Electrolyte film evaporation under the effect of externally applied electric field

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The effect of an externally applied electric field and bulk ion-concentration on the evaporation rate of an electrolyte film in contact with a wall is studied numerically. A mathematical model for steady state condition is developed to couple momentum, energy, and mass conservation equations together with the Poisson–Boltzmann equation that describes charge distribution in the liquid film. The model also describes the interaction with humid air flowing above the electrolyte film. The results show a non-uniform volumetric heat generation due to Joule heating that induces a temperature variation along the film length and in the direction away from the wall.

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1. Introduction

Liquid-film evaporation and condensation have been extensively studied in the past due to the interesting transport phenomena involved and the many applications of these processes in a variety of areas that include renewable energy, chemical distillation, air conditioning, cooling towers, drying, desalination, liquid film evaporation, heat and mass exchangers, and desiccant systems, among others. Mathematically, the process of falling film evaporation is similar to that of falling film condensation [1]. The heat transfer mechanism in liquid films occurs due to conduction and convection inside the film where phase change takes place at the air–liquid interface [2].

Chun and Seban [3] studied the process of heat transfer in evaporating liquid film on a heated vertical tube under non-nucleating conditions. Their purpose was to estimate the heat transfer coefficient. No temperature distribution inside the liquid film was presented in their study. The process of heat transfer to a liquid film draining off a vertical plate was studied numerically by Isenberg and Gutfinger [4]. The temperature distribution inside the liquid film was obtained under the assumption of creeping flow. The temperature distribution in the gas side was not calculated. The lack of experimental data prompted Chandra and Savery [5] to study heat and mass transfer over a vertical falling film under forced convection. The non-homogeneous boundary conditions were specified by using measured temperature and concentration distributions along the gas–liquid interface. The problem of evaporation and heating with turbulent falling liquid films was studied by Seban and Faghri [6]. An appropriate turbulence model was examined and more experimental results were obtained with essentially the same apparatus used by Chun and Seban [3]. The evaporation of water into a laminar stream of dry air, humid air, and superheated steam was studied numerically by Chow and Chung [7]. The focus of their study was to investigate the physical reasons for the existence of the inversion temperature and to discuss the suitability of the one-third-rule for defining the reference temperature and mass concentration. Their paper presented the mathematical formulation of the heat and mass transfer on the air side and at the liquid–air interface. Schröppel and Thiele [8] studied heat and mass transfer in laminar and turbulent boundary layer flows of gas over a vaporizing liquid film. The liquid film was considered stationary with a constant thickness. The heat and mass transfer effects on the liquid film were not presented. Yan et al. [9] experimentally studied the evaporative cooling of liquid film through interfacial heat and mass transfer in a vertical channel. The theoretical model for the same process was discussed in a separate paper by Yan and Lin [10]. The model was simplified by neglecting inertia in the momentum equation and the normal convection term in the heat equation for the liquid film flow. Making similar assumptions, Yan and Soong [11] numerically

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studied the convective heat and mass transfer along an inclined heated plate with film evaporation. Mezaache and Daguene [12] studied film evaporation along an inclined plate for various inlet conditions using a two-dimensional boundary-layer model. Jaballah et al. [13] studied convective heat and mass transfer involved conditions using a two-dimensional boundary-layer model. Jaballah and Levine et al. [15] developed an analytical approximation to the solution of the Poisson equation to determine the electrical potential and the electric double layer. The characteristics of electroosmotic flow (EOF) in rectangular microchannels were investigated numerically by Arulananand and Li [17]. The results demonstrated the significance of geometry effects, ionic concentration, and zeta potential on EOF. The energy equation was not included but it was indicated that for the conditions analyzed, the thermal effects were negligible. Hsu et al. [18] studied the electrokinetic flow through an elliptical microchannel under the basis of three boundary conditions: constant surface potential, constant surface charge density, and charge regulated surface. Jamaati et al. [19] considered the electric double layer. The characteristics of electroosmotic flow (EOF) in rectangular microchannels were investigated numerically by Arulananand and Li [17]. The results demonstrated the significance of geometry effects, ionic concentration, and zeta potential on EOF. The energy equation was not included but it was indicated that for the conditions analyzed, the thermal effects were negligible. Hsu et al. [18] studied the electrokinetic flow through an elliptical microchannel under the basis of three boundary conditions: constant surface potential, constant surface charge density, and charge regulated surface. Jamaati et al. [19] considered the effect of slip at the walls for pressure-driven electrokinetic flows in planar microchannels. Their study indicated that liquid slip at the walls could increase the induced voltage significantly. A mathematical model was developed to study the electrokinetic effects on pressure-driven liquid flow in a parallel slit by Chen et al. [20]. The model included the general Nernst–Planck equation to describe anion and cation distribution, the Poisson equation to determine the electrical potential profile, and the modified Navier–Stokes equation to determine the velocity flow field. The results showed that the presence of the EOF in the developing flow decreases the fluid velocities. It was also shown that in the developing regime the electrical potential and the electrical net charge density predicted by using the Nernst–Planck equation and the Boltzmann distributions were very different.

