Model compression as constrained optimization, with application to neural nets

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Compressing neural nets is an active research problem, given the large size of state-of-the-art nets for tasks such as object recognition, and the computational limits imposed by mobile devices. We give a general formulation of model compression as constrained optimization. This includes many types of compression: quantization, low-rank decomposition, pruning, lossless compression and others. Then, we give a general algorithm to optimize this nonconvex problem based on the augmented Lagrangian and alternating optimization. This results in a "learning-compression" algorithm, which alternates a learning step of the uncompressed model, independent of the compression type, with a compression step of the model parameters, independent of the learning task. This simple, efficient algorithm is guaranteed to find the best compressed model for the task under standard assumptions.

Motivation Neural nets have achieved spectacular practical successes in hard problems in computer vision, speech and language in recent years. However, these neural nets are very large (upwards of many millions of weights), which makes it difficult to deploy them in mobile phones or other devices with limited computation, memory, bandwidth or battery life. This motivates the need for compressing a neural net while minimally hurting its performance.

Neural nets are overparameterized in practice, so even simple compression approaches achieve remarkably high compression—such as quantizing the net or pruning small weights and then retraining the resulting net. Many algorithms have been proposed since the 1980s for neural net compression which consider a specific type of compression (such as pruning) and combine some type of rounding operation with the backpropagation algorithm. Although these works achieve significant compression, they have no guarantees of converging to an optimally compressed net, or at all.

What distinguishes our approach? We provide a generic formulation for the problem of optimally compressing a model, independent of the compression type. This puts the problem of compression in a sound mathematical footing, amenable to modern optimization techniques. Also, we give a generic training algorithm to find the compressed model with lowest loss. All this algorithm requires is the usual SGD training of the original model; and access to a black-box compression routine for the desired compression type (k-means for quantization, SVD for low-rank, etc.). The combination of these two elements in our learning-compression algorithm gives rise to a convergent algorithm with guarantees of finding the optimal model (under some standard assumptions).

This brings several practical advantages: a simple algorithm that can be easily integrated in a deep learning toolbox; general applicability, in being able to handle most compression techniques in a common framework; and performance, in achieving more compression for the same target loss than other, approximate algorithms. Our algorithm also opens the door for more sophisticated uses of compression along memory, time, energy and other dimensions constrained by the hardware.

Next, we summarize our general approach and mention its application to quantization and pruning. For further details see papers [1–3].

A constrained optimization formulation of model compression Assume we have previously trained a model with weights $\overline{\mathbf{w}} = \arg\min_{\mathbf{w}} L(\mathbf{w})$. This is our *reference* model, which represents the best loss we can achieve without compression. We define *compression* as finding a low-dimensional parameterization $\boldsymbol{\Delta}(\boldsymbol{\Theta})$ of $\mathbf{w}$ in terms of $Q < P$ parameters $\boldsymbol{\Theta}$. *We seek a* $\boldsymbol{\Theta}$ *such that its corresponding model has (locally) optimal loss. We denote this"optimal compressed" and write it as* $\boldsymbol{\Theta}^*$ *and* $\mathbf{w}^* \equiv \boldsymbol{\Delta}(\boldsymbol{\Theta}^*)$ (see fig. 1 right). We define *model compression as a constrained optimization* problem: $\min_{\mathbf{w},\boldsymbol{\Theta}} L(\mathbf{w})$ s.t. $\mathbf{w} = \boldsymbol{\Delta}(\boldsymbol{\Theta})$. Compression and decompression are usually seen as algorithms, but here we regard them as mathematical mappings in parameter space. The *decompression mapping* $\boldsymbol{\Delta}$: $\boldsymbol{\Theta} \in \mathbb{R}^Q \rightarrow \mathbf{w} \in \mathbb{R}^P$ maps a low-dimensional parameterization to uncompressed model weights. The *compression mapping* $\boldsymbol{\Pi}(\mathbf{w}) = \arg\min_{\boldsymbol{\Theta}} \|\mathbf{w} - \boldsymbol{\Delta}(\boldsymbol{\Theta})\|^2$ behaves as its "inverse" and appears in the C step of our learning-compression algorithm. The C (compression) step is itself carried out by an algorithm: SVD for low-rank compression, k-means for quantization, etc. The *feasible set* $\mathcal{C} = \{\mathbf{w} \in \mathbb{R}^P\colon\ \mathbf{w} = \boldsymbol{\Delta}(\boldsymbol{\Theta}) \text{ for } \boldsymbol{\Theta} \in \mathbb{R}^Q\}$ contains all high-dimensional models $\mathbf{w}$ that can be obtained by decompressing some low-dimensional model $\boldsymbol{\Theta}$. In our framework, *compression is equivalent to orthogonal projection on the feasible set*.

