THE MODELLING OF THE THERMAL SUBSYSTEM IN SPACECRAFT REAL–TIME SIMULATORS

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3rd Workshop on Simulators for European Space Programmes

ESTEC, Noordwijk, The Netherlands
15-17 November 1994
Outline

- Outline and introduction
- Methods for thermal modelling
  - Integrators
    * Example: ESATAN
  - Interpolators
    * Example: real-time satellite simulators (RTSS) developed at ESOC/FCSD/SIM
- Suitability of integrators for RTSS
- Conclusions and future work
Introduction

• Primary functions of the RTSSs developed at ESOC/FCSD/SIM:
  – Testing and validation of the Operations Control Centre facilities and flight control procedures
  – Training of the ground–network operators

• The mission of the thermal subsystem of an RTS is to provide thermal telemetry (TTM) in due time

• Minimal requirements for the thermal subsystem:
  – Temperature of the equipment stays between certain margins
  – Temperature tendencies with time are more or less correct

• Approaches for the modelisation of a thermal system:
  – Integrators
  – Interpolators
  – Finite–element method: not suitable for real–time

• Heat transfer modes in a satellite:
  – Conduction
  – Radiation
  – Convection (less important)

• The payload is considered to be uncoupled with the rest of the system
Integrators

Steps:

1. **Nodalisation** of the thermal system, which produces a lumped parameter network (LPN) consisting of:
   - **Nodes**: isothermal volumes able to store heat \([C_i \text{ (J/°C)}, Q_i \text{ (W)}, T_i \text{ (°C)})\]
   - **Links**: heat transfer paths \([K_{ij} \text{ (W/°C)}, R_{ij} \text{ (W/°C})]\)

2. **Application of the heat transfer equations** \((Q\) is the heat rate in watts):
   - Conduction: Fourier’s law:
     \[
     Q = -\kappa A \nabla T
     \]
   - Radiation: Stefan–Boltzmann law for a grey body:
     \[
     Q = \sigma \alpha \varepsilon F \Delta T_{\text{th}}
     \]
   and the principle of the conservation of energy:
   \[
   Q_{\text{in}} - Q_{\text{out}} = \Delta U = \Delta (\rho c V T)
   \]
   leads to the system
   \[
   C_i \frac{dT_i}{dt} = \sum_{i \neq j} K_{ij} (T_j - T_i) + \sum_{i \neq j} R_{ij} (T_{ij}^4 - T_i^4) + Q_i, \quad i, j = 1, \ldots, n
   \]  

(1)
3. **Integration** of system (1) w.r.t. the time:
   
   - Steady state: \( \frac{dT_i}{dt} = 0 \) for all \( i \). Solve a system of nonlinear equations
   
   - Transient: finite–differences methods commonly use a first–order approximation for the time derivative
     \[
     \left( \frac{dT_i}{dt} \right)_t = \frac{T_i(t + \Delta t) - T_i(t)}{\Delta t} + \mathcal{O}(\Delta t^2)
     \]
     and compute \( T_i(t_0 + (\Delta t)_1), T_i(t_0 + (\Delta t)_1 + (\Delta t)_2), \) etc.
     known \( T_i(t_0) \).
     \( \Delta t \) is the **timestep**.

**Example of actual implementation:**

- ESATAN (European Space Agency Thermal ANalyser), with ESARAD for the radiative calculations
Integrators (cont.)

Pros/cons:

- High accuracy (for “small” $\Delta t$)
- Also valid for non-nominal situations
- High CPU consumption
- Development of thermal model requires expertise
Interpolators

Steps:
1. Determination of parameters $P_j$ that affect the S/C thermal behaviour. S/C thermal context: $\mathbf{C}(t) = (P_1(t), \ldots, P_p(t))$, $P_j(t) \in \mathbb{R}$
2. Context discretisation: $\tilde{\mathbf{C}} = (\tilde{P}_1, \ldots, \tilde{P}_p)$, $\tilde{P}_j \in \{\tilde{P}_{j,\text{min}}, \ldots, \tilde{P}_{j,\text{max}}\}$
3. Off-line computation with a thermal analyser of steady states for each $\tilde{\mathbf{C}}$:
   \[
   \tilde{T}_\infty : \{\tilde{P}_{1,\min}, \ldots, \tilde{P}_{1,\max}\} \times \cdots \times \{\tilde{P}_{p,\min}, \ldots, \tilde{P}_{p,\max}\} \longrightarrow \mathbb{R}^m
   \]
   $\tilde{\mathbf{C}} = (\tilde{P}_1, \ldots, \tilde{P}_p) \xrightarrow{} \tilde{T}_\infty(\tilde{\mathbf{C}}) = (\tilde{T}_{1,\infty}, \ldots, \tilde{T}_{m,\infty})^T$
4. Interpolation for nondiscrete contexts: $\mathbf{T}_\infty(\mathbf{C}) = \mathcal{F}(P_1, \ldots, P_p)$
   - Global interpolation
   - Local interpolation among neighbouring discrete contexts
5. Real-time calculation of the S/C thermal map:
   \[
   T_i(t + \Delta t) = T_{i,\infty} + (T_i(t) - T_{i,\infty}) e^{-\frac{\Delta t}{\tau_i}}, \quad i = 1, \ldots, m
   \]
   The time constants $\tau_i$ are also determined with the help of a thermal analyser.
Interpolators (cont.)

