Sketching Sampled Data Streams

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Motivation & Goal

Motivation

- Multicore processors — How to use all the processing power?
  - Parallel algorithms
  - Side tasks (analytical, exploratory)

Goal

- Analyze data at wire speed — single pass, small memory
  - *Skew* of a relation read from the disk
  - *Correlation* between flows passing through a high-speed router

Class of Queries

Aggregates over Joins

- Equi-join J between relations F and G with the join constraint $F.a = G.a$
- Queries specified by a join and an aggregate: `COUNT`, `SUM`
  - size of join (dot product), self-join size (second frequency moment)

Example

- Stream F:

a	1	1	2	3	1	3

, frequency vector $\mathbf{f}$:

i	1	2	3
f_i	3	1	2

- Stream G:

a	3	1	3	1	1

, frequency vector $\mathbf{g}$:

i	1	2	3
g_i	3	0	2

$$\mathtt{COUNT}(F \bowtie_a G) = \mathbf{f}\mathbf{g}^T = \sum_i f_i g_i$$

$$= \begin{bmatrix} 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} = 13$$

Basic Sketching Technique (AGMS99)

Main Idea:

- Summarize frequency table by projecting it onto a random vector $\Rightarrow$ *sketch*
- Use sketches to recover query result

Random vectors:

- $\xi = [\xi_1 \ldots \xi_n]$ random vector of ± 1 values, called ξ **family**

Sketches:

- Sketch of F, $X_F = \mathbf{f}\xi^T$
- Sketch of G, $X_G = \mathbf{g}\xi^T$
- $X = X_F X_G$ estimates $\texttt{COUNT}(F \bowtie_a G)$ since

$$E[X] = E[\mathbf{f}\xi^T \xi \mathbf{g}^T] = \mathbf{f}E[\xi^T \xi]\mathbf{g}^T = \mathbf{f}I\mathbf{g}^T = \mathbf{f}\mathbf{g}^T$$

if $E[\xi^T \xi] = I$.
$\Rightarrow$ distinct elements of ξ must be pair-wise independent $\forall i \neq i',\ \xi_i^2 = 1,\ E[\xi_i \xi_{i'}] = 0$

Basic Sketching Technique

Example: $\xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \end{bmatrix} = \begin{bmatrix} -1 & +1 & -1 \end{bmatrix}$

$$X_F = \mathbf{f}\xi^T = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -4$$

$$X_G = \mathbf{g}\xi^T = \begin{bmatrix} 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -5$$

$$X = X_F X_G = (-4)(-5) = 20 \approx 13$$

- Error of estimate X due to its variance

$$\text{Var}[X] = \sum_{i \in I} f_i^2 \sum_{j \in I} g_j^2 + \left(\sum_{i \in I} f_i g_i \right)^2 - 2 \sum_{i \in I} f_i^2 g_i^2$$

$$\text{Var}(X) \leq 2\mathbf{ff}^T \mathbf{gg}^T = 2 \ \text{SJ}(F) \ \text{SJ}(G)$$

- ξ is a family of 4-wise independent random variables

Sketch Maintenance over Streams

- Choose seed s that generates ξ
- Stream F:

a	1	1	2	3	1	3

, frequency vector $\mathbf{f}$:

i	1	2	3
f_i	3	1	2

$$X_F = \mathbf{f}\xi^T = \sum_i f_i \xi_i = \sum_i (\xi_i + \cdots + \xi_i)$$
$$= \sum_{t \in F} \xi_{t.a} = \xi_1 + \xi_1 + \xi_2 + \xi_3 + \xi_1 + \xi_3$$
$$= h(s,1) + h(s,1) + h(s,2) + h(s,3) + h(s,1) + h(s,3)$$

- Stream G:

a	3	1	3	1	1

, frequency vector $\mathbf{g}$:

i	1	2	3
g_i	3	0	2

$$X_G = \mathbf{g}\xi^T = \sum_i g_i \xi_i = \sum_i (\xi_i + \cdots + \xi_i)$$
$$= \sum_{t \in G} \xi_{t.a} = \xi_3 + \xi_1 + \xi_3 + \xi_1 + \xi_1$$
$$= h(s,3) + h(s,1) + h(s,3) + h(s,1) + h(s,1)$$