In the literature related to liquid–film evaporation, heat transfer to the fluid is generally specified at the boundary, in terms of either steady-state fully-developed laminar flows. It was shown that for aqueous solutions of low ionic concentration and a solid surface of high zeta potential, the liquid flow and heat transfer in rectangular microchannels are significantly influenced by the presence of the electric double layer. The characteristics of electroosmotic flow (EOF) in rectangular microchannels were investigated numerically by Arulananand and Li [17]. The results demonstrated the significance of geometry effects, ionic concentration, and zeta potential on EOF. The energy equation was not included but it was indicated that for the conditions analyzed, the thermal effects were negligible. Hsu et al. [18] studied the electrokinetic flow through an elliptical microchannel under the basis of three boundary conditions: constant surface potential, constant surface charge density, and charge regulated surface. Jamaati et al. [19] considered the effect of slip at the walls for pressure-driven electrokinetic flows in planar microchannels. Their study indicated that liquid slip at the walls could increase the induced voltage significantly. A mathematical model was developed to study the electrokinetic effects on pressure-driven liquid flow in a parallel slit by Chen et al. [20]. The model included the general Nernst–Planck equation to describe anion and cation distribution, the Poisson equation to determine the electrical potential profile, and the modified Navier–Stokes equation to determine the velocity flow field. The results showed that the presence of the EOF in the developing flow decreases the fluid velocities. It was also shown that in the developing regime the electrical potential and the electrical net charge density predicted by using the Nernst–Planck equation and the Boltzmann distributions were very different.

In the literature related to liquid–film evaporation, heat transfer to the fluid is generally specified at the boundary, in terms of either
wall temperature or heat flux, or as a volumetric heat generation term. In the present work, the heat source results from an externally applied electric field which also has an effect on the fluid velocity distribution. The effect of the application of an electric field on the rate of evaporation and temperature distribution in an electrolyte film in contact with moist air is the focus of this study.

2. Mathematical formulation

Most surfaces acquire surface charge upon coming into contact with a polar medium. Some of the charging mechanisms are ionization, ion dissolution, and ion adsorption. The surface charge in an electrolyte solution influences the distribution of nearby ions in the solution. As a result of this electric interaction, ions of opposite charge (co-ions) are repelled from the surface while ions of same charge (counter-ions) are attracted toward the surface. This process of attraction and repulsion combined with the motion of ions leads to the formation of an electric double layer [1]. An externally applied electric field acts upon the charge distribution in the electric double layer and induces momentum transport and Joule heating, as shown in Fig. 1. The hydrodynamics and Joule heating due to externally applied electric field are formulated following the description in Ref. [21].

2.1. Poisson–Boltzmann equation

According to the theory of electrostatics, the electrical potential distribution, \( \psi(x,y) \), is governed by the Poisson equation,

\[
\nabla \cdot (\epsilon \nabla \psi) = \frac{\rho_e}{\epsilon_0}
\]

(1)

where \( \rho_e \) is the local charge density, \( \epsilon \) is the dielectric constant of the electrolyte which is considered to be a function of temperature, as described in Ref. [21].

\[
\epsilon(T) = 305.7 \exp \left( \frac{-T}{218} \right)
\]

(2)

In the case of a symmetric electrolyte with co-ions and counterions having the same charge valence, its ionic concentration distribution for both anions and cations is assumed to follow the Boltzmann distribution,

\[
n^+ = n_0 \exp \left( \frac{ze\psi}{k_b T} \right), \quad n^- = n_0 \exp \left( \frac{-ze\psi}{k_b T} \right)
\]