Global interpolation

Figure 1: Global interpolation for context $C = (\frac{3\pi}{4}, 0.3)$ over parameters $\hat{P}_1 \in \{0, \frac{\pi}{7}, \pi\}$ and $\hat{P}_2 \in \{0, 1\}$.

Local interpolation

Figure 2: Local bilinear interpolation for context $C = (\frac{3\pi}{4}, 0.3)$ over parameters $\hat{P}_1 \in \{0, \frac{\pi}{7}, \pi\}$ and $\hat{P}_2 \in \{0, 1\}$, using 4 neighbours (highlighted).
Interpolators (cont.)

Example of actual implementation:

- The thermal subsystem of the RTSs developed at ESOC/FCSD/SIM for the ISO, Italsat, Eureca and Pastel satellites (among others)

Pros/cons:

⊕ Low CPU consumption
⊕ Easy computation of steady states
⊕ Straightforward implementation
⊕ Low accuracy (interpolation error + inexact eq. for $T_i$)
⊕ Extrapolation (for contexts out of those foreseen) produces uncertain predictions
⊕ Necessary to store the tables that define $\tilde{T}_\infty$
Use of integrators in real–time satellite simulators

• Main reasons: better accuracy and applicability to non–nominal scenarios

• Involves two independent operations:
  - Actual integration of system (1), that provides with the S/C thermal map in due time
  - Update of the coefficients of system (1)
Performance of ESATAN solution routines

- Approximate formula for the CPU time in seconds employed by SOLVIT (steady-state by Newton–Raphson method):

\[
\Delta t_{CPU} \approx \frac{t_i(n, c)i}{MIPS}
\]

where \(n\) is the number of nodes and \(c\) the number of links of the model, \(i\) the number of iterations required to solve the algebraic system (1), \(t_i(n, c)\) the time per iteration and the CPU power is expressed in MIPS

- Objective: Minimise real–time fraction \(\frac{\Delta t_{CPU}}{\Delta t}\)
  - Increment \(\Delta t\). Drawbacks: the approximation error \(O(\Delta t^2)\) grows; some routines may not converge (SLFRWD); TTM period is 1-10 seconds
  - Decrease \(t_i(n, c)\) \(\Leftrightarrow\) Decrease \(n\) and \(c\) in the LPN. Drawback: the model is less accurate.

There exist algorithms for model reduction made by some companies, based on node grouping and node/link removal

- Increase MIPS: currently, the RTSs at SIM run on VAX 4000.90 (30 MIPS); near future: DEC/Alphas (> 100 MIPS)

- \(i\) (LOOPCT): depends on the matrix of system (1) and on the desired accuracy of the iterative method. The SOHO and ISO models require about 100 iterations with RELXCA between 0.01 and 0.001
Performance of ESATAN solution routines (cont.)

- Assuming \( i = 100 \) and that 10\% of the CPU time is allocated to the thermal solution routine:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( c )</th>
<th>( \Delta t_{\text{min}}(30 \text{ MIPS}) )</th>
<th>( \Delta t_{\text{min}}(100 \text{ MIPS}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>300</td>
<td>9.7</td>
<td>2.9</td>
</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>31</td>
<td>9.4</td>
</tr>
<tr>
<td>200</td>
<td>3000</td>
<td>93</td>
<td>28</td>
</tr>
<tr>
<td>500</td>
<td>10000</td>
<td>320</td>
<td>96</td>
</tr>
</tbody>
</table>

Table 1: Minimum timestep in seconds for real-time simulation of ESATAN models.
Update of the LPN coefficients

- Factors that influence the LPN coefficients:
  1. TCU command/telecommand: low frequency, not responsibility of the thermal subsystem
  2. Temperature: slow variations (great inertia)
  3. S/C geometry: great effect on $R_{ij}$
  4. S/C attitude and orbital coordinates: great effect on $Q_i$

3. and 4. are usually computed by a ray-tracing algorithm, which can’t run in real-time at all (minutes of CPU consumption) \(\Rightarrow\) Table interpolation for $R_{ij}, Q_i$
Conclusions

- SIM’s computer facilities allow for a hybrid method in real–time thermal simulation:
  - Numerical integration of a reduced ESATAN thermal model
  - Table interpolation for radiative couplings and external fluxes
- Benefits:
  - Improved accuracy of the generated thermal telemetry
  - Use of thermal mathematical models designed by thermal engineers right from the beginning
  - Reliable routines (ESATAN is validated through years of use)
  - Interpolation schemes can be simulated by an ad–hoc LPN with a clever election of coefficients

Future work

- ESATAN version tailored for real–time (?)
- Interface ESATAN ↔ real–time simulator
- Powerful model reduction algorithms with error estimate