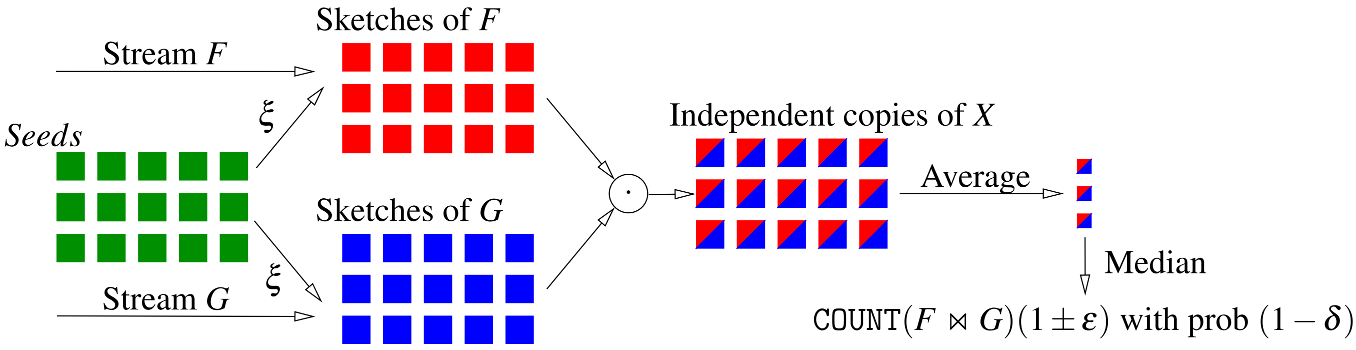
- Counters X_F and X_G need only log space in the size of the stream

Sketch Error Reduction

- Estimation of $\texttt{COUNT}(F \bowtie_a G)$ from single sketches of F and G is *too noisy*

Solution:

- Average $\frac{8\mathsf{Var}(X)}{\varepsilon^2 E^2[X]}$ independent copies of X to reduce error to ε
- Compute median of $2\log 1/\delta$ such averages to increase confidence to $1-\delta$



- Memory required independent of the size of the stream

Speed-Up Methods

- Hashing
  - Fast-AGMS sketches are faster and have better accuracy

- Pseudo-random number generating schemes
  - EH3 is as good as any 4-wise scheme + faster and denser

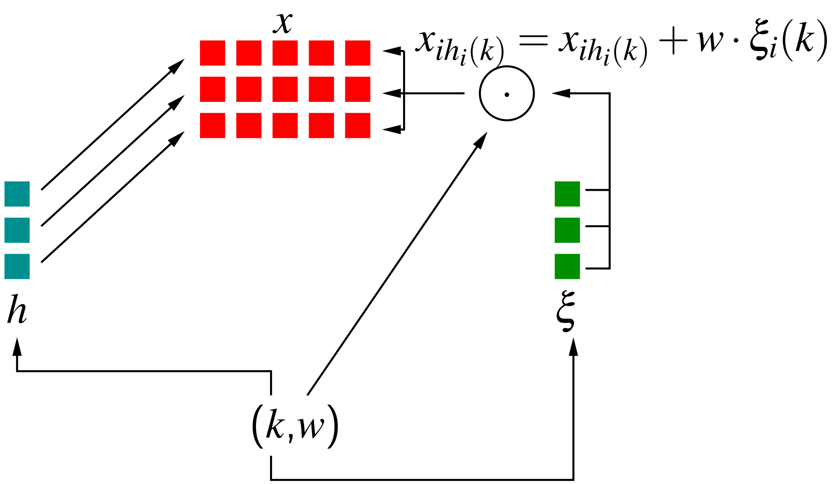
Fast-AGMS Sketches (CG05)

Randomization

- Vector h_i of 2-universal hash functions, $h_i : I \to B$
- Vector ξ_i of 4-wise independent ± 1 random variables, $\xi_i : I \to \{-1, +1\}$

