This set covers chapters 12–15 of the book *Numerical Optimization* by Nocedal and Wright.

From the book, the following exercises: 12.4, 12.5, 12.15, 12.16, 12.18, 12.19, 12.20, 12.21, 13.2, 13.3, 14.1. In addition, the exercises below. There are no Matlab programming exercises.

Exercise 1. Apply the KKT conditions (th. 12.1) to the problem

$$\min_{\mathbf{p}\in\mathbb{R}^n} f + \mathbf{g}^T\mathbf{p} + \frac{1}{2}\mathbf{p}^T\mathbf{B}\mathbf{p} \text{ s.t. } \|\mathbf{p}\|_2 \le \Delta.$$

Relate your results to theorem 4.3 (book, p. 78).

Exercise 2. Write a linear program in standard form (eq. (13.1)) to find a point $\mathbf{x} \in \mathbb{R}^2$ satisfying $2x_1+x_2 \le 10$, $\mathbf{x} \ge 0$ that minimises $|x_1 - 2x_2| + |-3x_1 - x_2|$. Use the KKT conditions to show that $\mathbf{x} = \binom{0}{0}$ is a solution.

Exercise 3. Determine the range of values for the parameter $a \in \mathbb{R}$ such that $\mathbf{x} = \binom{4}{3}$ is the optimal solution to $\max_{\mathbf{x}\in\mathbb{R}^2} ax_1 + x_2$ subject to $x_1^2 + x_2^2 \le 25,\ x_1 - x_2 \le 1,\ \mathbf{x} \ge 0$.

Exercise 4. Given the constrained optimisation problem

$$\min_{\mathbf{x}\in\mathbb{R}^3} (x_1+1)^2 + x_2^2 + x_3^2 \text{ subject to } x_3 = 0,\ x_1 \ge 0,\ x_1 + x_2 \ge 2$$

1. Solve it by writing the KKT and second-order conditions.
2. Solve the KKT system that results if we assume that the equality constraint is active and:
   - (i) No inequality constraints are active.
   - (ii) Exactly one inequality constraint is active (2 cases).
   - (iii) All inequality constraints are active.

   Verify the solution corresponds to one of the cases in (ii).