MAGIC: Model-Based Actuatio for Ground Irrigation Contro

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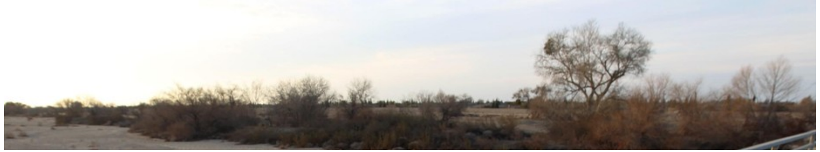
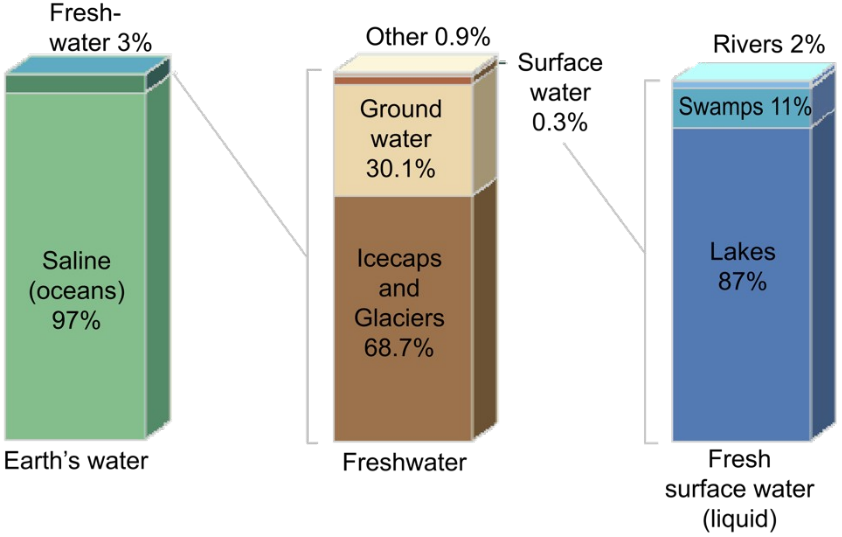
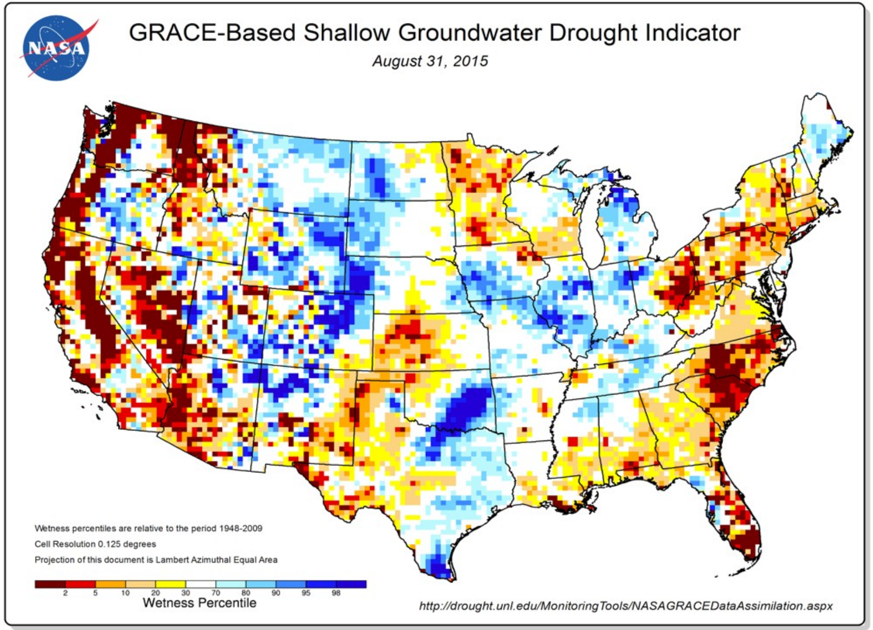
Fresh water is not abundant...



Distribution of Earth's Water





Lawn (Turf) Coverage

In continental United States alone:

- 128,000 square kilometers
    - Estimated 3x more than corn
- 9 Billion gallons/day to irrigate!!
    - 13,600 olympic swimming pools
    - 3.3 hours of Niagara Falls flow

Systems aren’t great

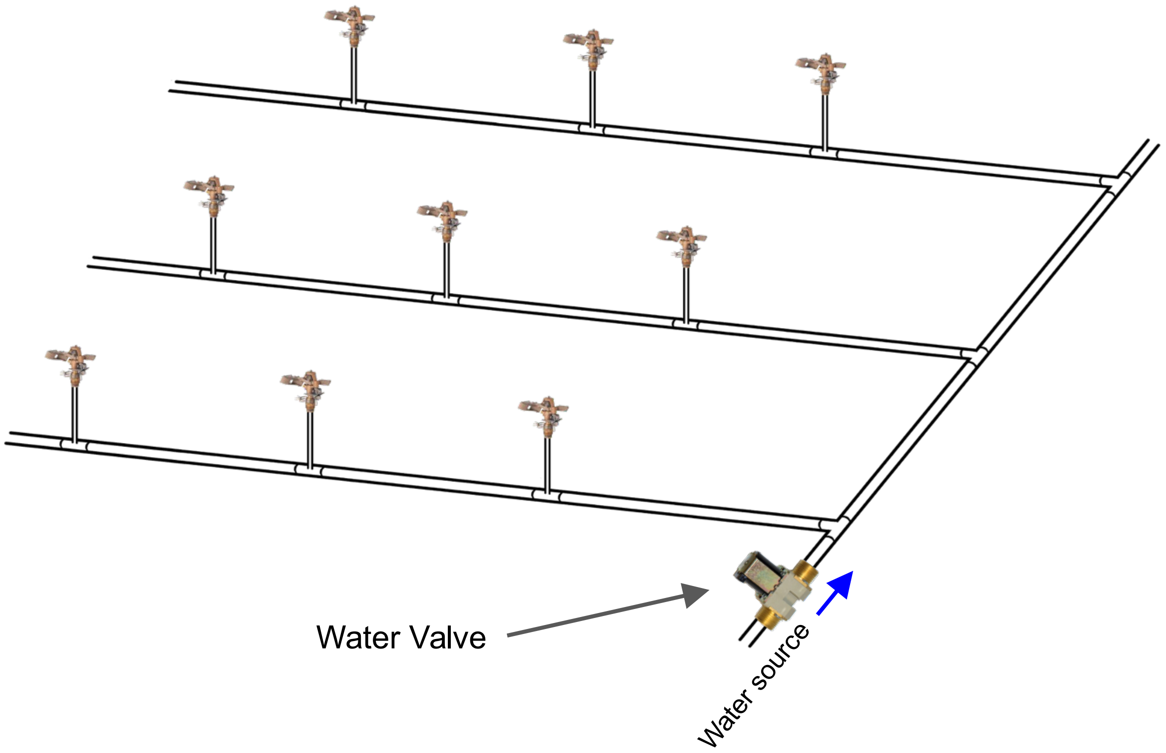
It’s easy to find irrigation systems that aren’t doing their jobs properly…

Primary offenses:

- Underwatering
  - Bad quality
- Overwatering
  - Bad efficiency

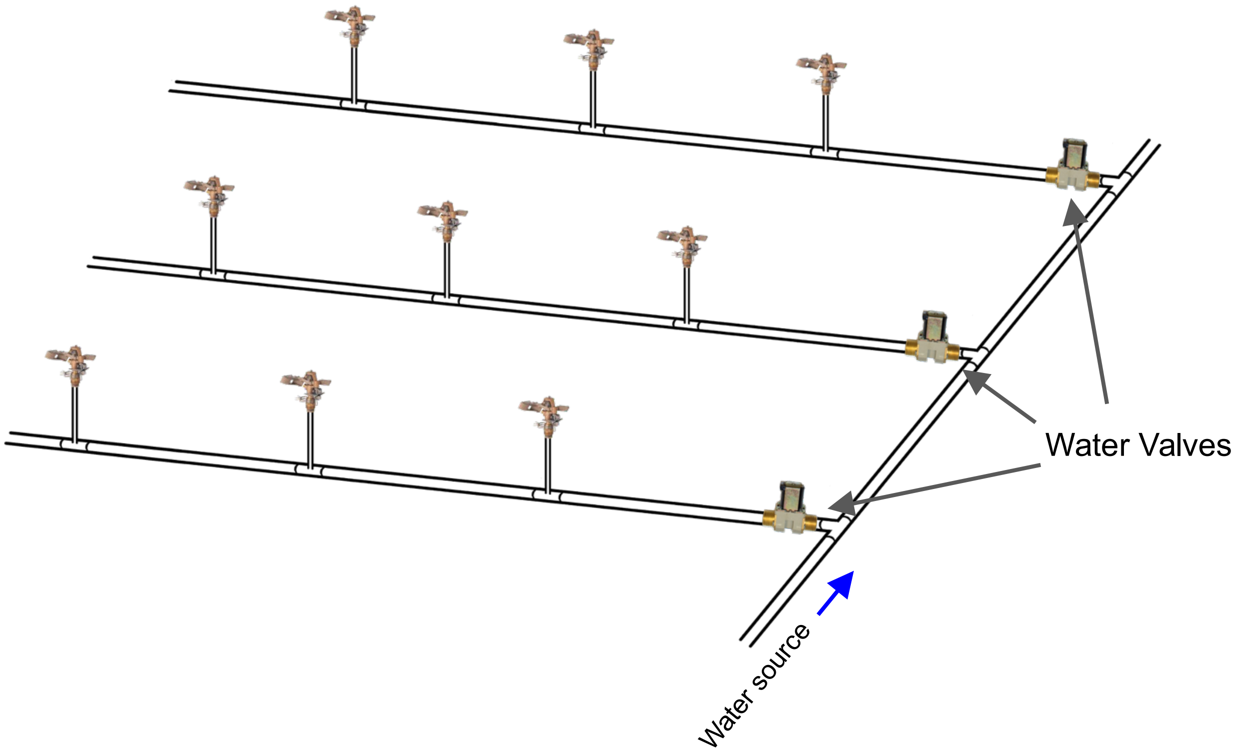
Irrigation System Architecture

Generally, valves are installed like this:



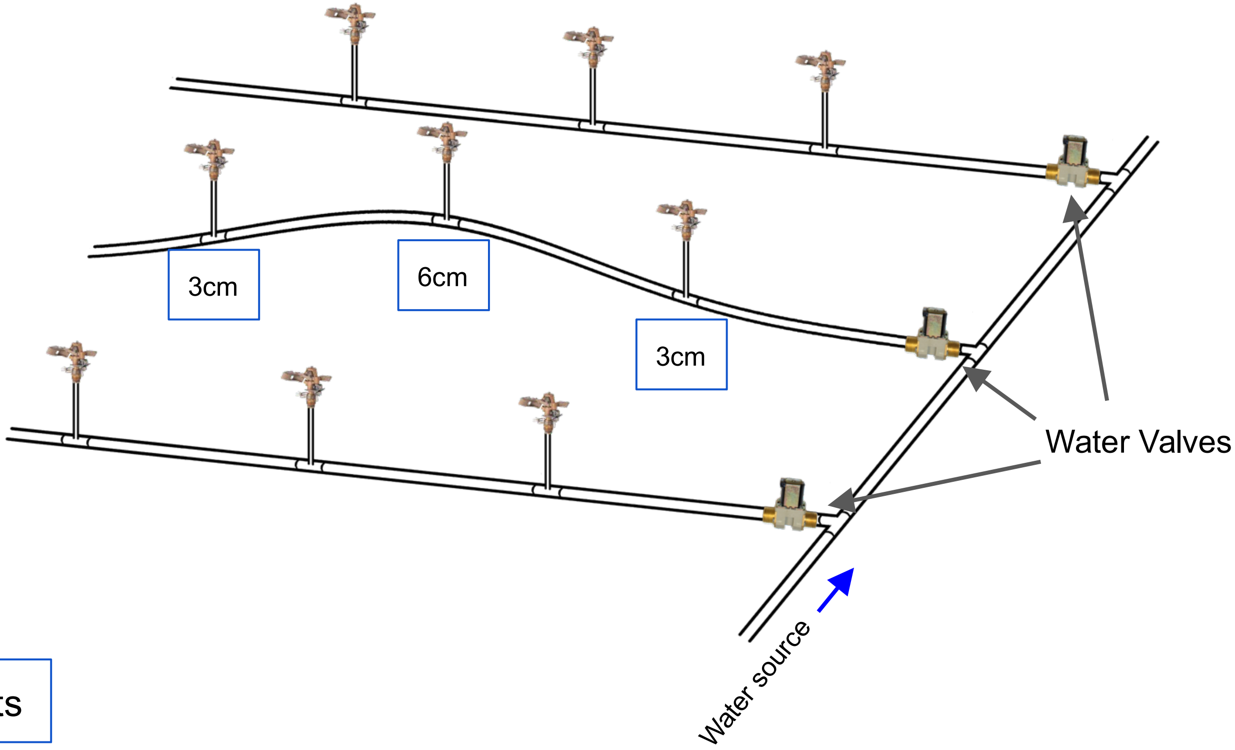
Irrigation System Architecture

Depending on the size of the system, the valves may be placed on each run, like this:



Irrigation System Architecture

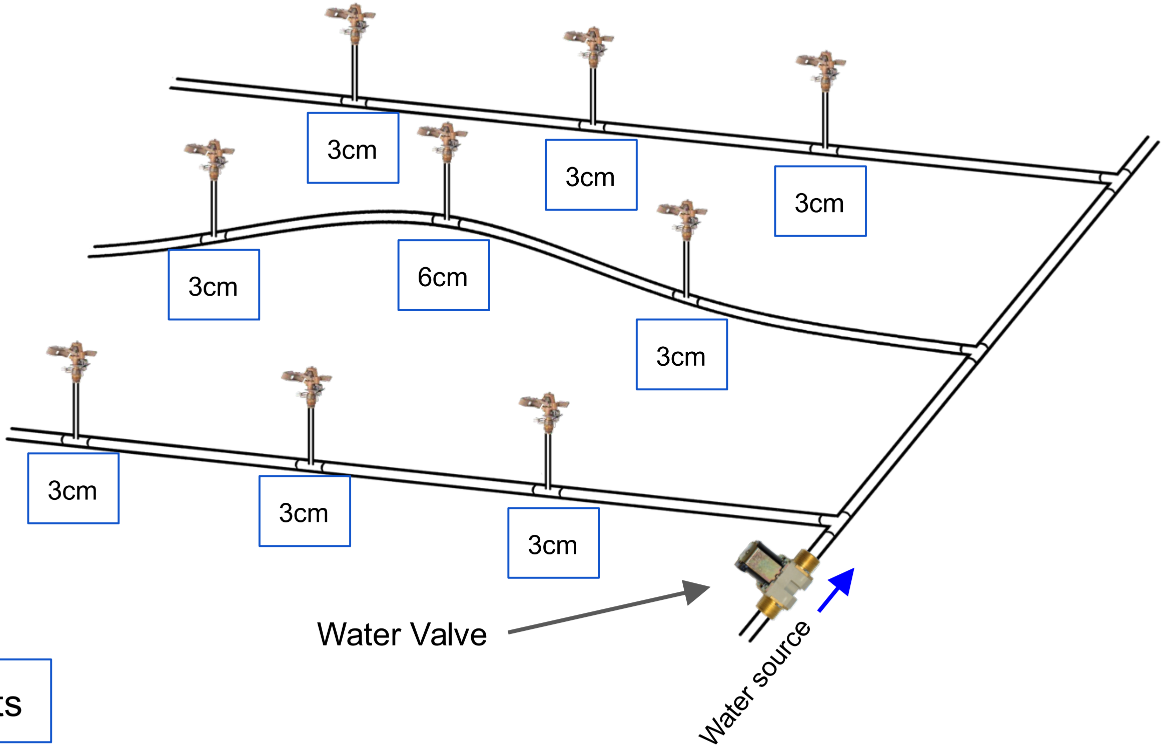
Water needs are not necessarily constant everywhere!



Water Requirements

Irrigation System Architecture

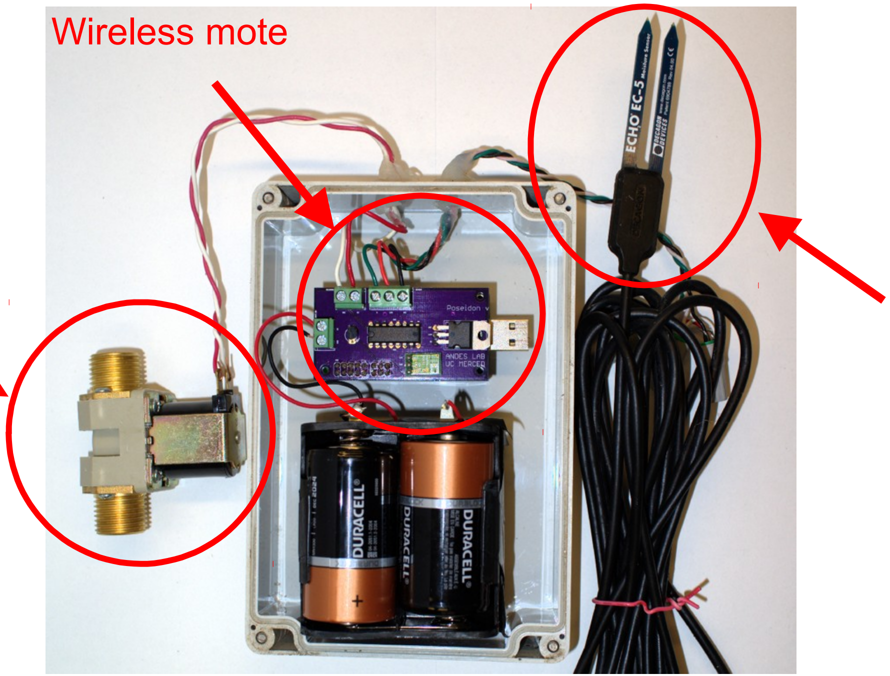
Water needs are not necessarily constant everywhere!



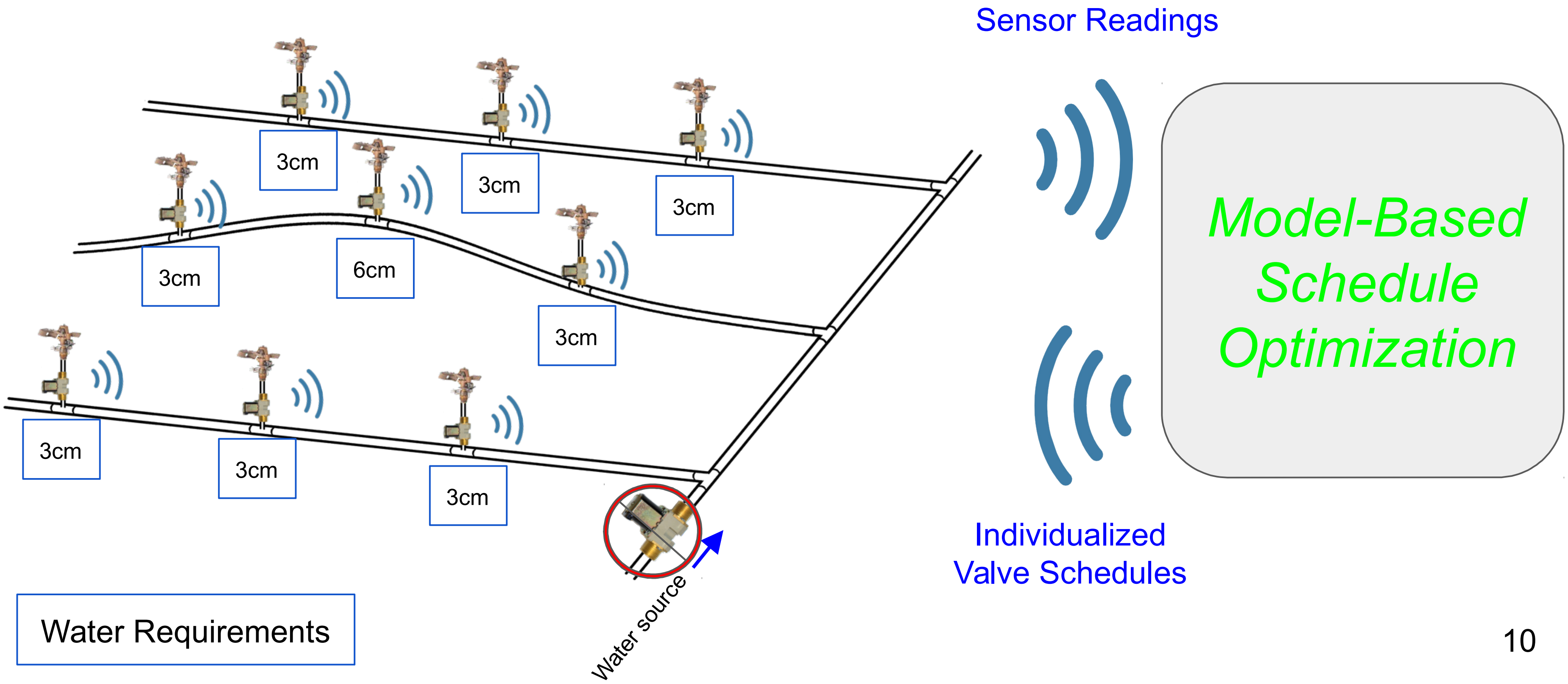
Water Requirements

Our proposed solution: *Distributed Actuation*

The MAGIC Node



Our proposed solution: *Distributed Actuation*



Water Requirements

Current Control Strategies

- Trial-and-error (most common by far)
- Sensor-based
- Weather-based
    - Rain-detection only
    - Evapotranspiration (ET)

"Give 45 minutes a try"

& Increase until turf seems to stay healthy

Control Valve

Current Control Strategies

- Trial-and-error (most common by far)
- Sensor-based
- Weather-based
  - Rain-detection only
  - Evapotranspiration (ET)

"Very Dry?
Irrigate 45 minutes"

"Already Saturated?
Irrigate 15 minutes"

Control Valve

Local Weather Station

Current Control Strategies

- Trial-and-error (most common by far)
- Sensor-based
- Weather-based
  - Rain-detection only
  - Evapotranspiration (ET)

State-of-the-art

“Based on today’s temperature, sun exposure, humidity, rain, and wind, you should irrigate 36 minutes”

Control Valve

How can we generate schedules for MAGIC nodes?