Update

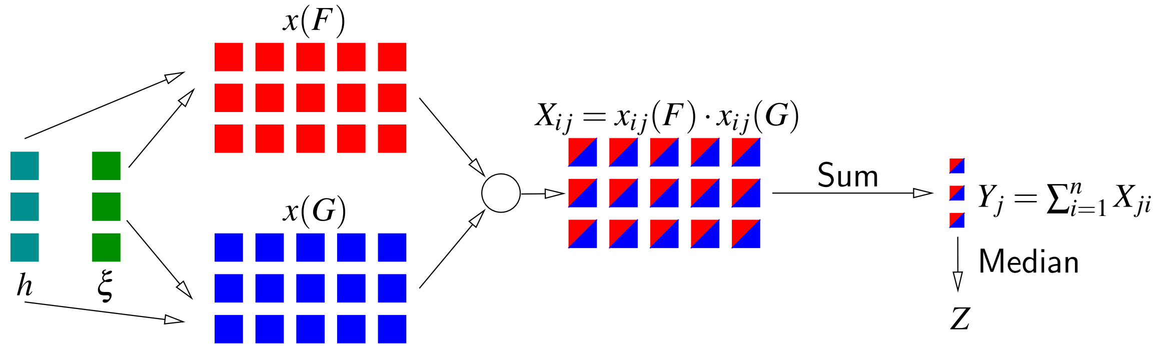
- Time $\approx$ no. of rows m



Fast-AGMS Sketches (CG05)

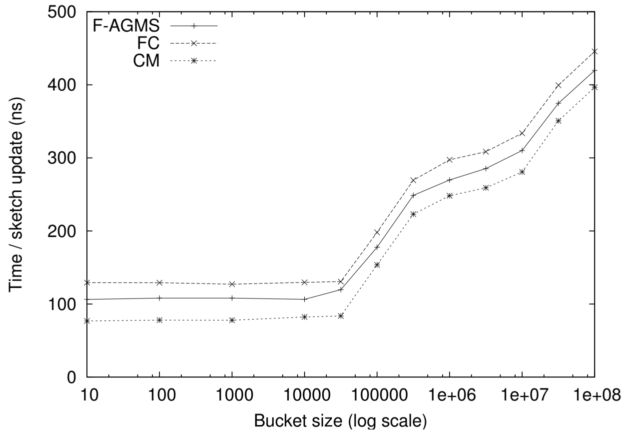
Size of Join Estimator

- $E[Z] = \bar{f} \odot \bar{g} = |F \bowtie G|$, $Z \in (\bar{f} \odot \bar{g} \pm \varepsilon ||\bar{f}||_2 ||\bar{g}||_2)$ with probability at least $1 - \delta$
- $||\bar{f}||_2 = \sqrt{\bar{f} \odot \bar{f}} = \sqrt{\sum_{i \in I} f_i^2}$, $||\bar{g}||_2 = \sqrt{\bar{g} \odot \bar{g}} = \sqrt{\sum_{i \in I} g_i^2}$
- Sketch size: $B = n = \mathcal{O}(\frac{1}{\varepsilon^2})$ and $m = \mathcal{O}(\log \frac{1}{\delta})$



Update Time

Setup: sketch size (row=1), Xeon 2.8 GHz, 512 KB cache, 4 GB main memory



- 100 ns / sketch $\times$ 10 sketches $\approx 1\ \mu$s
- 1 million integers / second $\Rightarrow$ 4 MB / second

Desired rate

- 100 MB / second $\Longrightarrow$ 25X

Sampling

- Stream F: | a | 1 1 2 3 1 3 |, frequency vector $\mathbf{f}$:

i	1	2	3
f_i	3	1	2

- Sample F': | a | 1 3 1 |, sampled frequency vector $\mathbf{f}'$:

i	1	2	3
f'_i	2	0	1

- Stream G: | a | 3 1 3 1 1 |, frequency vector $\mathbf{g}$:

i	1	2	3
g_i	3	0	2

- Sample G': | a | 3 1 |, sampled frequency vector $\mathbf{g}'$:

i	1	2	3
g'_i	1	0	1

$$X = C \cdot \mathbf{f}'\mathbf{g}'^{\mathbf{T}} = C \cdot \begin{bmatrix} 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = C \cdot 3 \approx 13$$

