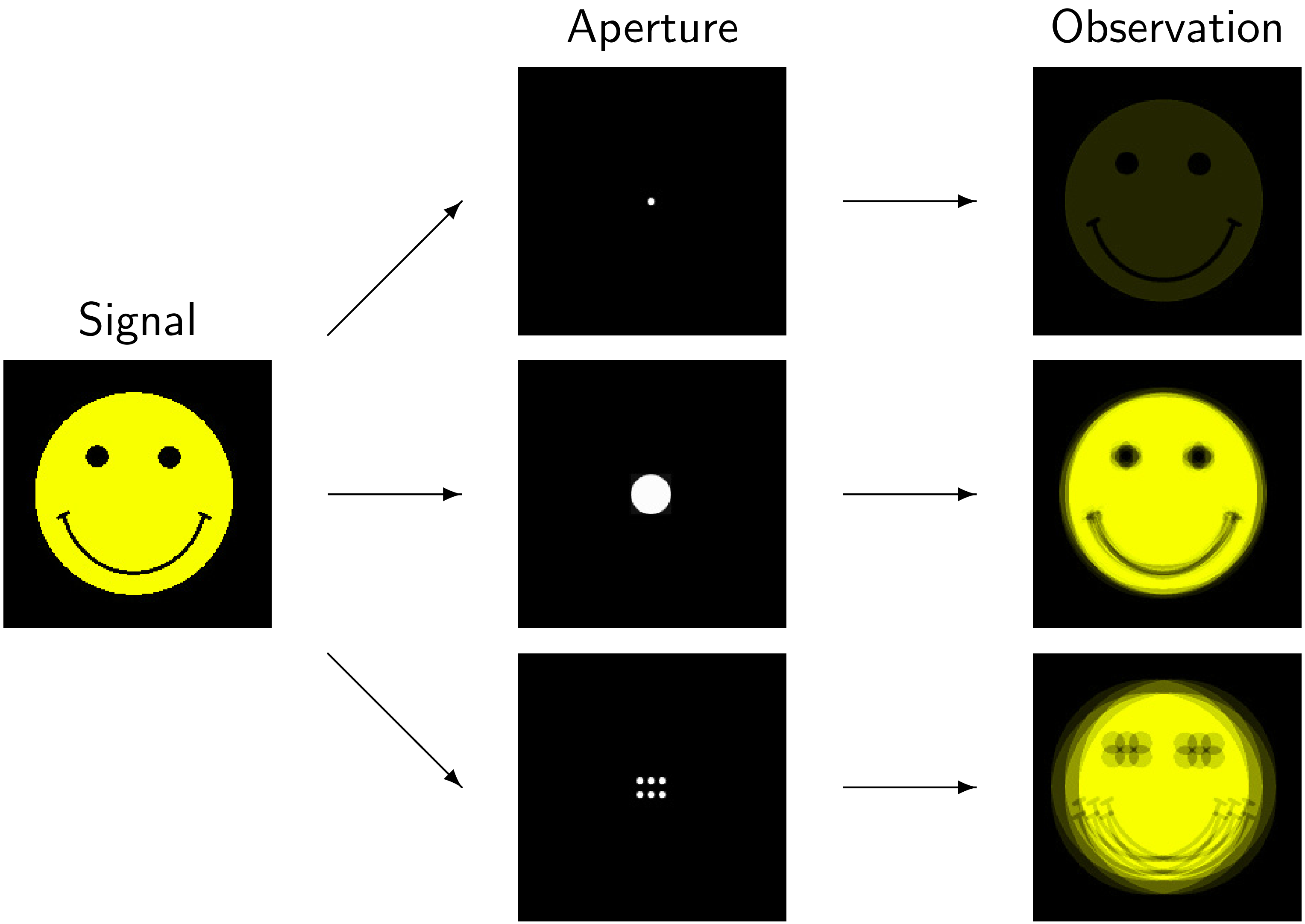
Small pinholes allow little light $\Longrightarrow$ dark observations.

Aperture imaging



Larger pinholes allow more light but leads to decrease in resolution $\Longrightarrow$ blurry observations.

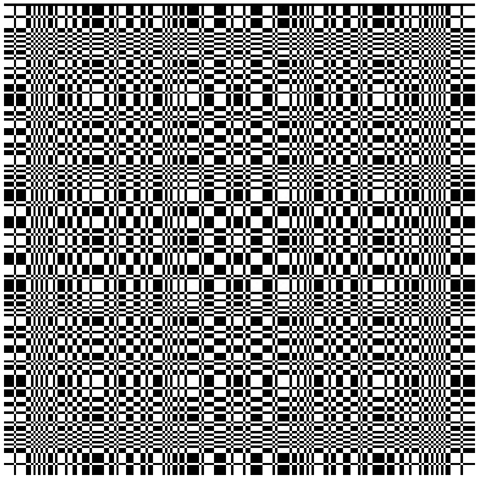
Aperture imaging



Multiple small pinholes $\Longrightarrow$ overlapping observations.