(3)

where \( k_b \) is the Boltzmann constant, \( e \) is the fundamental charge, and \( z \) is the charge valence of the ions. Once the concentration distributions of cations and anions are known, the local charge density can be calculated as:

\[
\rho_e(x,y) = ze \left( n^+ - n^- \right) = -2z\epsilon_0 n_0 \sinh \left( \frac{ze\psi}{k_b T} \right)
\]

(4)

The Poisson–Boltzmann equation is obtained from Eqs. (1) and (4) as:

\[
\nabla \cdot (\epsilon \nabla \psi) = \frac{2z\epsilon_0 n_0 \sinh \left( \frac{ze\psi}{k_b T} \right)}{\epsilon_0}
\]

(5)

The distribution of the externally applied electric potential is governed by the Laplace equation,

\[
\nabla \cdot (\lambda \nabla \phi) = 0
\]

(6)

where \( \lambda \) is the electrical conductivity of the electrolyte defined below as given in Ref. [21]:

\[
\lambda(T) = \lambda^+ (T) \eta^+ + \lambda^- (T) \eta^-
\]

(7)

\[
\lambda^+ (T) = \lambda_0^+ + 0.025\lambda_0^+ (T - 298)
\]

(8)

\[
\lambda^- (T) = \lambda_0^- + 0.025\lambda_0^- (T - 298)
\]

(9)

where \( \lambda^+ (T) \) and \( \lambda^- (T) \) are ionic conductivities of cations and anions of the electrolyte at temperature \( T \). The terms \( \lambda_0^+ \) and \( \lambda_0^- \) are ionic conductivities of cations and anions of the electrolyte at 298 K.

2.2. Conservation equations in liquid film

In the presence of an applied electric field parallel to the surface, the flow of an electrolyte solution is influenced by the interaction between net charge density in the EDL and the applied electric field, and is governed by steady state, incompressible Navier–Stokes equations with an added term to include the effect of electric force. Thus, under the assumption of laminar flow, the governing equations take the following form:

2.2.1. Continuity equation

\[
\nabla \cdot \bar{u}_i = 0
\]

(10)

2.2.2. Momentum equations

The Navier–Stokes equation including the effect of the electric field becomes:

\[
\rho \bar{u}_i \cdot \nabla \bar{u}_j = -\nabla p_i + \nabla \cdot (\mu_i \nabla \bar{u}_i) + \rho_0 \vec{E}
\]

(11)

where \( \vec{E} = -\nabla \psi \) is the strength of the local applied electric field, and \( \mu_i \) is the temperature dependent viscosity of the electrolyte solution.

2.2.3. Energy equation

The energy equation neglecting viscous dissipation and taking into account Joule heating effect can be expressed as:

\[
\rho_l C_p l (\bar{u}_i \cdot \nabla T_i) = \nabla \cdot (k \nabla T_i) + \dot{q}
\]

(12)

where \( k \) is the temperature-dependent thermal conductivity of the electrolyte solution, \( \dot{q} \) is the volumetric Joule heating which
consists of two parts: one due to the local applied electric field imposed on the conductive solution \((E_l)\), and the other due to the net charge density moving with the flow \((\nu_0 \rho_e)\). According to the Ohm’s law, the Joule heating can be expressed as:

\[
\dot{q} = \frac{(u_0 \rho_e + E_l)^2}{\lambda} \tag{13}
\]

2.3. Conservation equations on air side

The evaporation of liquid film adds vapor to the air flowing above the film. The heat and mass transfer on the air side and at the interface can be formulated using conservation of momentum, energy, mass, and concentration [12].

2.3.1. Continuity equation

\[
\nabla \cdot \vec{u}_a = 0 \tag{14}
\]

2.3.2. Momentum equations

\[
\rho_a \vec{u}_a \cdot \nabla \vec{u}_a = -\nabla p_a + \nabla \cdot (\mu_a \nabla \vec{u}_a) \tag{15}
\]

where \(\mu_a\) is the temperature dependent viscosity of air. It is to be observed that no electric field is applied on the air side.

2.3.3. Energy equation

\[
\rho_a C_{p,a}(\vec{u}_a \cdot \nabla T_a) = \nabla \cdot \left( k_a \nabla T_a \right) + \rho_a D_a (C_{p,v} - C_{p,a}) \frac{\partial m_v}{\partial y} \frac{\partial T_a}{\partial y} \tag{16}
\]

where, the last term represents energy transport by mass diffusion under the assumption \(dh = C_d dT\). Temperature dependent values of \(\mu_a, \rho_a, k_a, \) and \(k_a\) are taken from Ref. [22] corresponding to water and air.

