This set covers chapters 12–15 of the book *Numerical Optimization* by Nocedal and Wright, 2nd ed.

From the book, the following exercises: 12.5–6, 12.14, 12.16, 12.18–22, 13.1, 13.5, 14.1, 14.10. In addition, the exercises below. There are no Matlab programming exercises. However, you may find it useful to plot the objective function and constraints with `fcontour`.

### III.1. KKT conditions and trust regions.

Apply the KKT conditions (th. 12.1) to the problem

$$
\min_{p \in \mathbb{R}^n} f(p) + g^T p + \frac{1}{2} p^T B p \quad \text{s.t.} \quad \|p\|_2 \leq \Delta.
$$

Relate your results to theorem 4.1 (book, p. 70).

### III.2. Nonsmooth objective function and linear constraints.

Write a linear program in standard form (eq. (13.1)) to find a point $x \in \mathbb{R}^2$ satisfying $2x_1 + x_2 \leq 10$, $x \geq 0$ that minimises $|x_1 - 2x_2| + |{-3}x_1 - x_2|$. Use the KKT conditions to show that $x = (0, 0)$ is a solution.


Determine the range of values for the parameter $a \in \mathbb{R}$ such that $x = (4, 3)$ is the optimal solution to $\max_{x \in \mathbb{R}^2} ax_1 + x_2$ subject to $x_1^2 + x_2^2 \leq 25$, $x_1 - x_2 \leq 1$, $x \geq 0$.

### III.4. Quadratic-programming problem.

Given the constrained optimisation problem

$$
\min_{x \in \mathbb{R}^3} (x_1 + 1)^2 + x_2^2 + x_3^2 \quad \text{subject to} \quad x_3 = 0, \ x_1 \geq 0, \ x_1 + x_2 \geq 2
$$

1. Solve it by writing the KKT and second-order conditions.
2. Solve the KKT system that results if we assume that the equality constraint is active and:
   (i) No inequality constraints are active.
   (ii) Exactly one inequality constraint is active (2 cases).
   (iii) All inequality constraints are active.

Verify the solution corresponds to one of the cases in (ii).

### III.5. Duality.

Consider the LP

$$
\min_{x \in \mathbb{R}^2} x_1 + 2x_2 \quad \text{s.t.} \quad x_1 + x_2 = 1, \ x_1 \geq \frac{1}{2}, \ x_2 \geq 0.
$$

Write the dual, solve both (primal and dual), draw them and show that the solutions agree and have the same objective value (strong duality). Pick a point that is primal-dual feasible and show that weak duality holds for it. Show that the Wolfe dual is equivalent to the dual.

### III.6. Interior-point methods.

Consider the LP

$$
\min_{x \in \mathbb{R}} x \quad \text{s.t.} \quad x \geq 0.
$$

1. Write the KKT conditions and find the solution.
2. Determine the central path $C$ and draw it.
3. Assuming the complementarity conditions equal $\sigma \mu$, write the function $F$, its Jacobian $J$, compute the full Newton step ($\alpha = 1$) and show it jumps directly to the central path from any initial point that is strictly feasible.
4. Assuming we always take full steps ($\alpha_k = 1 \ \forall k$) starting from a point on the central path and with $\sigma_k = \sigma \ \forall k$, determine whether the interior-point method converges to the solution in the following cases: $\sigma \in (0, 1)$, $\sigma = 0$, $\sigma = 1$. Determine the convergence rate, and how many iterations are required to reduce $\mu_k$ below $\epsilon \mu_0$. 