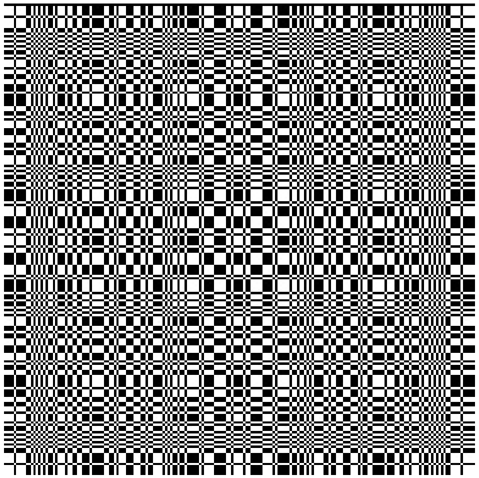
Modified Uniformly Redundant Array (MURA)

A MURA pattern p consists of specified openings that has a corresponding decoding pattern $\widetilde{p}$:



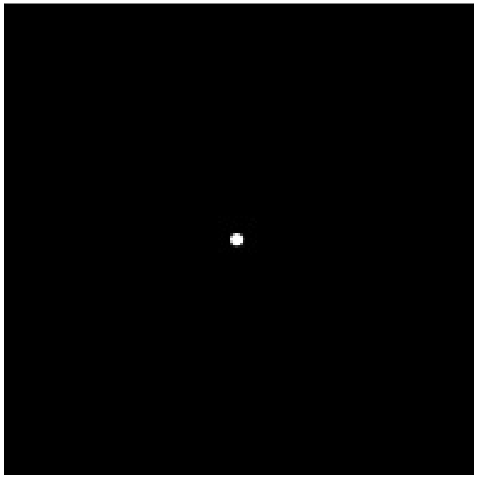
Coded aperture p

$*$



Decoding pattern $\widetilde{p}$

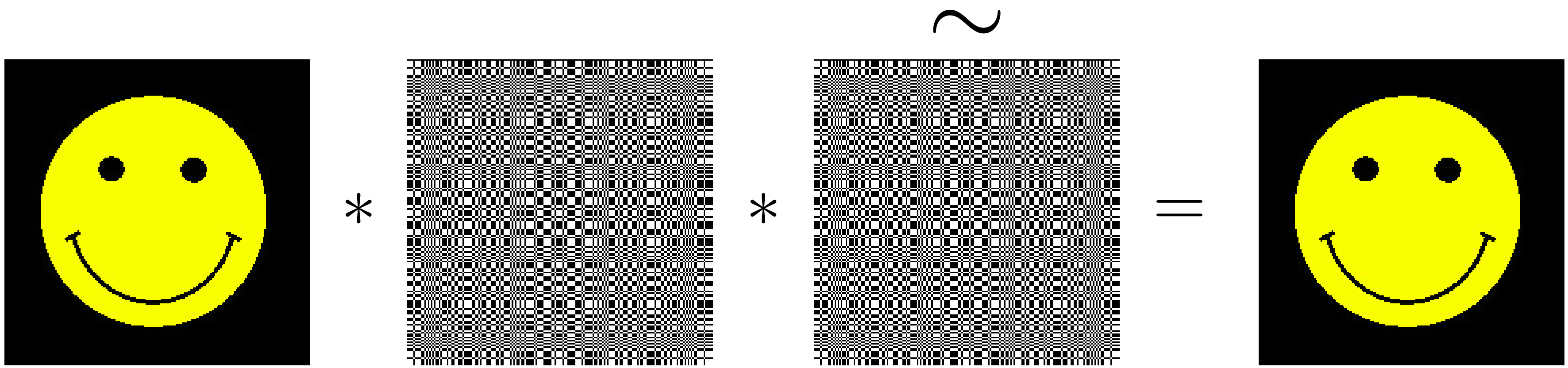
$=$



δ

Gottesman and Fenimore (1989)

Modified Uniformly Redundant Array (MURA)



Gottesman and Fenimore (1989)