In steady state, vapor mass fraction in air is given by,

\[
\vec{u}_a \cdot \nabla m_v = D_a \nabla^2 m_v \tag{17}
\]

2.4. Boundary conditions

The boundary conditions described in Table 1 along with the parameters defined in Table 2 were utilized to solve the set of governing equations. It is assumed that only convective fluxes are present at the outlet for heat and mass transfer. In addition, the interfacial vapor mass flux \(\omega_i\) is given by:

\[
\omega_i = \left( \frac{\rho_a D_a}{1 - m_v} \frac{\partial m_v}{\partial y} \right)_i \tag{18}
\]

where the subscript \(i\) denotes evaluation at the interface.

Assuming the air–vapor mixture to be an ideal gas and the interface to be in thermodynamic equilibrium, the vapor mass fraction can be written as:

\[
m_{v,i} = \frac{M_v p_{v,i}}{M_a (p_a - p_{v,i})} + M_i p_{v,i} \tag{19}
\]

where \(p_{v,i}\) is the partial pressure of the saturated vapor at the gas–liquid interface. For the case of water vapor, the partial pressure is given by Bertrand’s equation [23]:

\[
\log(p_{v,i}) = 17.443 - \frac{2795}{T_i} - 3.868 \log(T_i) \tag{20}
\]

2.5. Non-dimensional equations

The above set of equations are non-dimensionalized using the following dimensionless variables and parameters:

\[
\begin{align*}
\varphi &= x/H_l, \quad y = y/H_l, \quad T_f = T_l/T_r, \quad \theta = (T - T_{in})/T_{in}, \\
\varphi &= \psi/\psi_l, \quad \theta = \phi/\phi_l, \quad \vec{u}_f = \vec{u}_l/\vec{u}_r, \quad \vec{r}_f = \vec{r}_l/\vec{r}_r, \\
\vec{F} &= -\nabla \varphi, \quad \lambda = \lambda_l/\lambda_r, \quad k_f = k_l/k_r, \quad \mu_f = \mu_l/\mu_r, \\
\bar{w} &= \omega_l/\omega_r, \quad \bar{r} = r/r, \quad Re_f = \frac{\rho_l u_l H_l}{\mu_l}, \quad Pr_f = \frac{\mu_f C_p f}{k_f r}, \\
G &= \frac{D_a (C_{p,v} - C_{p,a})}{H_l u_l C_{p,a}}, \quad A = \frac{\rho_e u_l H_l}{\phi_l r_l}, \quad B = \frac{2\bar{w} n_z r_e \phi_l}{\rho_l u_l^2}, \\
F &= H_l \left( \frac{2\bar{w} n_z r_e^2 e^2}{\epsilon \epsilon_0 k_0 T_l} \right), \quad \alpha = E_0/E_r, \quad Q_l = \frac{\omega_l h_l H_l}{\phi_l r_l}, \quad K = \frac{k_{l,f}}{k_{l,r}}, \\
R &= \frac{\rho_a}{\rho_l}, \quad U = \frac{\mu_{a,f}}{\mu_{l,r}}, \quad \beta = n_0/n_r. \tag{21}
\end{align*}
\]

where,

\[
T_r = \frac{\phi_l^2 \lambda_r}{k_{l,r}} \quad u_r = \frac{\epsilon_0 \epsilon_r \phi_l |\vec{E}|}{\mu_{l,r}}, \quad \psi_l = \frac{k_l T_l}{\epsilon_0}, \quad \rho_{e,r} = 2\bar{w} n_z r_e.
\]

\[
\lambda_r = \lambda_l \eta + \lambda_0 \eta, \quad \phi_l = \alpha E_l, \quad Pr_f = \rho_l u_l^2, \quad \omega_r = \frac{\rho_a D_a}{H_l}
\]

The governing equations in non-dimensional form using the above set of variables and parameters are given below.

