FREE-FORM NON-RIGID IMAGE REGISTRATION:
USING GENERALIZED ELASTIC NETS

Andriy Myronenko, Xubo Song and Miguel Á. Carreira-Perpiñán

Department of Computer Science & Electrical Engineering, OGI, OHSU
Portland, OR, USA

1 ABSTRACT

We introduce a novel probabilistic approach for nonrigid image registration using generalized elastic nets, a model previously used for topographic maps. The idea of the algorithm is to adapt an elastic net (a constrained Gaussian mixture) in the spatial-intensity space of one image to fit the second image. The resulting net directly represents the correspondence between image pixels in a probabilistic manner. The centroids implicitly represent a quadratic prior on its centroids. The centroids represent the correspondence between image pixels in a probabilistic way and recover the underlying image deformation. We regularize the net with a differential prior and develop an efficient optimization algorithm using linear conjugate gradients. The nonparametric formulation allows for complex transformations having local deformation.

2 METHOD

The elastic net is a Gaussian mixture model (GMM) with a quadratic prior on its centroids. The centroids represent a nonlinear manifold that probabilistically maps an image in spatial-intensity space.

- We represent two images I₁ and I₂ in the spatial-intensity space.
- The elastic net Y is initialized with each centroid representing the spatial-intensity value of one pixel in I₁.
- The net is adapted by adjusting the centroids to fit data X (image I₂ in spatial-intensity space) by MAP estimation.
- the final centroid locations (Y), when E is minimized, directly show the displacement of each pixel in I₁ deformed into I₂.

Minimize: \( E(Y) = -\sum_{n=1}^{N} \log \sum_{m=1}^{M} e^{-\frac{1}{2}||y_n - m||^2} + \frac{1}{2} \text{tr}(Y^T S Y) \)

where \( S = \beta_1 D_1^T D_1 + \beta_2 D_2^T D_2 \), and \( D_1, D_2 \) are first- and second-order derivatives. Weights \( \beta_1 \) and \( \beta_2 \) control tearing and folding regularization. \( \|D_1 Y\|^2 = \text{tr}((D_1 Y)^T(D_1 Y)) = \text{tr}(Y^T S Y) \)

<table>
<thead>
<tr>
<th>Deform.</th>
<th>GEN</th>
<th>Intensity</th>
<th>ITK</th>
</tr>
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<tbody>
<tr>
<td>STD</td>
<td>RMSE (pixels)</td>
<td>RMSE</td>
<td>RMSE</td>
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<tr>
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<td>3.0</td>
<td>1.0062</td>
<td>0.0059</td>
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3 EXPERIMENTAL RESULTS

4 CONCLUSION

We have developed a probabilistic nonrigid image registration method based on the generalized elastic net. The resulting formulation allows for complex and localized deformations flexibly without prior knowledge about the type of transformation required, and to use sophisticated regularizers (e.g. higher-order derivatives and linear combinations of them). The structured, sparse nature of the regularizer matrix allows an efficient optimization with linear conjugate gradients, faster than thin-plate splines. The method accurately registers images with nonlinear local deformations, and has robustness to image intensity distortion. The method accommodates arbitrary features (e.g. gradient information and color components), spatial dimensions (e.g. 3D, 4D), and images of different spatial resolutions.

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