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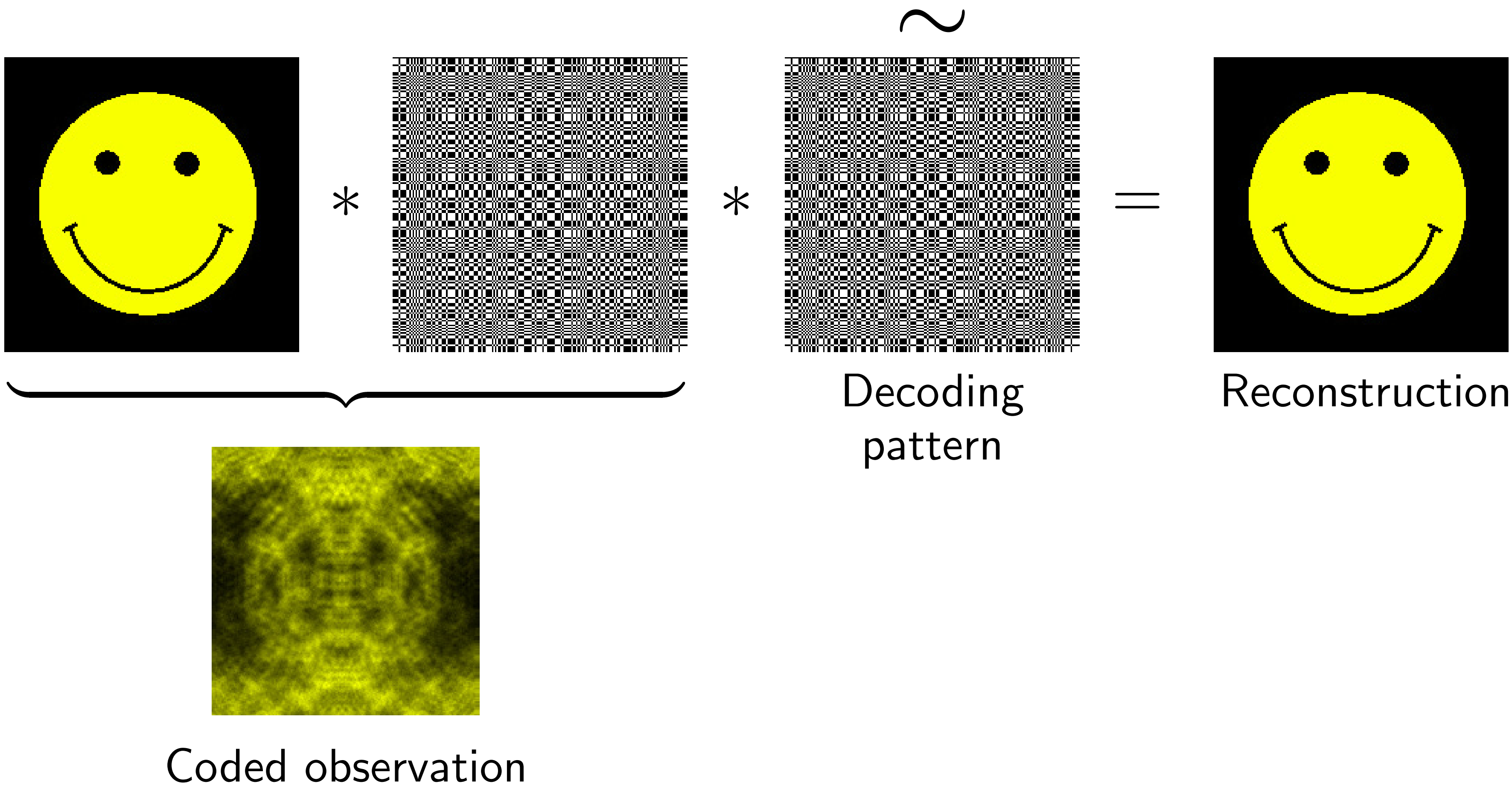
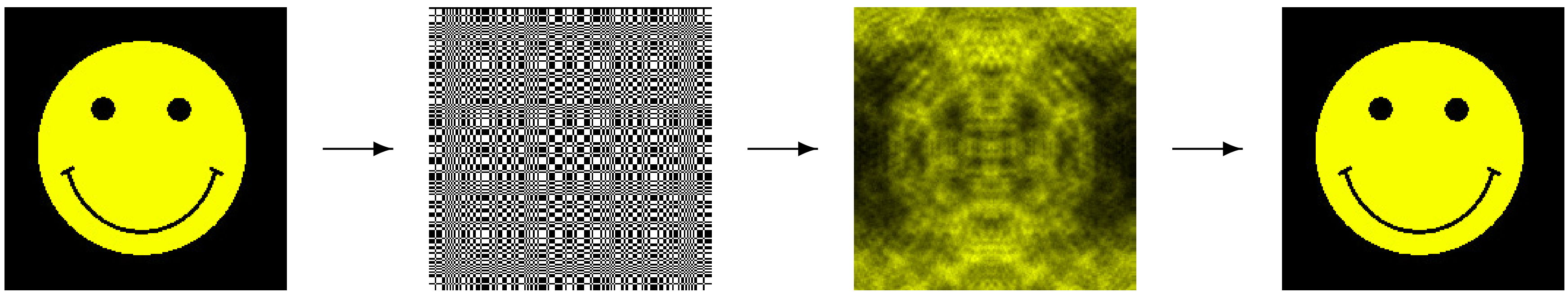


MURA patterns are $\sim$50% open $\Longrightarrow$ coded observations are much brighter than those from small pinhole cameras.

Gottesman and Fenimore (1989)

