This set covers chapters 12–15 of the book *Numerical Optimization* by Nocedal and Wright.

From the book, the following exercises: 12.4, 12.5, 12.15, 12.16, 12.18, 12.19, 12.20, 12.21, 13.2, 13.3, 14.1. In addition, the exercises below. There are no Matlab programming exercises.

**Exercise 1.** Apply the KKT conditions (th. 12.1) to the problem

\[
\min_{p \in \mathbb{R}^n} f + g^T p + \frac{1}{2} p^T B p \quad \text{s.t.} \quad \|p\|_2 \leq \Delta.
\]

Relate your results to theorem 4.3 (book, p. 78).

**Exercise 2.** Write a linear program in standard form (eq. (13.1)) to find a point \(x \in \mathbb{R}^2\) satisfying \(2x_1 + x_2 \leq 10, x \geq 0\) that minimises \(|x_1 - 2x_2| + |-3x_1 - x_2|\). Use the KKT conditions to show that \(x = (0, 0)\) is a solution.

**Exercise 3.** Determine the range of values for the parameter \(a \in \mathbb{R}\) such that \(x = (4, 3)\) is the optimal solution to \(\max_{x \in \mathbb{R}^2} ax_1 + x_2\) subject to \(x_1^2 + x_2^2 \leq 25, x_1 - x_2 \leq 1, x \geq 0\).

**Exercise 4.** Given the constrained optimisation problem

\[
\min_{x \in \mathbb{R}^3} (x_1 + 1)^2 + x_2^2 + x_3^2 \quad \text{subject to} \quad x_3 = 0, x_1 \geq 0, x_1 + x_2 \geq 2
\]

1. Solve it by writing the KKT and second-order conditions.

2. Solve the KKT system that results if we assume that the equality constraint is active and:

   (i) No inequality constraints are active.

   (ii) Exactly one inequality constraint is active (2 cases).

   (iii) All inequality constraints are active.

   Verify the solution corresponds to one of the cases in (ii).