2.5.1. Electric potential

\[
\nabla \cdot (\sigma \nabla \varphi) = F^2 \sinh \left( \frac{\varphi}{T} \right) \tag{22}
\]

\[
\nabla \cdot (\lambda \nabla \theta) = 0 \tag{23}
\]

2.5.2. Liquid film

\[
\nabla \cdot \vec{u}_l = 0 \tag{24}
\]

\[
\vec{u}_l \cdot \nabla \vec{u}_l = -\nabla p_l + \frac{1}{Re_l} \left[ \nabla \cdot (\vec{u}_l \nabla \vec{u}_l) \right] - B \vec{F} \sinh \left( \frac{\varphi}{T_l} \right) \tag{25}
\]

\[
\vec{u}_l \cdot \nabla T_l = \frac{1}{Re_l Pr_l} \left[ \nabla \cdot (k_l \nabla T_l) \right] + \frac{1}{Re_l Pr_l} \left( \frac{A \eta \eta + E_l^2}{\lambda} \right) \tag{26}
\]

2.5.3. Air film

\[
\nabla \cdot \vec{u}_a = 0 \tag{27}
\]
Table 1
Boundary conditions.

<table>
<thead>
<tr>
<th>Inlet</th>
<th>Dimensional</th>
<th>Non-dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 - T_a = T_m )</td>
<td>( T_1 - T_a = T_{in}/T_i )</td>
<td></td>
</tr>
<tr>
<td>( p_1 = 0 )</td>
<td>( p_1 = 0 )</td>
<td></td>
</tr>
<tr>
<td>( u_a = 2u_r )</td>
<td>( u_a = 2u_{in} )</td>
<td></td>
</tr>
<tr>
<td>( m_r = m_{in} )</td>
<td>( m_r = m_{in} )</td>
<td></td>
</tr>
<tr>
<td>( \phi = E \varepsilon )</td>
<td>( \phi = E \varepsilon )</td>
<td></td>
</tr>
<tr>
<td>Wall</td>
<td>Dimensional</td>
<td>Non-dimensional</td>
</tr>
<tr>
<td>( u_w = -u_r )</td>
<td>( u_w = -u_{in} )</td>
<td></td>
</tr>
<tr>
<td>( \phi = 0 )</td>
<td>( \phi = 0 )</td>
<td></td>
</tr>
<tr>
<td>Interface ((y - H_i))</td>
<td>Dimensional</td>
<td>Non-dimensional</td>
</tr>
<tr>
<td>( u_{1i} = u_{a,i}, v_{1i} = v_{a,i} )</td>
<td>( u_{1i} = u_{a,i}, v_{1i} = v_{a,i} )</td>
<td></td>
</tr>
<tr>
<td>( p_{1,1} - p_{a,i} )</td>
<td>( p_{1,1} - p_{a,i} )</td>
<td></td>
</tr>
<tr>
<td>( \mu_{ij} )</td>
<td>( \mu_{ij} )</td>
<td></td>
</tr>
<tr>
<td>( \phi_l, \phi_r )</td>
<td>( \phi_r )</td>
<td></td>
</tr>
</tbody>
</table>

\( \vec{\nabla} \cdot \vec{n} = \nabla \cdot (\vec{n} \cdot \vec{T}) = 0 \)  
\( \vec{n} \cdot \nabla \vec{T} = \frac{1}{Re_{da}} \left[ \nabla \cdot (\vec{n} \cdot \nabla \vec{T}) \right] + \frac{G \partial m_v \partial T_a}{\partial y \partial y} \)  
\( \vec{n} \cdot \nabla \vec{n} = \frac{1}{ScRe_{da}} \nabla^2 m_v \)

Table 2
Values of various parameters used in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{w,1} )</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>4181.3</td>
</tr>
<tr>
<td>( C_{w,p} )</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>1012</td>
</tr>
<tr>
<td>( C_{w,v} )</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>1864</td>
</tr>
<tr>
<td>( D_a )</td>
<td>m(^2) s(^{-1})</td>
<td>(2.56 \times 10^{-5})</td>
</tr>
<tr>
<td>( E_a )</td>
<td>V m(^{-1})</td>
<td>30</td>
</tr>
<tr>
<td>( h_{fg} )</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>2.26 \times 10^6</td>
</tr>
<tr>
<td>( h_{ev} )</td>
<td>W m(^{-1}) K(^{-1})</td>
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</tr>
<tr>
<td>( h_{r} )</td>
<td>W m(^{-1}) K(^{-1})</td>
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</tr>
<tr>
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<td>kg mol(^{-1})</td>
<td>18 \times 10(^{-3})</td>
</tr>
<tr>
<td>( M_r )</td>
<td>kg mol(^{-1})</td>
<td>29 \times 10(^{-3})</td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>(m^{-3})</td>
<td>(6.022 \times 10^{23})</td>
</tr>
<tr>
<td>( T_{in} )</td>
<td>K</td>
<td>300</td>
</tr>
<tr>
<td>( \lambda_0 )</td>
<td>m(^2) S mol(^{-1})</td>
<td>(0.08 \times 10^{-4})</td>
</tr>
<tr>
<td>( \lambda_f )</td>
<td>m(^2) S mol(^{-1})</td>
<td>(76.31 \times 10^{-4})</td>
</tr>
<tr>
<td>( \mu_a )</td>
<td>Pa s</td>
<td>7.98 \times 10^{-4}</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>Pa s</td>
<td>1.98 \times 10^{-5}</td>
</tr>
<tr>
<td>( \mu_a )</td>
<td>kg m(^{-1}) s(^{-1})</td>
<td>1000</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>kg m(^{-1}) s(^{-1})</td>
<td>1.204</td>
</tr>
<tr>
<td>( \rho_e )</td>
<td>kg m(^{-1}) s(^{-1})</td>
<td>1.204</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>kg m(^{-1}) s(^{-1})</td>
<td>8.967</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td>Pa</td>
<td>78.4</td>
</tr>
</tbody>
</table>