MURA aperture imaging

Signal Aperture Observation Reconstruction



f p y $\hat{f}$

The observation y is given by

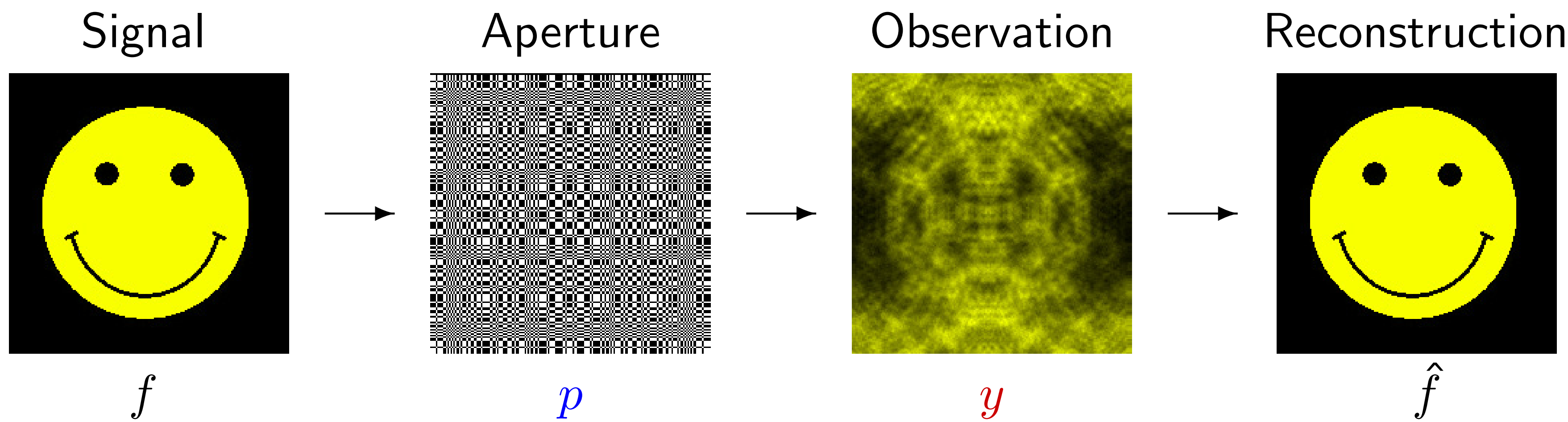
$$y = f * p + w$$

where w is zero-mean white Gaussian noise. The MURA reconstruction is given by

$$\hat{f}_{\mathsf{MURA}} = y * \tilde{p}$$

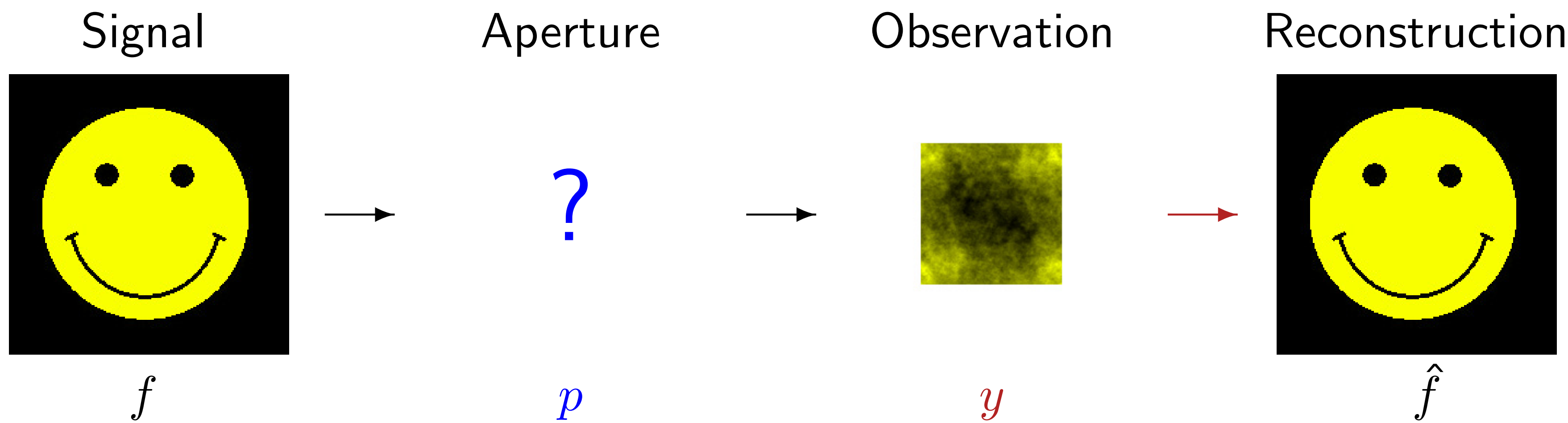
where $\tilde{p}$ is the decoding pattern. This reconstruction method is linear in y.

Coded aperture imaging



- MURA patterns are optimal assuming linear reconstruction and no downsampling.
- Few guiding principles for coded aperture mask design for nonlinear reconstructions.
- Low resolution observations useful for lower bandwidth and storage requirements, for smaller focal plane arrays.

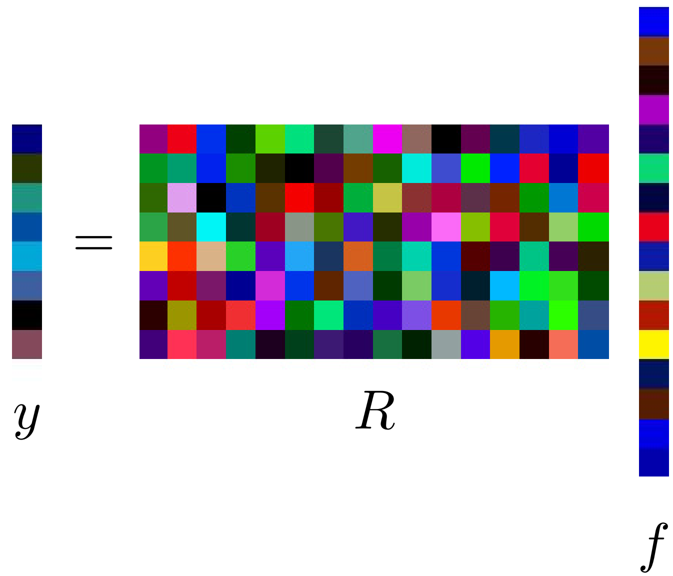
Coded aperture imaging



This talk: How to design coded aperture, p, for nonlinear reconstruction of signal from low-resolution noisy observations y.

Compressive Sensing

Recover signal f from limited observations $y \in \mathbb{R}^k$:



with (underdetermined) projection matrix $R \in \mathbb{R}^{k \times n}$ and $k \ll n$.

Highly accurate estimates of f can be obtained with high probability if

- f is sparse in some basis W, *i.e.*, $f = W\theta$ with θ mostly zeros.
- RW is sufficiently "nice" (RIP, details to follow).

