This set covers chapter 3 of the AIMA textbook (4th ed.). References (figures, etc.) are to the AIMA textbook. In all exercises, *explain your answers.* **Total possible points: 100.**

Exercise 1 (10 points). Consider the A* algorithm using a perfect heuristic function h, i.e., for any node (state) n, $h(n)$ equals the minimal cost from n to a goal state. Modify the pseudocode for A* (fig. 3.7) so it improves in space and/or time complexity as much as possible. The better the solution the higher the grade. Explain your answer.

Exercise 2 (17 points). Consider a certain heuristic function $h(n)$ that satisfies the following properties (on a state graph where all costs are positive):

1. $h(n) = 0$ if n is a goal state.
2. For any nodes n, m such that an action that takes us from n to m has a cost $c_{nm} > 0$, we have that $h(m) \geq h(n) - c_{nm}$.

Prove that h is an admissible heuristic. *Hint*: use induction on the number of edges of a shortest path.

Exercise 3 (42 points). Consider a vacuum world as follows. We have a 2×2 grid with cells $\{(0,0),(0,1),(1,0),(1,1)\}$. The agent can move one step in any direction (U)p or (D)own with cost 1, (L)eft or (R)ight with cost 2; or (S)uck dirt with cost 10. Any of the cells may or may not have dirt. A goal state has no dirt in any cell.

1. (4 points) Draw the state-space graph as in fig. 3.2. How many states are there?
2. (2 points) For this problem, what is better: tree search or graph search? Explain.
3. Let the initial state be: agent at (0,0); dirt only at (1,0). For each of the following search algorithms, run it showing your work (the relevant data structures, path costs and parent (predecessor) in the path). Draw the final search tree, give the path found to the goal and its cost. When needed to break ties in picking actions, use this order: S,U,D,L,R; also, if the lowest value of the evaluation function $f(n)$ in a priority queue is achieved by multiple states, pop the oldest one (i.e., the one that entered the queue first).
   (a) (4 points) BFS.
   (b) (4 points) DFS.
   (c) (4 points) UCS.
4. (12 points) Let the initial state be: agent at (0,0); dirt only at (0,0) and (1,1). Repeat as in the previous point.
5. (6 points) Repeat point 3 but now assuming the actions U and D are not allowed.
6. (6 points) Repeat point 4 but now assuming the actions U and D are not allowed.

Exercise 4 (31 points). Consider an infinite 2D grid where the initial state is the origin (0,0), the goal state is $(u,v) \in \mathbb{Z}^2$ and the only possible actions are Up, Down, Left, Right (picked in that order when needed to break ties), each with a cost of 1. As in the previous exercise, break ties in the priority queue by popping the oldest state.

1. (1 point) What is the branching factor b? Explain.
2. (3 points) How many distinct states are there at depth $i > 0$? Explain.
3. (4 points) How many nodes (at most) will BFS expand if using tree search? And if using graph search?
4. Run each of the following algorithms for a goal $(u,v) = (2,1)$, giving for each the sequence of states as they are explored (either as a list or by marking it on the grid), and the cost of the path found:
   (a) (4 points) BFS.
   (b) (4 points) DFS.
5. (3 points) Consider $h_2(x,y) = (u-x)^2 + (v-y)^2$ at a state (x,y). Is this an admissible heuristic? Explain.
6. (6 points) Run A* using as heuristic h_2 for a goal $(u,v) = (2,1)$. How many nodes does A* expand? Give the cost for the path found. Is it optimal?
7. (6 points) Repeat the previous two points for the function $h_1(x,y) = |u-x| + |v-y|$.