Helps, but extra effort required of the groundskeepers wouldn't be sustainable

This method does have potential, but some flaws exist...

Weather data won't provide us info for valves that are close spatially

Reactive control using sensors - what moisture conditions are we reacting to?

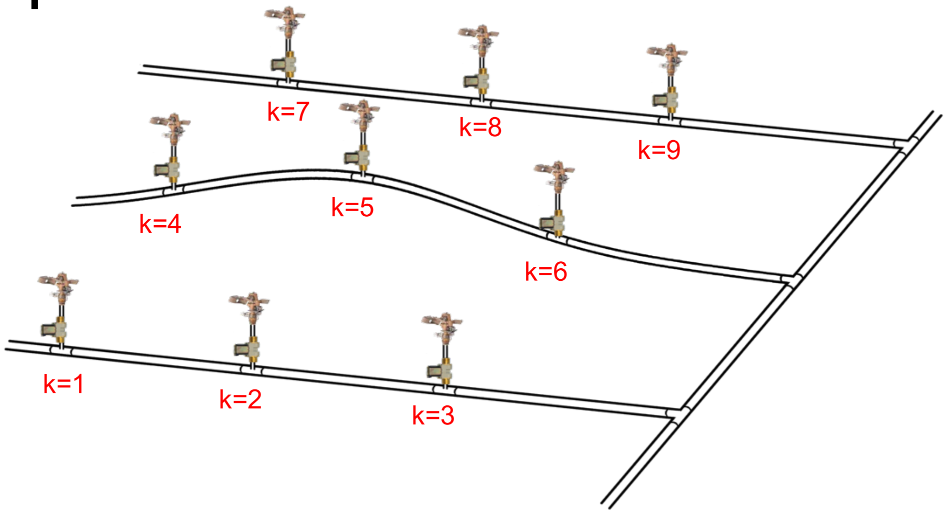
Reactive control using sensors - what moisture conditions are we reacting to?

Sensors + model of moisture movement - what moisture conditions are we reacting to?

Model-based Schedule Optimization

F

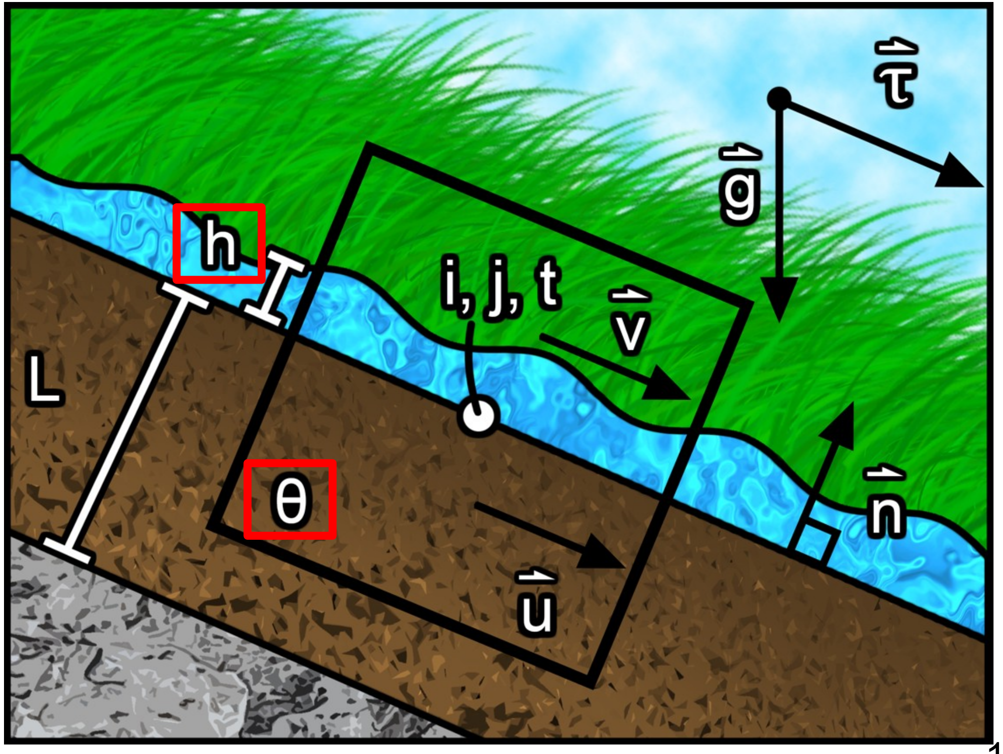
	k=1	2	3	...	K
t=0	0	1	1	1	1
1	0	1	1	0	1
2	1	0	0	0	0
...	1	0	0	0	0
N_t	1	1	0	1	1



$$\min \sum_{k=1}^{K} \sum_{t=0}^{N_t} f_{kt}$$

Soil moisture movement model

- PDE Model built from first principles
- Full details/justifications in the paper



Soil moisture movement model

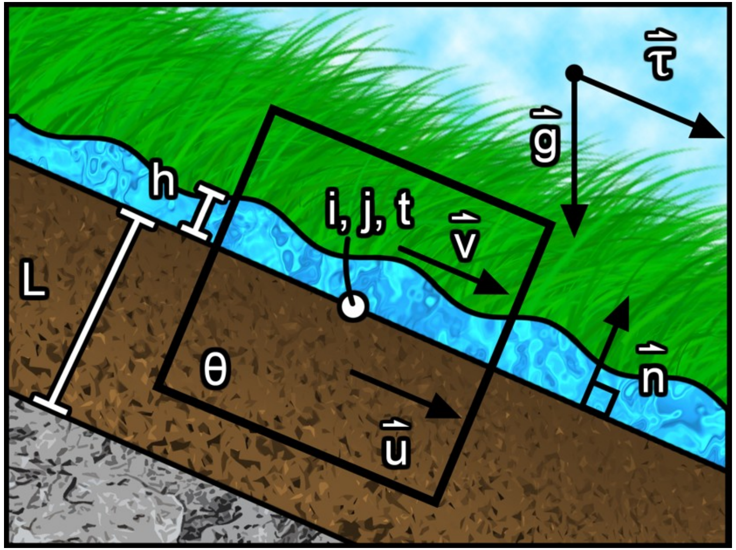
$$\boxed{\frac{\partial h}{\partial t} = F_s}$$

K sprinklers in total

Does sprinkler k reach me?

$$F_s = \sum_{k=1}^{K} c_k f_k(t)$$

Is sprinkler k on or off?



Soil moisture movement model

$$\frac{\partial h}{\partial t} = F_s \boxed{- \mu h K(\theta)}$$

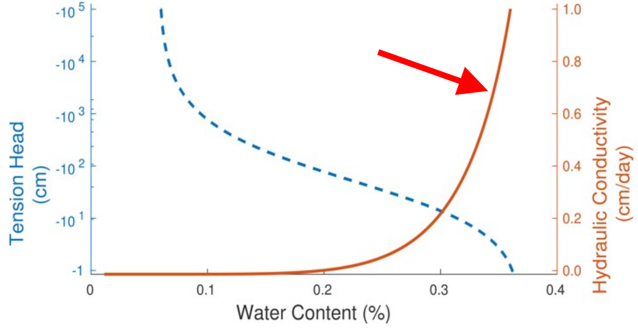
$$\frac{\partial \theta}{\partial t} = \boxed{\zeta h K(\theta)}$$

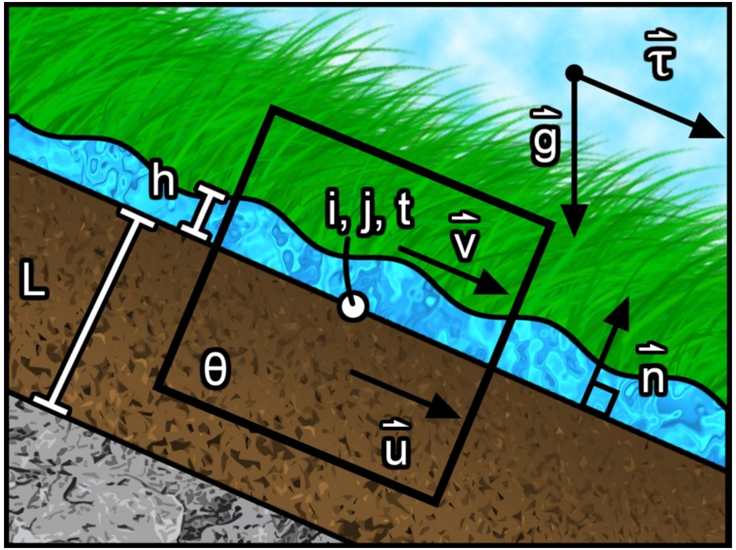
$$\zeta = 1/L^2 \phi_s$$

$$\mu = \zeta L \phi_s / \phi_g$$

$$F_s = \sum_{k=1}^{K} c_k f_k(t)$$

Conversion factors between height (H) and volumetric content (Θ)