Candès et al. (2006), Donoho (2006), Baraniuk (2007)

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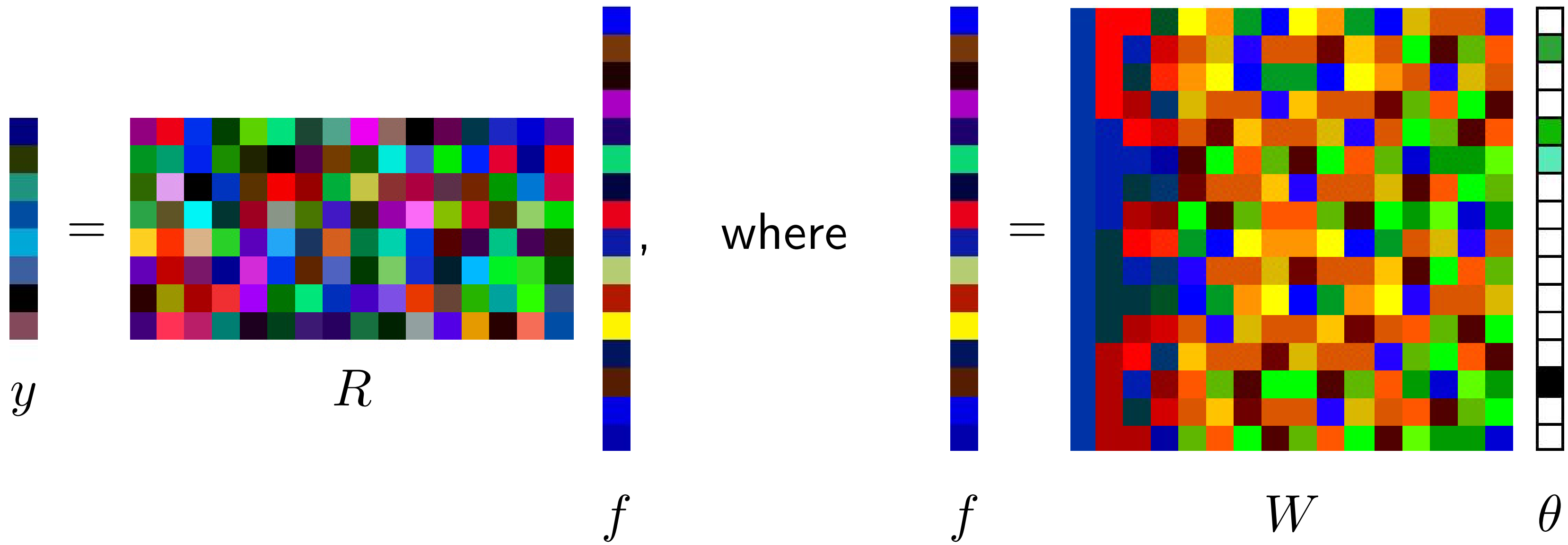

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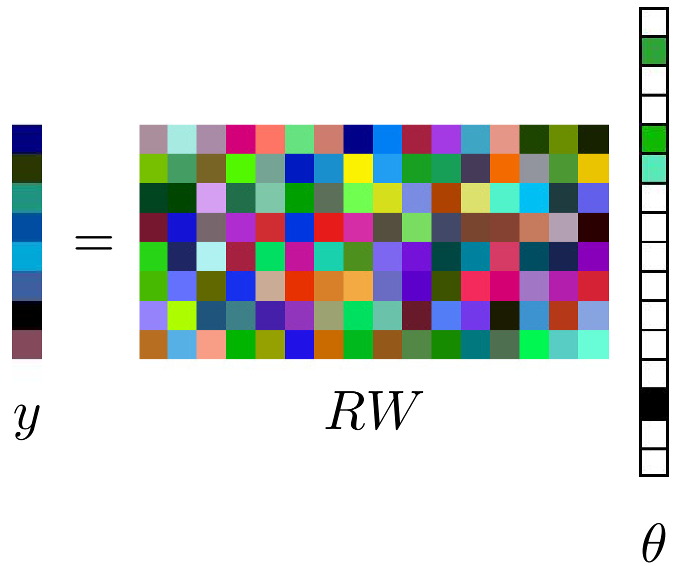

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Candès et al. (2006), Donoho (2006), Baraniuk (2007)

$\ell^2 - \ell^1$ minimization

Recover the signal f by solving the nonlinear optimization problem

$$\hat{\theta} = \underset{\theta}{\text{argmin}} \; \frac{1}{2}\|y - RW\theta\|_2^2 + \tau\|\theta\|_1$$

$$\hat{f} = W\hat{\theta}$$

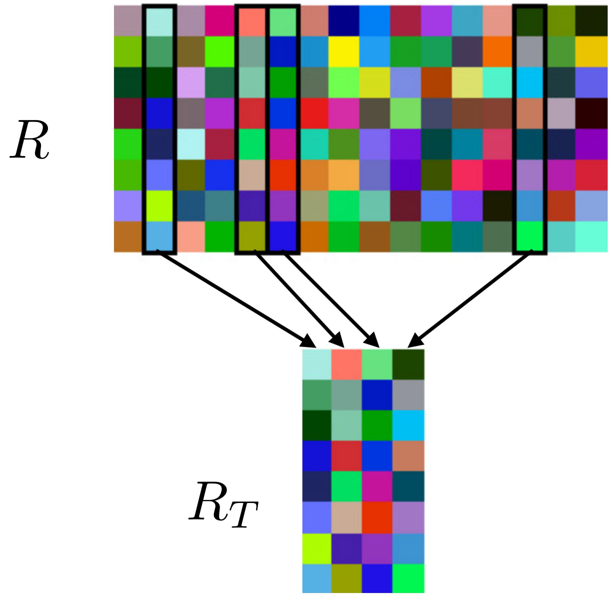
where

- ℓ^2 term minimizes the least-squares error.
- ℓ^1 term drives small components of θ to zero.
- $\tau > 0$ is a regularization parameter to make problem well-posed.
- $\ell^2 - \ell^1$ minimization (or equivalent variants) is the right problem to solve.

