$Z = (X – \text{mean}(X)) / \text{sd}(X)$

$Z = 1/\text{sd}(X) * X – 1/\text{sd}(X) * \text{mean}(X)$

$= -(1/\text{sd}(X)*\text{mean}(X)) + 1/\text{sd}(X) * X$ (form: $a + b*X$)

New mean: $-1/\text{sd}(X)*\text{mean}(X) + 1/\text{sd}(X)*\text{mean}(X) = 0$

New sd: $1/\text{sd}(X) * \text{sd}(X) = 1$

So Z scores, by the rule for change in mean and sd under linear transformation, have mean 0 and sd 1.