Our framework includes well-known types of compression (and combinations thereof), such as:

- *Low-rank compression* defines $\boldsymbol{\Delta}(\mathbf{U},\mathbf{V}) = \mathbf{U}\mathbf{V}^T$, where we write the weights in matrix form with $\mathbf{W}$ of $m \times n$, $\mathbf{U}$ of $m \times r$ and $\mathbf{V}$ of $n \times r$, and with $r < \min(m,n)$. The compression mapping is given by the singular value decomposition (SVD) of $\mathbf{W}$.
- *Quantization* uses a discrete mapping $\boldsymbol{\Delta}$ given by assigning each weight to one of K codebook values. The compression mapping is given by k-means (or by a form of rounding if we use a fixed codebook, such as $\{-1,+1\}$ or $\{-1,0,+1\}$).
- *Pruning* defines $\mathbf{w} = \boldsymbol{\Delta}(\boldsymbol{\theta}) = \boldsymbol{\theta}$ where $\mathbf{w}$ is real and $\boldsymbol{\theta}$ is constrained to have few nonzero values, e.g. by using a sparsifying norm such as $\|\boldsymbol{\theta}\|_0 \le \kappa$ or $\|\boldsymbol{\theta}\|_1 \le \kappa$. The compression mapping involves some kind of thresholding.

A "Learning-Compression" (LC) algorithm Although the above constrained problem can be solved with a number of nonconvex optimization algorithms, it is key to profit from parameter separability, which we achieve with penalty methods (here, the quadratic penalty method) and alternating optimization. This results in an algorithm that alternates two generic steps while slowly driving the penalty parameter $\mu \to \infty$:

- **L (learning) step:** $\min_{\mathbf{w}} L(\mathbf{w}) + \frac{\mu}{2}\|\mathbf{w} - \boldsymbol{\Delta}(\boldsymbol{\Theta})\|^2$. This is a regular training of the uncompressed model but with a quadratic regularization term. *This step is independent of the compression type.*
- **C (compression) step:** $\min_{\boldsymbol{\Theta}} \|\mathbf{w} - \boldsymbol{\Delta}(\boldsymbol{\Theta})\|^2 \Leftrightarrow \boldsymbol{\Theta} = \boldsymbol{\Pi}(\mathbf{w})$. This means finding the best (lossy) compression of $\mathbf{w}$ (the current uncompressed model) in the ℓ_2 sense (orthogonal projection on the feasible set), and corresponds to our definition of the compression mapping $\boldsymbol{\Pi}$. *This step is independent of the loss, training set and task.*

The LC algorithm defines a continuous path $(\mathbf{w}(\mu), \boldsymbol{\Theta}(\mu))$ which, under some mild assumptions, converges to a stationary point (typically a minimizer) of the constrained problem. The beginning of this path, for $\mu \to 0^+$, corresponds to training the reference model and then compressing it disregarding the loss (*direct compression*), a simple but suboptimal approach popular in practice.

Optimizing the L and C steps The L step can be solved by stochastic gradient descent (clipping the step sizes so they never exceed $\frac{1}{\mu}$). The C step can be solved by calling a compression routine corresponding to the desired compression type. For example, for quantization with an adaptive codebook we can run k-means with a codebook of size K weights [2]; for pruning, we can prune all but the top-k weights (where k depends on the sparsifying norm used) [3].

Experiments The figure (left and middle panels) shows compression results (as a tradeoff curve of error vs compression level) for a well-known benchmark, the LeNet neural nets [4]. We can use a single bit per weight (for quantization) or remove nearly 99% of the weights (for pruning) with no error degradation over the reference. See the cited references [2, 3] for more results.

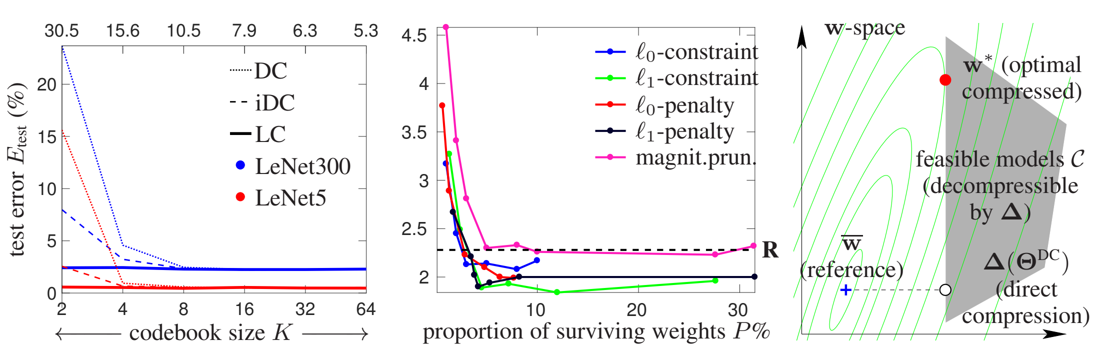

Figure 1: *Left*: error-compression curves for LeNet neural nets using quantization. *Middle*: same but using pruning. *Right*: illustration of the idea of model compression by constrained optimization.

References

[1] M. Á. Carreira-Perpiñán. Model compression as constrained optimization, with application to neural nets. Part I: General framework. arXiv:1707.01209 [cs.LG], July 5 2017.

[2] M. Á. Carreira-Perpiñán and Y. Idelbayev. Model compression as constrained optimization, with application to neural nets. Part II: Quantization. arXiv:1707.04319 [cs.LG], July 13 2017.

[3] M. Á. Carreira-Perpiñán and Y. Idelbayev. Model compression as constrained optimization, with application to neural nets. Part III: Pruning. arXiv, Sept. 2017.

[4] Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition. *Proc. IEEE*, 86(11):2278–2324, Nov. 1998.