Candès and Tao (2005), Haupt and Nowak (2006)

Restricted Isometry Property (RIP)

A matrix R satisfies the Restricted Isometry Property of order m if submatrices R_T of R are almost an isometry, *i.e.*, for some constant δ_m,



$$(1-\delta_m)\|z\|_2^2 \leq \|R_T z\|_2^2 \leq (1+\delta_m)\|z\|_2^2$$

- Example: Elements of R are drawn from a zero-mean Gaussian distribution – not realizable in most optical systems.
- Verifying the RIP for a particular matrix cannot be done computationally.

Candès and Tao (2005)

Projection matrix R

In our setup, the observation y is given by

$$y = \underbrace{D}_{\text{Downsampling operator}}(\underbrace{f}_{\text{Signal}} * \underbrace{p}_{\text{Coded aperture}}) + \underbrace{w}_{\text{Gaussian noise}},$$

Projection matrix R

In our setup, the observation y is given by

$$y = \overbrace{D(f * p)}^{Rf} + w,$$

Downsampling operator
Signal
Coded aperture
Gaussian noise

Then
$$Rf = D(f * p) = (D\mathcal{F}^{-1}C_p\mathcal{F})f$$

Inverse Fourier transform
Transfer function
Fourier transform

Projection matrix R

In our setup, the observation y is given by

$$y = \overbrace{D(f * p)}^{Rf} + w,$$

Downsampling operator — Signal — Coded aperture — Gaussian noise

Then

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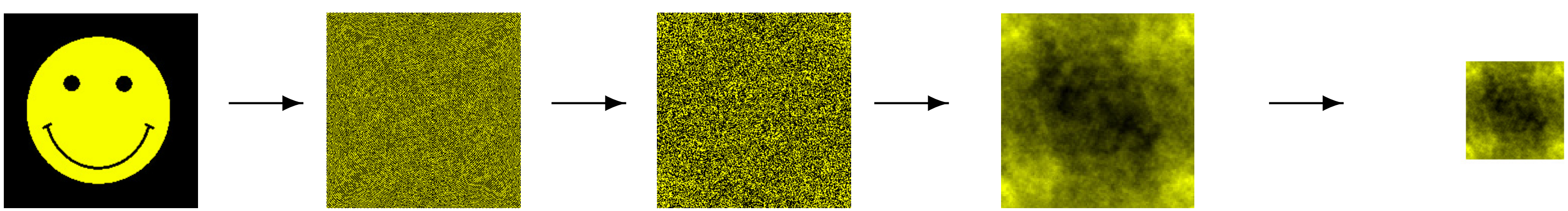
Inverse Fourier transform — Transfer function — Fourier transform



f → $\mathcal{F}f$ → $C_p\mathcal{F}f$ → $\mathcal{F}^{-1}C_p\mathcal{F}f$ → $D\mathcal{F}^{-1}C_p\mathcal{F}f$

Projection matrix R

In our setup, the observation y is given by

$$y = \overbrace{D(f * p)}^{Rf} + w,$$

Downsampling operator | Signal | Coded apertu | Gaussian noise

Then

$$Rf = D(f * p) = (\overbrace{D\mathcal{F}^{-1}C_p\mathcal{F}}^{A})f$$

Projection matrix R

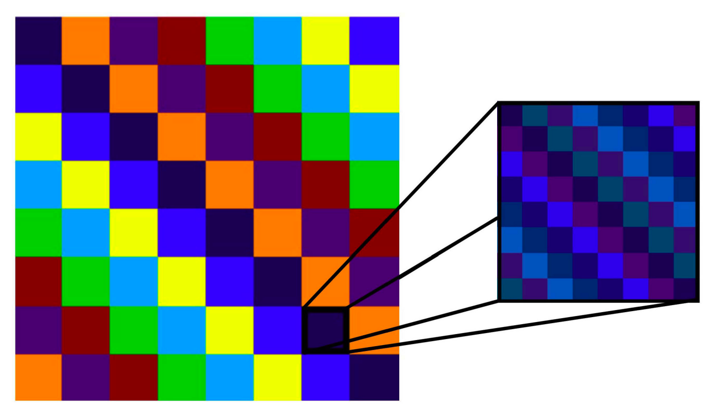
In our setup, the observation y is given by

$$y = \overbrace{D(f * p)}^{Rf} + w,$$

Downsampling operator | Signal | Coded aperture | Gaussian noise

Then $$Rf = D(f * p) = (D\overbrace{\mathcal{F}^{-1}C_p\mathcal{F}}^{A})f$$

The matrix $A = \mathcal{F}^{-1}C_p\mathcal{F}$ is block-circulant with circulant blocks:

$A =$ 

n blocks

Projection matrix R

Theorem: [Bajwa et al (2007)] If R is circulant whose entries are drawn from an appropriate probability distribution, R satisfies the RIP with high probability.

Proposed compressive coded aperture:

- $R = DA$ is "pseudo-circulant" $\Longrightarrow R$ also satisfies the RIP with high probability.
- RW, where $W =$ Haar wavelet transform, also satisfies this property.