3. Results and discussion

The Poisson–Boltzmann equation together with the coupled momentum, energy, and mass transfer equations along with the specified boundary conditions were implemented and solved in 2D using commercial finite element software COMSOL Multiphysics 3.5 [24]. A total of 18,000 and 6000 quadrilateral elements in the liquid film and air, respectively, were used in the simulations. The solver UMFPACK, which solves general systems of the form \(Ax = b\) using the nonsymmetric-pattern multifrontal method and direct LU factorization of the sparse matrix \(A\), was used for the simulations. COMSOL Multiphysics estimates the error of the solution during each iteration. Once the error estimate is small enough, as determined by the convergence criterion \(p|M^{-1}(b - Ax)| < tol|M^{-1}b|\), where \(p\) is a factor in error estimate (default is 400), tol is the relative tolerance specified at 10\(^{-6}\), and \(M = LU\) with \(L\) and \(U\) being the LU factors computed by the solver, the software terminates the computations and returns a solution [24]. Grid independence was verified by refining the grid until no significant difference was observed in the results. Also, to verify the validity of results from the software, the normalized velocity profile in an electroosmotic flow in a capillary under the effect of externally applied electric field was compared to the analytical results of the Rice–Whitehead solution discussed in Ref. [25]. Even though the
case chosen for comparison is that of internal flow as opposed to external flow, the main purpose of such a comparison is to establish that the software is able to handle the complex interaction between externally applied electric field, electric double layer, and the resulting hydrodynamics. The flows in both cases are driven by externally applied electric field which acts on the electric charge distribution established inside the electric double layer. The comparison of the results shows excellent agreement between the analytical and numerical solutions, as shown in Fig. 2, where "κ" is the inverse of Debye length, "a" is the radius of a single long and uniform capillary containing an electrolyte.

In the present work, the electric field acts on the charge distribution in the double layer along with the ions present in the region outside the double layer. The motion of the charges due to the presence of the electric field gives rise to an electric current that causes the temperature in the liquid film to increase due to Joule heating effect. Since the surface potential has a negative value, the charge distribution in the double layer is predominantly positive. The length of the domain analyzed is \( L = 0.005 \) m, the initial thickness of the liquid film \( H_l = 0.0001 \) m, and the height of the air section is \( H_a = 0.0004 \) m, with a surface potential \( \psi_w = -0.075 \) V.

The dimensions and parameters are taken to be approximately the same as in Ref. [26].

A uniform velocity profile is imposed at the air inlet and no velocity profile is imposed at the inlet of the electrolyte film. The flow in the liquid film is driven by the applied electric field which varies along the length of the film according to Eq. (23). This electric field acts on the charge distribution inside the electric double layer resulting in the velocity profile depicted in Fig. 3, where \( \alpha \) and \( \beta \) represent dimensionless electric field and dimensionless number concentration, respectively. As the net charge distribution is confined within the electric double layer, the interaction between electrostatics and hydrodynamics is resolved by means of using a fine mesh near the wall surface. Representative values for the Reynolds number, inside the electric double layer and in the air adjacent to the interface, are \( Re_l = 2.5 \times 10^{-4} \) and \( Re_a = 3 \times 10^{-5} \), respectively. The Prandtl numbers for liquid and air are \( Pr_l = 5.6 \) and \( Pr_a = 0.8 \), respectively. The small values of the Reynolds number indicate the extremely small thickness of the electric double layer formed adjacent to the wall.