Soil moisture movement model

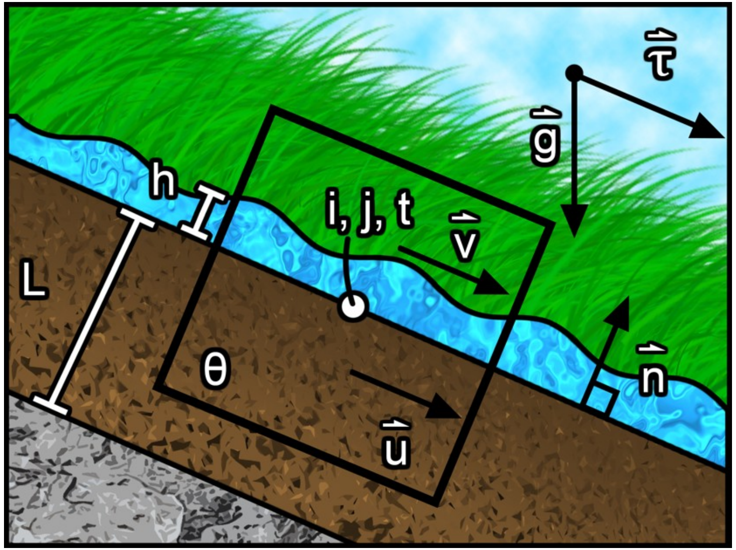
Surface fluid velocity

$$\frac{\partial h}{\partial t} = \boxed{-\nabla \cdot (h\vec{v})} + F_s - \mu h K(\theta)$$

$$\frac{\partial \theta}{\partial t} = \boxed{-\nabla \cdot (\theta\vec{u})} + \zeta h K(\theta)$$

Sub-Surface fluid velocity

$$F_s = \sum_{k=1}^{K} c_k f_k(t)$$



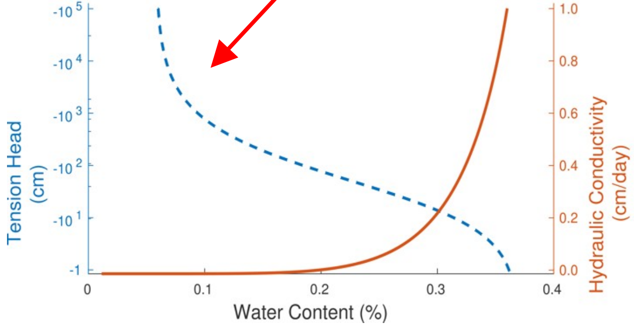
Soil moisture movement model

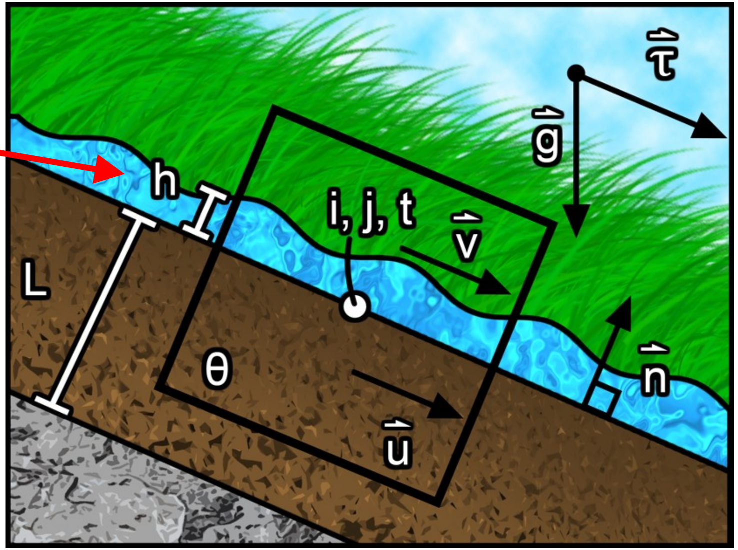
Darcy’s Law: Flow through porous media

$$\vec{u} = \frac{\kappa}{\eta}(-\nabla P + \vec{\tau})$$

Sub-surface velocity:

$$P = \rho g(\boxed{h} + \boxed{L\theta} + \psi(\theta))$$





Soil moisture movement model

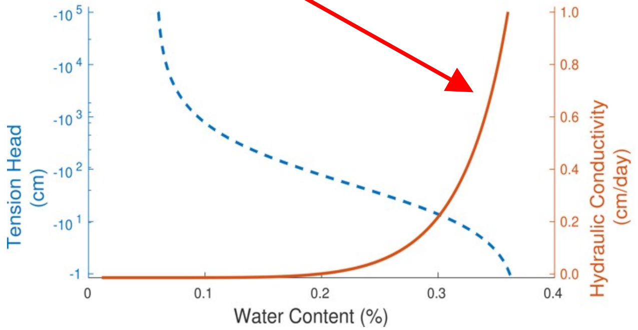
Darcy's Law: Flow through porous media

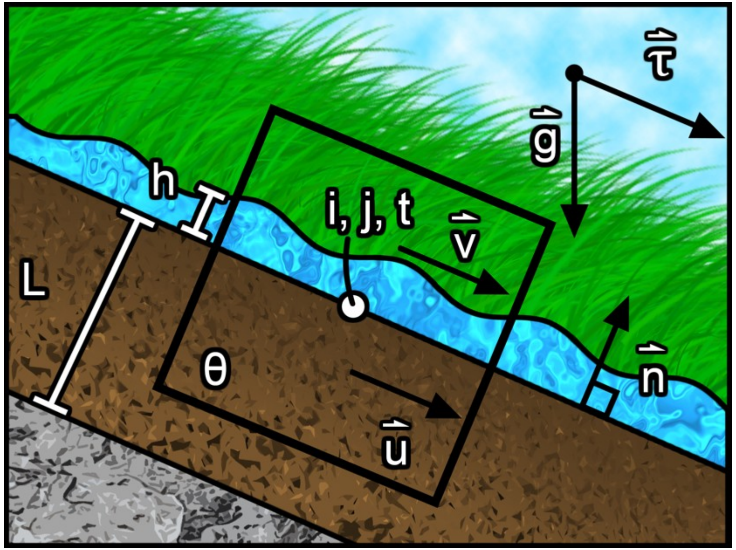
$$\vec{u} = \frac{\kappa}{\eta}(-\nabla P + \vec{\tau})$$

Sub-surface velocity:

$$P = \rho g(h + L\theta + \psi(\theta))$$

$$\boxed{\begin{aligned} K(\theta) &= \rho g \kappa(\theta)/\eta \\ \vec{u} &= -K(\theta)\nabla h + K(\theta)\vec{\tau} - K(\theta)(L + \psi'(\theta))\nabla\theta \end{aligned}}$$





Soil moisture movement model

Darcy’s Law: Flow through porous media

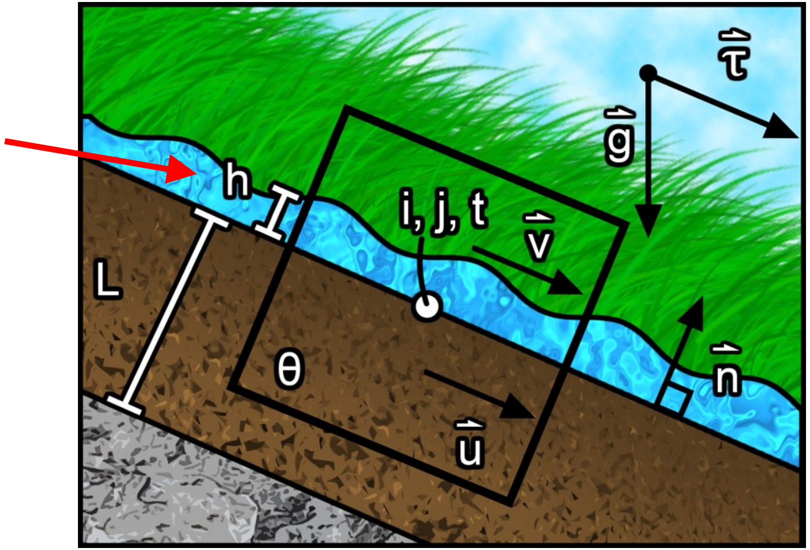
$$\vec{u} = \frac{\kappa}{\eta}(-\nabla P + \vec{\tau})$$

Sub-surface velocity:

$$P = \rho g(h + L\theta + \psi(\theta))$$
$$K(\theta) = \rho g \kappa(\theta)/\eta$$
$$\vec{u} = -K(\theta)\nabla h + K(\theta)\vec{\tau} - K(\theta)(L + \psi'(\theta))\nabla\theta$$

Surface velocity:

$$\vec{v} = \frac{\kappa_g}{\eta}(-\rho g \nabla \boxed{h} - \vec{\tau})$$



Soil moisture movement model

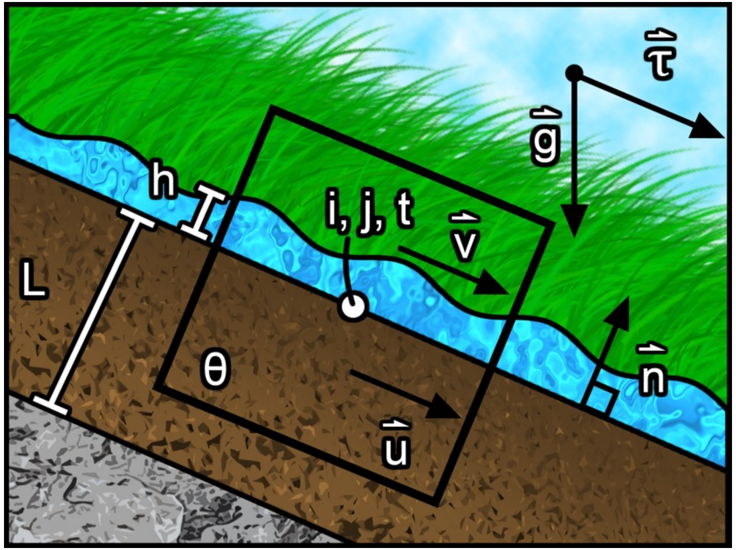
Bringing the terms all together, the final model is defined as follows:

$$\frac{\partial h}{\partial t} = -\nabla \cdot (h\vec{v}) + F_s - \mu h K(\theta) \qquad F_s = \sum_{k=1}^{K} c_k f_k(t)$$

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot (\theta \vec{u}) + \zeta h K(\theta)$$

$$\vec{u} = -K(\theta)\nabla h + K(\theta)\vec{\tau} - K(\theta)(L + \psi'(\theta))\nabla\theta$$

$$\vec{v} = \frac{\kappa_g}{\eta}(-\rho g \nabla h - \vec{\tau})$$



Soil moisture movement model

Non-linearities in model problematic for optimization

- No guarantee of global minimum (non-convex feasible set)
- Non-linear optimization is considerably slower