Bajwa et al. (2007)

Computing p from block-circulant A

General approach:

1. Draw elements of A randomly from Gaussian distribution (subject to a symmetry constraint).
2. Set $A = \mathcal{F}^{-1} C_p \mathcal{F}$.
3. Solve for p.

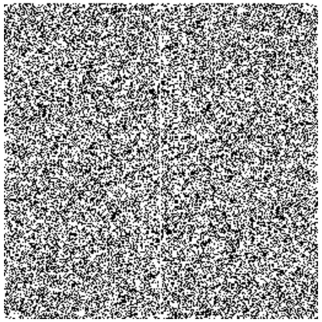
Issue: A is very large — solving for p non-trivial computationally but possible by exploiting structure of $\mathcal{F}^{-1} C_p \mathcal{F}$.

Computing p from block-circulant A

Mask p must be physically realizable:

- p = real-valued $\Longrightarrow \mathcal{F}(p)$ = circularly symmetric $\Longrightarrow A$ = symmetric ($A = A^T$).
- p = non-negative $\Longrightarrow$ Shift $p \Longrightarrow R$ is no longer zero mean — this can be compensated for in the reconstruction procedure.
- Rescale p so that its values $\in [0, 1]$.

Example:

$p =$ 

Gradient Projection for Sparse Reconstruction (GPSR)

The $\ell^2 - \ell^1$ minimization problem

$$\widehat{\theta} = \underset{\theta}{\operatorname{argmin}} \; \frac{1}{2}\|y - RW\theta\|_2^2 + \tau\|\theta\|_1$$

is solved using the Gradient Projection for Sparse Reconstruction (GPSR) algorithm. GPSR is

- fast, efficient, and accurate.
- shown to outperform many state-of-the-art methods for CS minimization.

Numerical experiment:

Compare three methods for reconstruction: (1) no coding, (2) proposed coding, and (3) coding with rounded values (0 or 1) for simplicity.

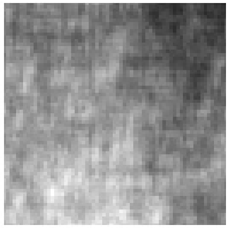
Figueiredo et al. (2007)

Numerical experiments

Original image

Uncoded observation

No coding
MSE $= 0.1011$



Coded observation

Proposed coding
MSE $= 0.0867$

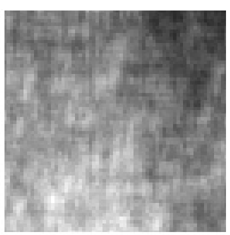
Coding with 0 and 1
MSE $= 0.0897$

Numerical experiments

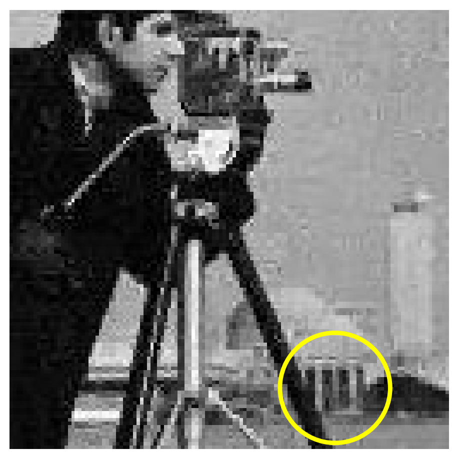
Original image

Uncoded observation

No coding
MSE = 0.1011



Coded observation



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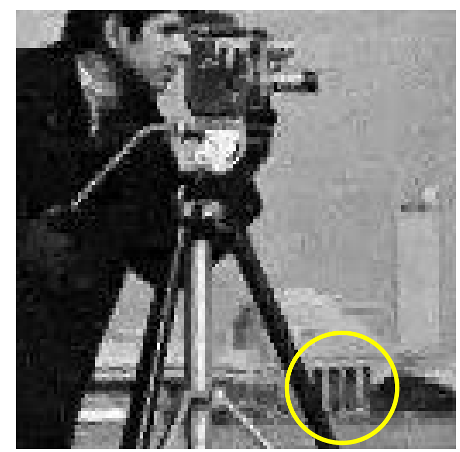

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Original image

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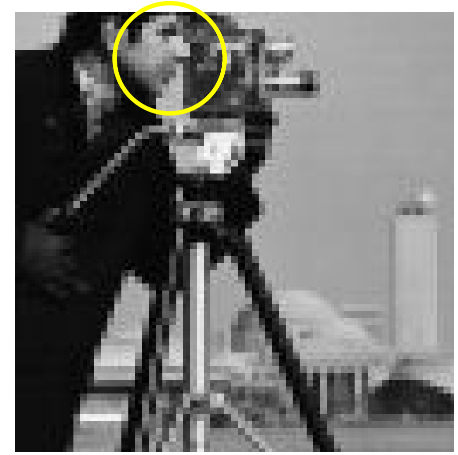

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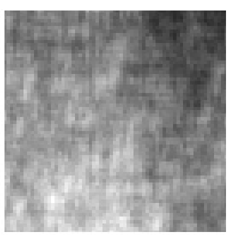
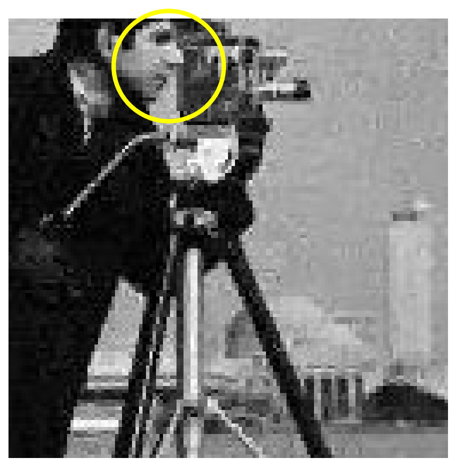