In order to show the effect of the applied electric field on the temperature distribution, Fig. 4 depicts the temperature

\[ f(x) = \frac{1}{x^2} + 2x \]

Fig. 2. Validation of results from COMSOL simulation against analytical results for the normalized velocity profile from the Rice–Whitehead solution discussed in Ref. [25]. \( x \) is the inverse of Debye length and \( a \) is the radius of a single long and uniform capillary containing an electrolyte.

Fig. 3. Velocity profiles in liquid film and humid air domains at location \( \tau = 30 \).

Fig. 4. Temperature variation in liquid film and air along the length of the domain at \( \tau = 0.5 \) and \( \tau = 3 \), respectively. Results are obtained for \( \beta = 5 \) and different values of dimensionless applied electric field, \( \alpha \).

Fig. 5. Vapor mass fraction in air along the length of the domain at \( \tau = 3 \) for \( \beta = 5 \) and different values of dimensionless applied electric field, \( \alpha \).
distribution in the liquid film and humid air along the length of the domain at $\gamma = 0.5$ and $\gamma = 3$ from the wall. The applied electric field plays a role in both, the momentum as well as the energy conservation equations as described by Eqs. (25) and (26), respectively. Volumetric heat generation increases with the increase in the applied electric field resulting in higher temperatures in both liquid and air. The non-dimensional outlet temperature, $\theta$, of the liquid film increased by 0.038, 0.081, and 0.125 when non-dimensional external electric fields, $\alpha$, of 1/3, 2/3, and 1 were applied, respectively. As a result of the volumetric heat generation, the rate of evaporation along the liquid film length also augments. This can be observed by analyzing the vapor mass fraction of the air near the liquid–air interface.

The variation of the vapor mass fraction along the length of the liquid film at $\gamma = 3$ is shown in Fig. 5. At a constant dimensionless number concentration, $\beta = 5$, the increase in non-dimensional applied electric field, $\alpha$, from 1/3 to 2/3 raised the vapor mass fraction from 0.049 kgv/kga to 0.12 kgv/kga. An increase of $\alpha$ to a value of 1 increased the vapor mass fraction to 0.224 kgv/kga. It is observed that mass flow rate of liquid is much larger than the mass liquid that evaporates, so the changes in the thickness of the liquid film were neglected in this analysis.

The effect of bulk ion-concentration on the temperature along the length of the electrolyte–film and air domains at constant external electric field ($\alpha = 1$) is shown in Fig. 6. The bulk ion-concentration, $\beta$, affects charge density in the electric double layer as well as the electrical conductivity of the electrolyte as described by Eqs. (4) and (7). $\beta$ plays a minor role in the hydrodynamics of the liquid film since the effect relates only to the electric double layer, as described by Eq. (25). However, $\beta$ has a substantial role in Joule heating through Eq. (26) since it affects the entire domain of the electrolyte. As the bulk ion-concentration increases, volumetric Joule heating increases resulting in an increase in the rate of evaporation as shown in Fig. 7.

The presence of the electric double layer near the wall surface causes more Joule heating near the wall, thus, raising its temperature. On the other hand, evaporation occurring at the liquid free surface decreases the film temperature at the air–liquid interface causing a temperature gradient along the thickness of the film. The temperature distribution across the liquid film and humid air is shown in Fig. 8 for values of $\alpha = 1$ and $\beta = 1$ at an arbitrary location $\pi = 30$. The decrease in temperature in the direction toward the liquid–air interface results in the decrease in the electric conductivity of the electrolyte and causes a slight reduction in the rate of Joule heating across the film thickness. Therefore, a non-uniform volumetric heat generation distribution is obtained.

4. Conclusions

Evaporation of an electrolyte film in the presence of an externally applied electric field is studied. The only source of heat generation is the volumetric Joule heating due to the presence of the applied electric field. Also, the flow is solely driven by the interaction of applied electric field with charge distribution in electric double layer. The presence of the electric field results in a coupling of the momentum, energy, and mass transfer equations that are solved simultaneously with the Poisson–Boltzmann equation that describes the electric potential distribution. The results show that the rate of evaporation at the air–liquid interface is a strong function of the applied electric field and bulk ion-concentration.

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References