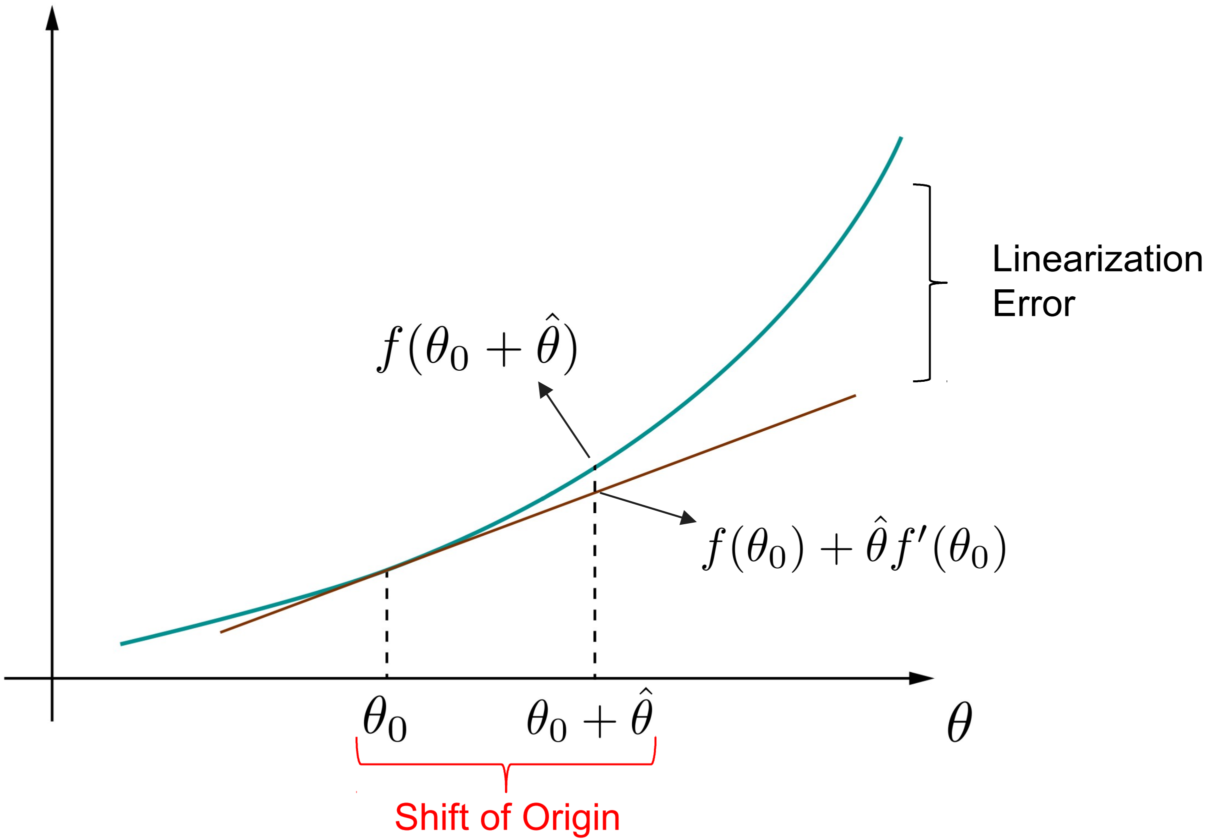
$$\frac{\partial h}{\partial t} = -\nabla \cdot (\boxed{h\vec{v}}) + F_s - \mu \boxed{hK(\theta)} \qquad F_s = \sum_{k=1}^{K} c_k f_k(t)$$

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot (\boxed{\theta\vec{u}}) + \zeta \boxed{hK(\theta)}$$

$$\vec{u} = -\boxed{K(\theta)\nabla h} + \boxed{K(\theta)}\vec{\tau} - \boxed{K(\theta)(L + \psi'(\theta))\nabla\theta}$$

$$\vec{v} = \frac{\kappa_g}{\eta}(-\rho g \nabla h - \vec{\tau})$$

Model Linearization



Model Linearization

$$\theta = \overbrace{\theta_0}^{\text{Constant}} + \overbrace{\hat{\theta}}^{\text{Linear}} \longrightarrow \text{Replicated on each model variable}$$

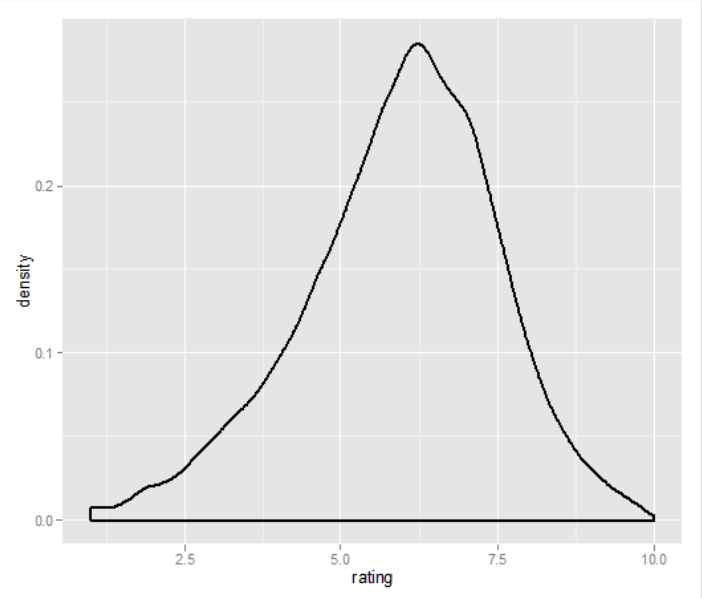
$$\varphi(\theta) \;=\; K(\theta)(L + \psi'(\theta))$$ (Substituted for readability)

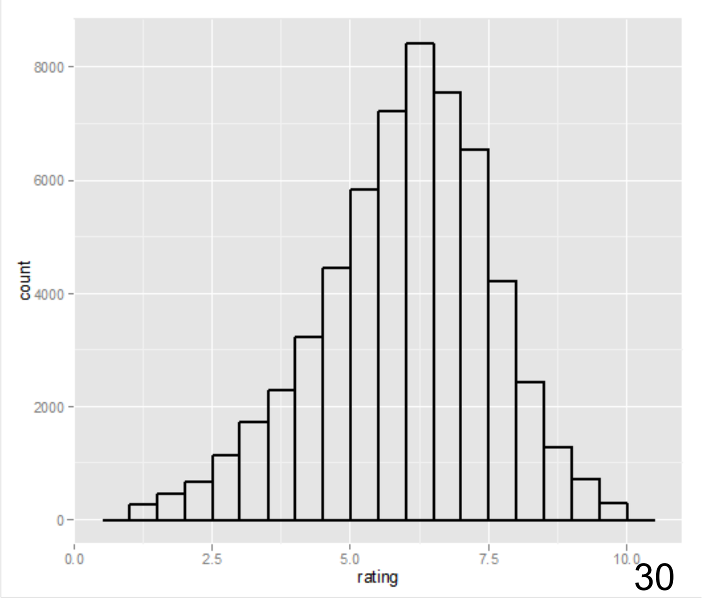
$$\begin{aligned}
\frac{\partial h}{\partial t} &= -\nabla \cdot (\hat{h}\vec{v}_0 + h_0\hat{\vec{v}}) + F_s - \eta\Big(h_0K(\theta_0) + h_0K'(\theta_0)\hat{\theta} + \hat{h}K(\theta_0) + \cancel{\underline{\hat{h}K'(\theta_0)\hat{\theta}}}\Big) \\
\frac{\partial \theta}{\partial t} &= -\nabla \cdot (\hat{\theta}\vec{u}_0 + \theta_0\hat{\vec{u}}) + \zeta\Big(h_0K(\theta_0) + h_0K'(\theta_0)\hat{\theta} + \hat{h}K(\theta_0) + \cancel{\underline{\hat{h}K'(\theta_0)\hat{\theta}}}\Big) \\
\vec{u} &= -K(\theta_0)\nabla\hat{h} - K(\theta_0)\nabla h_0 - K'(\theta_0)\hat{\theta}\nabla h_0 - \cancel{\underline{K'(\theta_0)\hat{\theta}\nabla\hat{h}}} + K(\theta)\vec{\tau} - \varphi(\theta_0)\nabla\theta_0 - \varphi(\theta_0)\nabla\hat{\theta} - \varphi'(\theta_0)\hat{\theta}\nabla\theta_0 - \cancel{\underline{\varphi'(\theta_0)\hat{\theta}\nabla\hat{\theta}}} \\
\vec{v} &= -\alpha_h\nabla h + \vec{\tau}
\end{aligned}$$

Model discretization

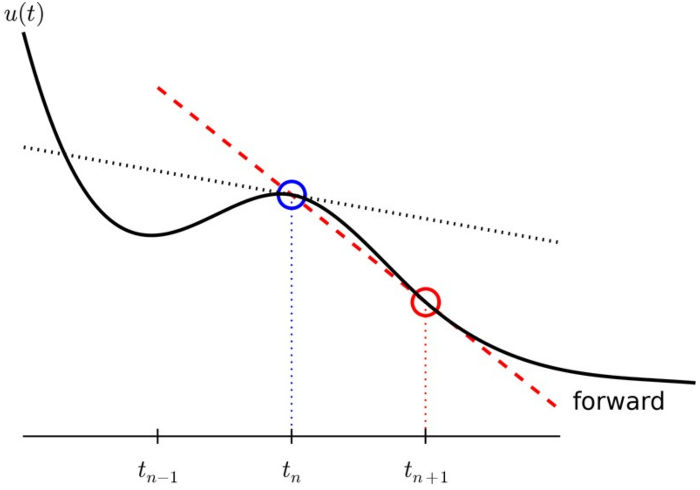
Moving variables from a continuous to a discrete space

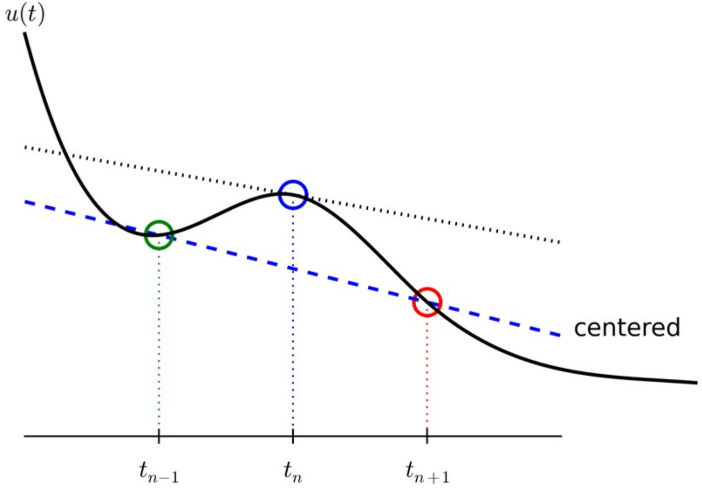
For example:





Model discretization





$$\frac{du}{dt} \approx \frac{u(t_{n+1}) - u(t_n)}{\Delta t}$$

$$\frac{du}{dt} \approx \frac{u(t_{n+1}) - u(t_{n-1})}{2\Delta t}$$

Linear, discretized

$$\frac{h_{i,j,t+1}-h_{i,j,t}}{\Delta t} = -\frac{1}{2\Delta x}\Big((\hat{h}v_0^x + h_0\hat{v}^x)_{i+1,j,t} - (\hat{h}v_0^x + h_0\hat{v}^x)_{i-1,j,t} + (\hat{h}v_0^y + h_0\hat{v}^y)_{i,j+1,t} - (\hat{h}v_0^y + h_0\hat{v}^y)_{i,j-1,t}\Big)$$
$$+F_s - \eta\Big(h_0K(\theta_0) + h_0K'(\theta_0)\hat{\theta} + \hat{h}K(\theta_0) + \hat{h}K'(\theta_0)\hat{\theta}\Big)$$

$$\frac{\theta_{i,j,t+1}-\theta_{i,j,t}}{\Delta t} = -\frac{1}{2\Delta x}\Big((\hat{\theta}u_0^x + \theta_0\hat{u}^x)_{i+1,j,t} - (\hat{\theta}u_0^x + \theta_0\hat{u}^x)_{i-1,j,t} + (\hat{\theta}u_0^y + \theta_0\hat{u}^y)_{i,j+1,t} - (\hat{\theta}u_0^y + \theta_0\hat{u}^y)_{i,j-1,t}\Big)$$
$$+\zeta\Big(h_0K(\theta_0) + h_0K'(\theta_0)\hat{\theta} + \hat{h}K(\theta_0) + \hat{h}K'(\theta_0)\hat{\theta}\Big)$$

$$u^x_{i,j,t} = -\frac{K(\theta_{0\,i,j,t})}{2\Delta x}(\hat{h}_{i+1,j,t} - \hat{h}_{i-1,j,t} + h_{0\,i+1,j,t} - h_{0\,i-1,j,t}) - K'(\theta_{0\,i,j,t})\hat{\theta}_{i,j,t}(h_{0\,i+1,j,t} - h_{0\,i-1,j,t}) + K(\theta_{i,j,t})\vec{\tau}^x$$
$$-\frac{\varphi(\theta_{0\,i,j,t})}{2\Delta x}(\theta_{0\,i+1,j,t} - \theta_{0\,i-1,j,t} + \hat{\theta}_{i+1,j,t} - \hat{\theta}_{i-1,j,t}) - \varphi'(\theta_{0\,i,j,t})(\theta_{0\,i+1,j,t} - \theta_{0\,i-1,j,t})\hat{\theta}_{i,j,t}$$

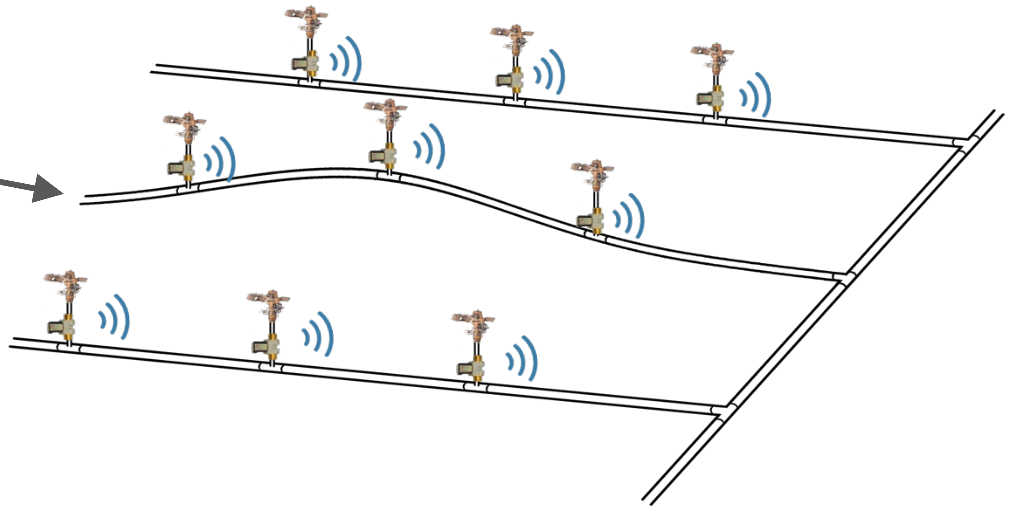
$$u^y_{i,j,t} = -\frac{K(\theta_{0\,i,j,t})}{2\Delta x}(\hat{h}_{i,j+1,t} - \hat{h}_{i,j-1,t} + h_{0\,i,j+1,t} - h_{0\,i,j-1,t}) - K'(\theta_{0\,i,j,t})\hat{\theta}_{i,j,t}(h_{0\,i,j+1,t} - h_{0\,i,j-1,t}) + K(\theta_{i,j,t})\vec{\tau}^y$$
$$-\frac{\varphi(\theta_{0\,i,j,t})}{2\Delta x}(\theta_{0\,i,j+1,t} - \theta_{0\,i,j-1,t} + \hat{\theta}_{i,j+1,t} - \hat{\theta}_{i,j-1,t}) - \varphi'(\theta_{0\,i,j,t})(\theta_{0\,i,j+1,t} - \theta_{0\,i,j-1,t})\hat{\theta}_{i,j,t}$$

$$v^x_{i,j,t} = -\alpha_h\left(\frac{h_{i+1,j,t}-h_{i-1,j,t}}{2\Delta x}\right) + \vec{\tau}$$

$$v^y_{i,j,t} = -\alpha_h\left(\frac{h_{i,j+1,t}-h_{i,j-1,t}}{2\Delta x}\right) + \vec{\tau}$$

Initial / Boundary Conditions

Get initial soil moisture conditions
from current soil measurements!



Dirichlet boundary conditions

- Fix function value on boundary
- Others could be used...

Boundary conditions fixed on H, θ chosen to be as small as reasonable

Optimization Problem Definition

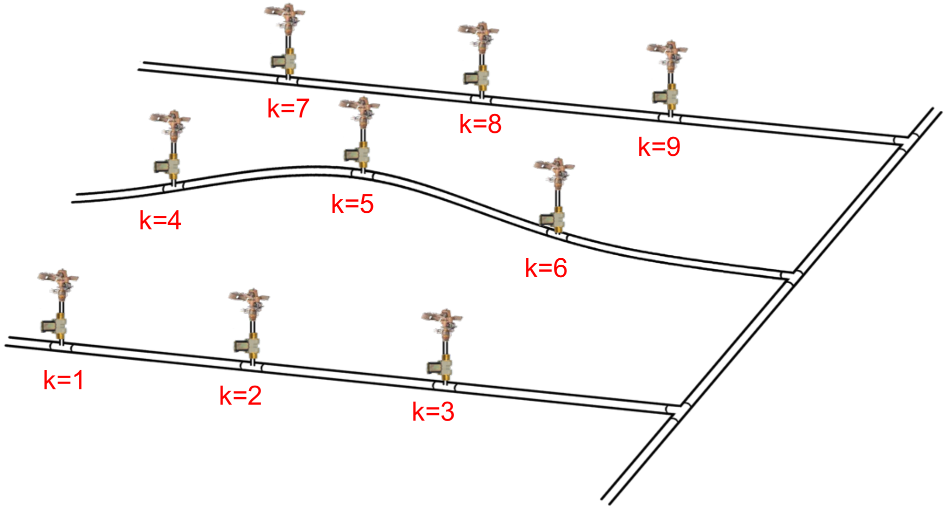
$$\min_{\{f_{kt}, h_{ijt}, \theta_{ijt}, u^x_{ijt}, u^y_{ijt}, v^x_{ijt}, v^y_{ijt}\}_{i,j,t=0}^{N_x, N_y, N_t}} \sum_{k=1}^{K} \sum_{t=0}^{N_t} \boxed{f_{kt}} \quad \text{s.t.}$$

(Sprinkler limitations) $0 \leq \boxed{f_{kt}} \leq 1 \quad k=1,\ldots,K; \; t=0,\ldots,N_t$

$$\theta_\text{l} \leq \theta_{ijt} \leq \theta_\text{u} \quad i=0,\ldots,N_x; \; j=0,\ldots,N_y; \; t=0,\ldots,N_t$$

$$\theta_\text{pwp} \leq \theta_{ijN_t} \quad i=0,\ldots,N_x; \; j=0,\ldots,N_y$$

PDE Model Equations



Optimization Problem Definition

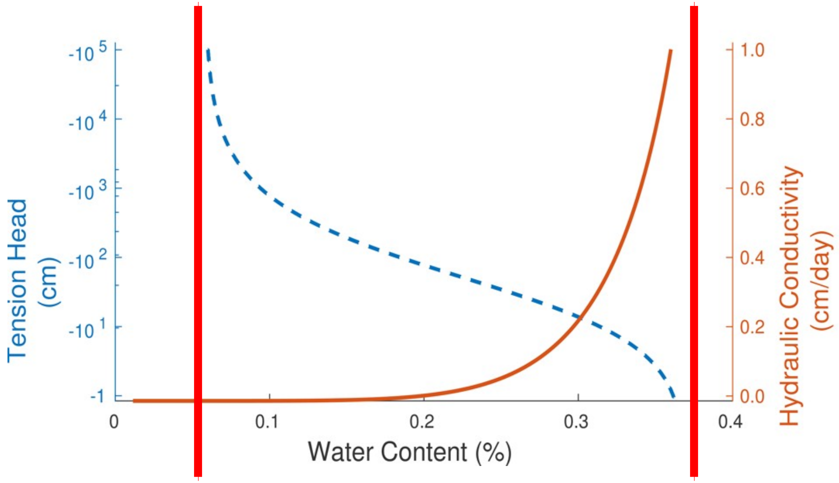
$$\min_{\{f_{kt},h_{ijt},\theta_{ijt},u^x_{ijt},u^y_{ijt},v^x_{ijt},v^y_{ijt}\}_{i,j,t=0}^{N_x,N_y,N_t}} \sum_{k=1}^{K}\sum_{t=0}^{N_t} f_{kt} \quad \text{s.t.}$$

$$0 \leq f_{kt} \leq 1 \qquad \begin{matrix} k=1,\ldots,K \\ t=0,\ldots,N_t \end{matrix}$$