Coded observation



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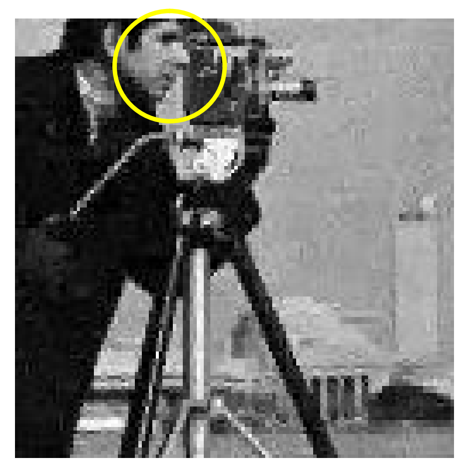
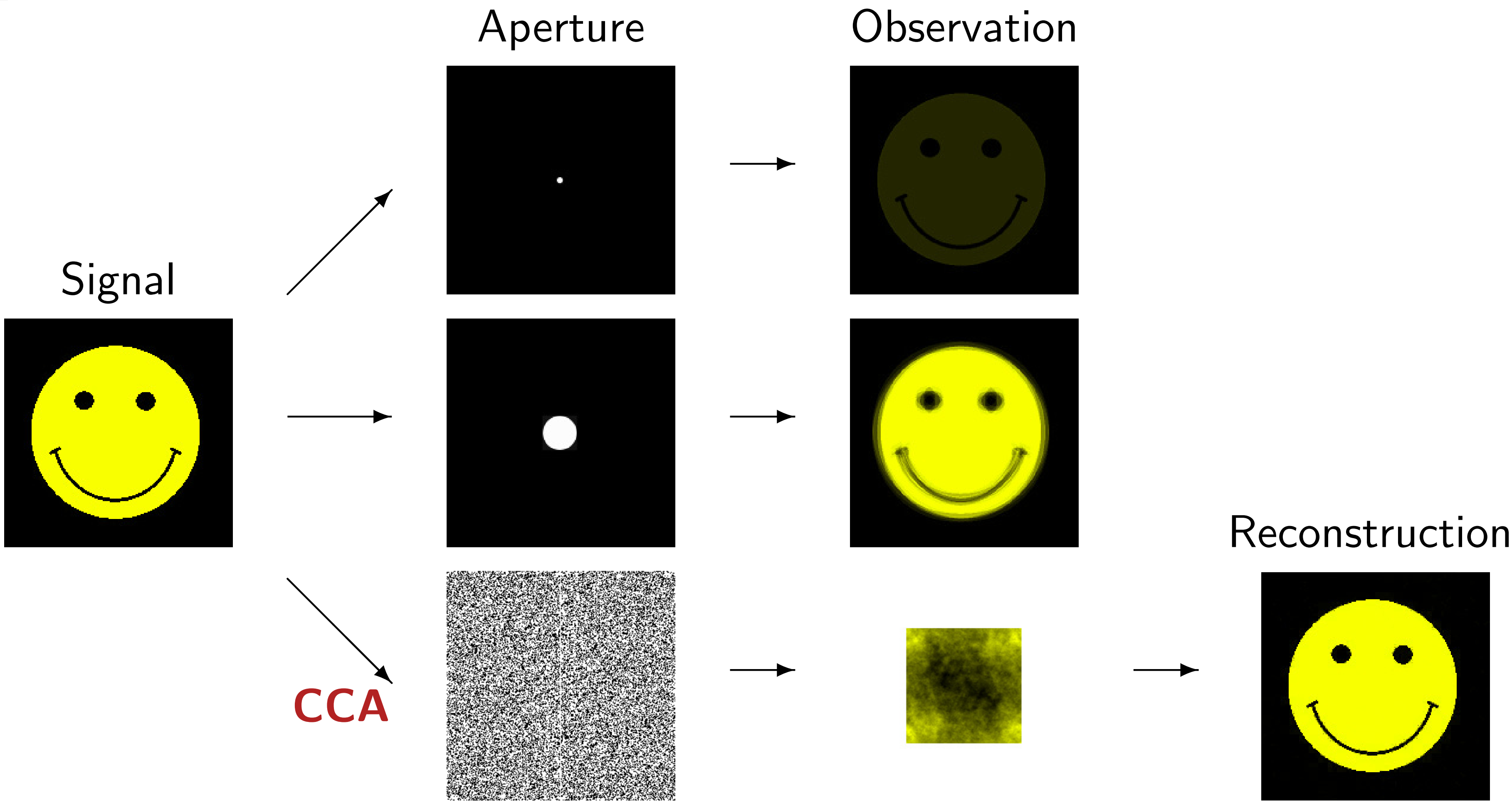


Coding with 0 and 1
MSE $= 0.0897$

Summary: Compressive Coded Aperture

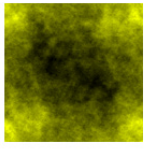


Compressive Coded Aperture (**CCA**) enhances image reconstruction from low-resolution observations using nonlinear methods.

Thank you.

Have a nice day.

Thank you.



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