(Soil limitations) $$\boxed{\theta_\mathrm{l}} \leq \theta_{ijt} \leq \boxed{\theta_\mathrm{u}} \qquad \begin{matrix} i=0,\ldots,N_x \\ j=0,\ldots,N_y \\ t=0,\ldots,N_t \end{matrix}$$

$$\theta_\mathrm{pwp} \leq \theta_{ijN_t} \qquad \begin{matrix} i=0,\ldots,N_x \\ j=0,\ldots,N_y \end{matrix}$$

PDE Model Equations



Optimization Problem Definition

$$\min_{\{f_{kt},h_{ijt},\theta_{ijt},u^x_{ijt},u^y_{ijt},v^x_{ijt},v^y_{ijt}\}_{i,j,t=0}^{N_x,N_y,N_t}} \sum_{k=1}^{K}\sum_{t=0}^{N_t} f_{kt} \quad \text{s.t.}$$

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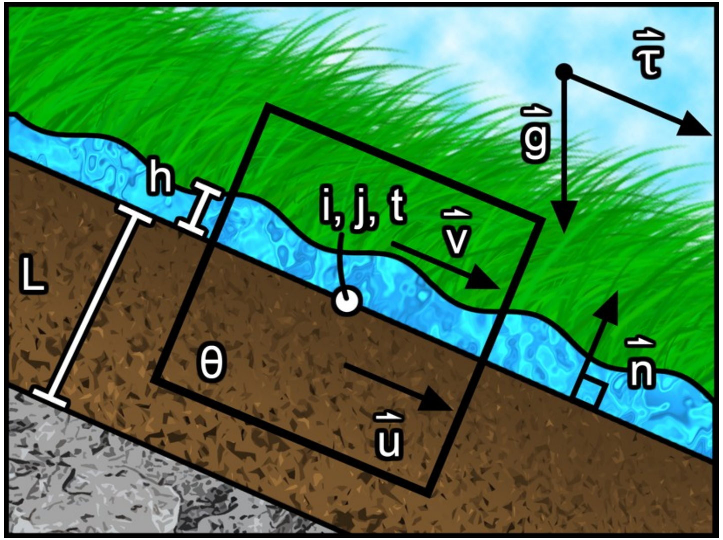
(Plant limitations) $$\boxed{\theta_\mathrm{pwp}} \leq \theta_{ijN_t} \qquad \begin{matrix} i=0,\dots,N_x \\ j=0,\dots,N_y \end{matrix}$$

PDE Model Equations



Optimization Problem Definition

$$\min_{\{f_{kt}, h_{ijt}, \theta_{ijt}, u^x_{ijt}, u^y_{ijt}, v^x_{ijt}, v^y_{ijt}\}_{i,j,t=0}^{N_x, N_y, N_t}} \sum_{k=1}^{K} \sum_{t=0}^{N_t} f_{kt} \quad \text{s.t.}$$

$$0 \leq f_{kt} \leq 1 \qquad \begin{matrix} k=1,\ldots,K \\ t=0,\ldots,N_t \end{matrix}$$

$$\theta_{\mathrm{l}} \leq \theta_{ijt} \leq \theta_{\mathrm{u}} \qquad \begin{matrix} i=0,\ldots,N_x \\ j=0,\ldots,N_y \\ t=0,\ldots,N_t \end{matrix}$$

$$\theta_{\mathrm{pwp}} \leq \theta_{ijN_t} \qquad \begin{matrix} i=0,\ldots,N_x \\ j=0,\ldots,N_y \end{matrix}$$

(Model constraints)

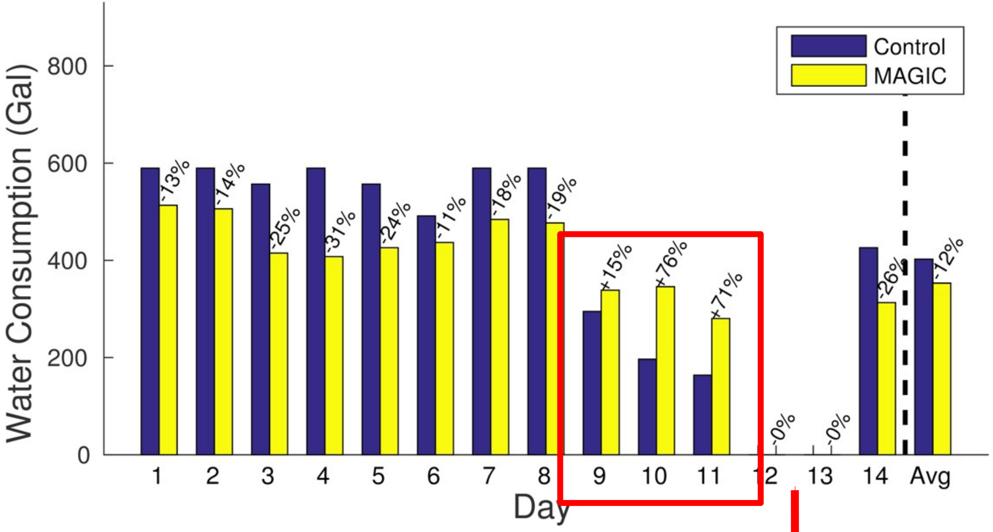
PDE Model Equations

$$\begin{aligned}
\frac{h_{i,j,t+1}-h_{i,j,t}}{\Delta t} &= -\frac{1}{2\Delta x}\Big((\hat{h}v_0^x + h_0\hat{v}^x)_{i+1,j,t} - (\hat{h}v_0^x + h_0\hat{v}^x)_{i-1,j,t} + (\hat{h}v_0^y + h_0\hat{v}^y)_{i,j+1,t} - (\hat{h}v_0^y + h_0\hat{v}^y)_{i,j-1,t}\Big) \\
&\quad + F_s - \eta\Big(h_0K(\theta_0) + h_0K'(\theta_0)\hat{\theta} + \hat{h}K(\theta_0) + \hat{h}K'(\theta_0)\hat{\theta}\Big) \\
\frac{\theta_{i,j,t+1}-\theta_{i,j,t}}{\Delta t} &= -\frac{1}{2\Delta x}\Big((\hat{\theta}u_0^x + \theta_0\hat{u}^x)_{i+1,j,t} - (\hat{\theta}u_0^x + \theta_0\hat{u}^x)_{i-1,j,t} + (\hat{\theta}u_0^y + \theta_0\hat{u}^y)_{i,j+1,t} - (\hat{\theta}u_0^y + \theta_0\hat{u}^y)_{i,j-1,t}\Big) \\
&\quad + \zeta\Big(h_0K(\theta_0) + h_0K'(\theta_0)\hat{\theta} + \hat{h}K(\theta_0) + \hat{h}K'(\theta_0)\hat{\theta}\Big) \\
u^x_{i,j,t} &= -\frac{K(\theta_{0\,i,j,t})}{2\Delta x}(\hat{h}_{i+1,j,t} - \hat{h}_{i-1,j,t} + h_{0\,i+1,j,t} - h_{0\,i-1,j,t}) - K'(\theta_{0\,i,j,t})\hat{\theta}_{i,j,t}(h_{0\,i+1,j,t} - h_{0\,i-1,j,t}) + K(\theta_{i,j,t})\vec{\tau}^x \\
&\quad - \frac{\varphi(\theta_{0\,i,j,t})}{2\Delta x}(\theta_{0\,i+1,j,t} - \theta_{0\,i-1,j,t} + \hat{\theta}_{i+1,j,t} - \hat{\theta}_{i-1,j,t}) - \varphi'(\theta_{0\,i,j,t})(\theta_{0\,i+1,j,t} - \theta_{0\,i-1,j,t})\hat{\theta}_{i,j,t} \\
u^y_{i,j,t} &= -\frac{K(\theta_{0\,i,j,t})}{2\Delta x}(\hat{h}_{i,j+1,t} - \hat{h}_{i,j-1,t} + h_{0\,i,j+1,t} - h_{0\,i,j-1,t}) - K'(\theta_{0\,i,j,t})\hat{\theta}_{i,j,t}(h_{0\,i,j+1,t} - h_{0\,i,j-1,t}) + K(\theta_{i,j,t})\vec{\tau}^y \\
&\quad - \frac{\varphi(\theta_{0\,i,j,t})}{2\Delta x}(\theta_{0\,i,j+1,t} - \theta_{0\,i,j-1,t} + \hat{\theta}_{i,j+1,t} - \hat{\theta}_{i,j-1,t}) - \varphi'(\theta_{0\,i,j,t})(\theta_{0\,i,j+1,t} - \theta_{0\,i,j-1,t})\hat{\theta}_{i,j,t} \\
v^x_{i,j,t} &= -\alpha_h\left(\frac{h_{i+1,j,t}-h_{i-1,j,t}}{2\Delta x}\right) + \vec{\tau} \\
v^y_{i,j,t} &= -\alpha_h\left(\frac{h_{i,j+1,t}-h_{i,j-1,t}}{2\Delta x}\right) + \vec{\tau}
\end{aligned}$$

System Deployment

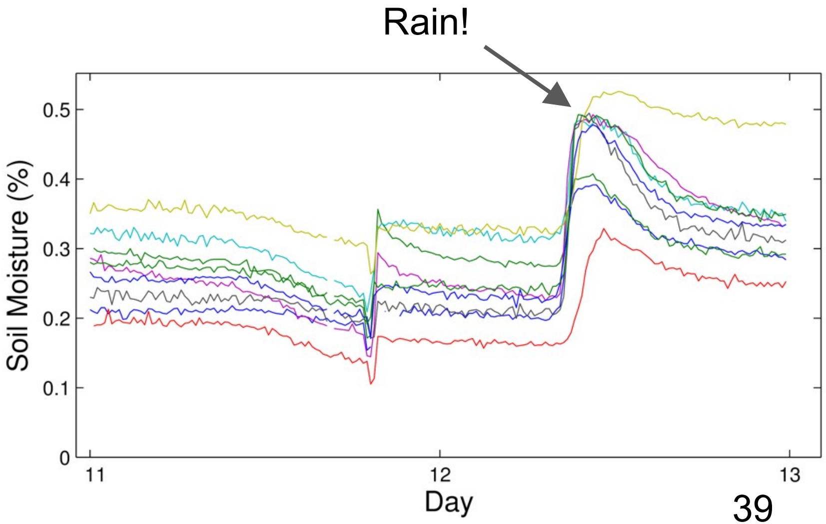
UC Merced’s “Bowl”

System Analysis - Water Consumption



12.3% less water used
vs evapotranspiration

23.4% less water used
vs trial-and-error



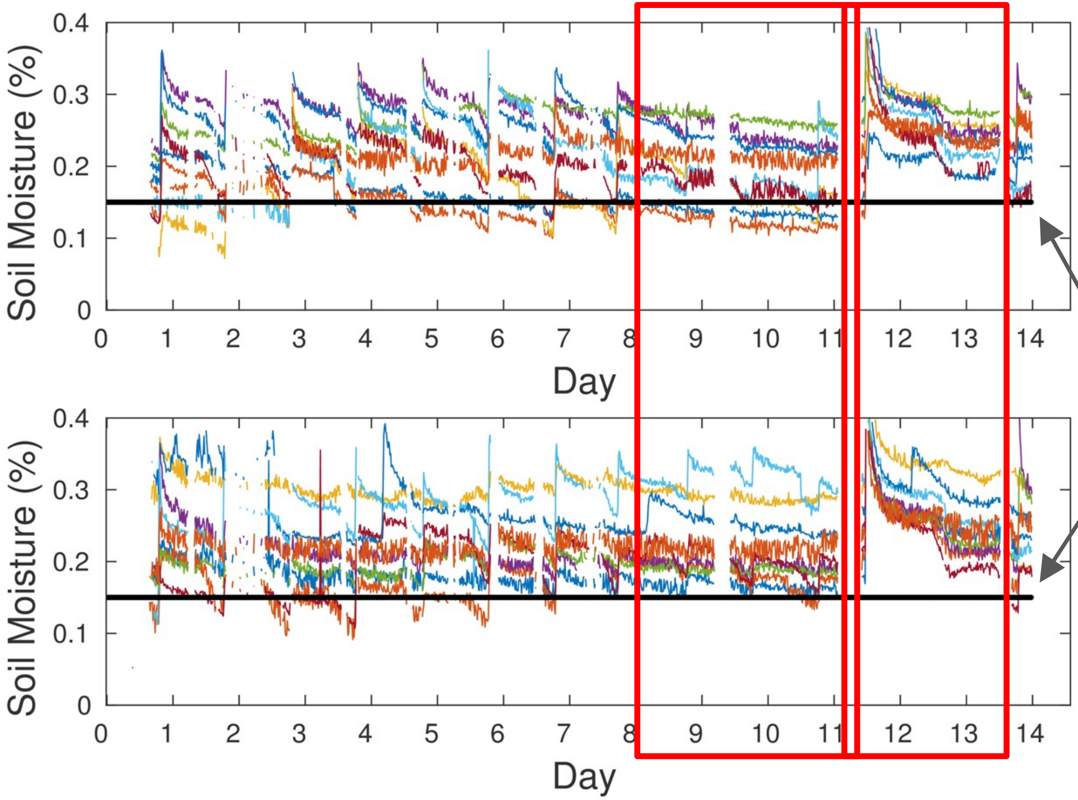
System Analysis - Quality of Irrigation

Evapotranspiration (Control)

68.1 Combined hours below

MAGIC

16.7 Combined hours below



Below this threshold, turf start withering!

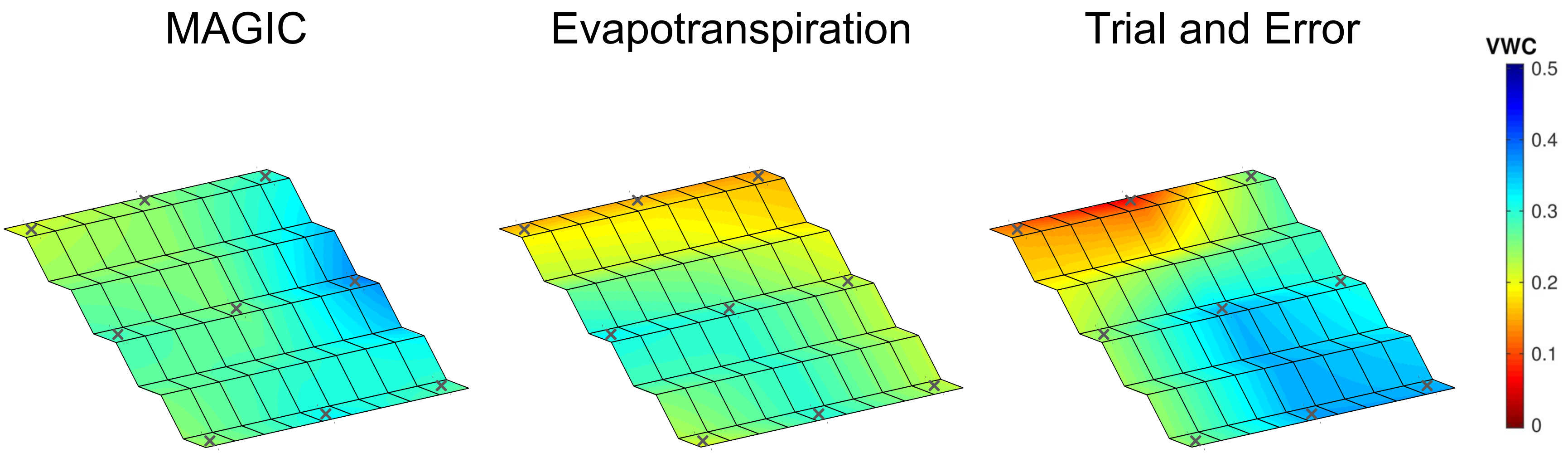
4.08x less

vs ET

3.23x less

vs trial-and-error

System Analysis - Moisture Distribution

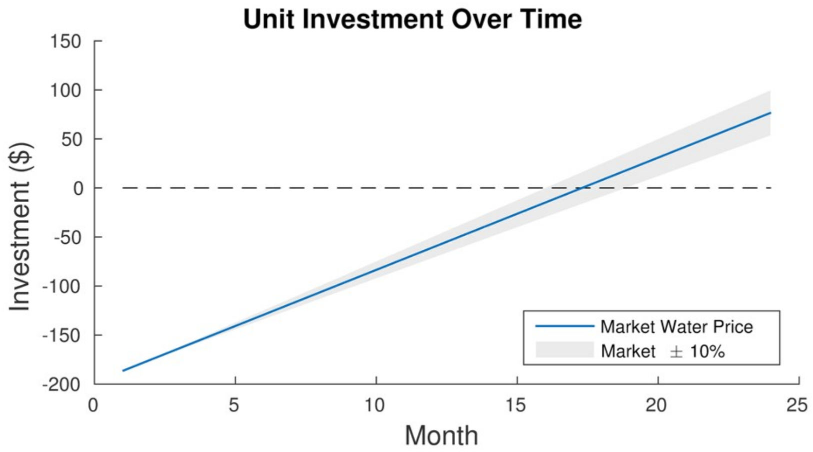


Average water distribution, interpolated

Return on Investment (ROI) analysis

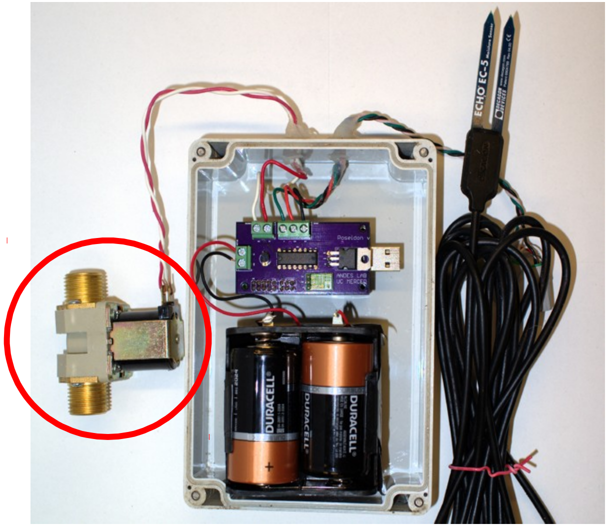
Unit Cost

Component	Price
Mote	\$37.57
Moisture Sensor	\$110
Batteries	\$4
Solenoid	\$15
Waterproof Enclosure	\$10
Manufacture & Assembly	\$10
	\$186.57



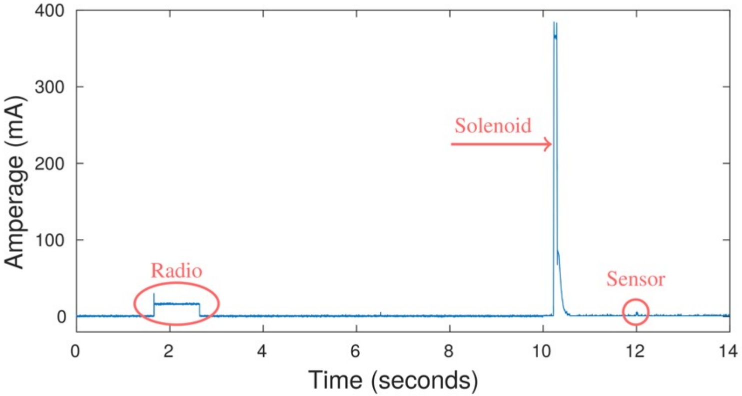
Even with 20% business markup, ROI in 18-23 months!

Node power management



2+ year lifetime (current prototype)

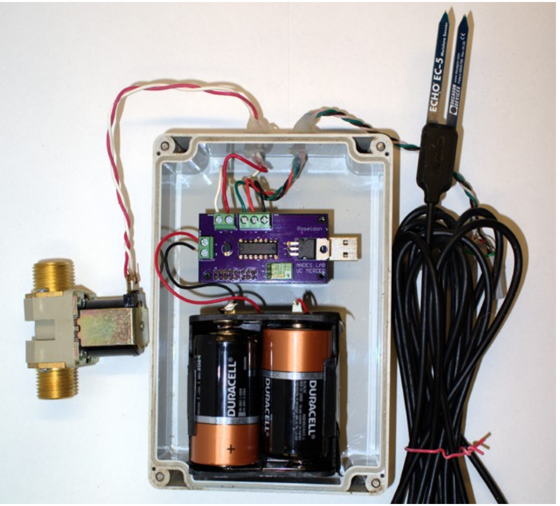
14+ year lifetime with smarter radio use



Future goals

- Explicit weather inclusion
- Sensor error -> model functionality
- Optimal sprinkler placement in new irrigation systems
- Data-driven model generation

Conclusions



- *Distributed Actuation* - MAGIC node allows us to actuate individual sprinkler heads according to schedules we provide
- *Model-Based Schedule Optimization* – Compute schedules that minimize water usage but satisfy minimum moisture requirements
- Across 7-weeks of deployment, MAGIC system is found to improve irrigation efficiency 12.3% - 23.4% while improving irrigation quality up to 4x !!