Sampling

- Sample at the tuple level
- Analyze in the frequency domain
- Random frequency vector (moment generating function)
  - Bernoulli $\rightarrow$ Binomial
  - WR $\rightarrow$ Multinomial
  - WOR $\rightarrow$ Multivariate hypergeometric

$$X = C \sum_{i \in I} f_i' g_i'$$

$$E[X] = C \sum_{i \in I} E[f_i'] E[g_i']$$

$$\mathrm{Var}[X] = C^2 \left[\sum_{i \in I} \sum_{j \in I} E[f_i' f_j'] E[g_i' g_j'] - \left(\sum_{i \in I} E[f_i'] E[g_i'] \right)^2 \right]$$

Sketches over Sampled Streams

- Stream F: | a | 1 1 2 3 1 3 |, frequency vector $\mathbf{f}$:

i	1	2	3
f_i	3	1	2

- Sample F': | a | 1 3 1 |, sampled frequency vector $\mathbf{f}'$:

i	1	2	3
f'_i	2	0	1

- Stream G: | a | 3 1 3 1 1 |, frequency vector $\mathbf{g}$:

i	1	2	3
g_i	3	0	2

- Sample G': | a | 3 1 |, sampled frequency vector $\mathbf{g}'$:

i	1	2	3
g'_i	1	0	1

- $\xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \end{bmatrix} = \begin{bmatrix} -1 & +1 & -1 \end{bmatrix}$

$$X'_F = \mathbf{f}'\xi^T = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -3 \qquad X'_G = \mathbf{g}'\xi^T = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -2$$

$$X = C \cdot X'_F X'_G = C \cdot (-3)(-2) = C \cdot 6 \approx 13$$

Sketching a Sample

- Build the sketch over the non-materialized sample

$$X = C \cdot \sum_{i \in I} f_i' \xi_i \cdot \sum_{j \in I} g_j' \xi_j$$

$$E[X] = C \cdot \sum_{i \in I} E[f_i'] E[g_i']$$

$$\begin{aligned} \mathrm{Var}[X] = \quad & C^2 \cdot \left[\sum_{i \in I} E\left[f_i'^2\right] \sum_{j \in I} E\left[g_j'^2\right] + 2 \cdot \sum_{i \in I} \sum_{j \in I} E[f_i' f_j'] E[g_i' g_j'] \right. \\ & \left. -2 \cdot \sum_{i \in I} E\left[f_i'^2\right] E\left[g_i'^2\right] - \left(\sum_{i \in I} E[f_i'] E[g_i'] \right)^2 \right] \end{aligned}$$

Averaging Multiple Sketches

- Sketches share the same sample $\rightarrow$ correlation

$$\mathrm{Var}\left[\frac{1}{n}\cdot\sum_{k=1}^{n}X_k\right]=\frac{1}{n}\left[\mathrm{Var}\left[X_k\right]+(n-1)\cdot\mathrm{Cov}_{k\neq l}\left[X_k,X_l\right]\right]$$

$$\mathrm{Var}\left[\frac{1}{n}\cdot\sum_{k=1}^{n}X_k\right]=C^2\cdot\left[\sum_{i\in I}\sum_{j\in I}E\left[f_i'f_j'\right]E\left[g_i'g_j'\right]-\left(\sum_{i\in I}E\left[f_i'\right]E\left[g_i'\right]\right)^2\right.$$
$$\left.+\frac{1}{n}\left(\sum_{i\in I}E\left[f_i'^2\right]\sum_{j\in I}E\left[g_j'^2\right]+\sum_{i\in I}\sum_{j\in I}E\left[f_i'f_j'\right]E\left[g_i'g_j'\right]-2\cdot\sum_{i\in I}E\left[f_i'^2\right]E\left[g_i'^2\right]\right)\right]$$

$$\mathrm{Var}_{\text{sketch over samples}}=\mathrm{Var}_{\text{sketch}}+\mathrm{Var}_{\text{sampling}}+\mathrm{Var}_{\text{interaction}}$$

Bernoulli Sampling

- p, q are sampling probabilities in F, G

$$\text{Var}\left[\frac{1}{n}\cdot\sum_{k=1}^{n} X_k\right] =$$

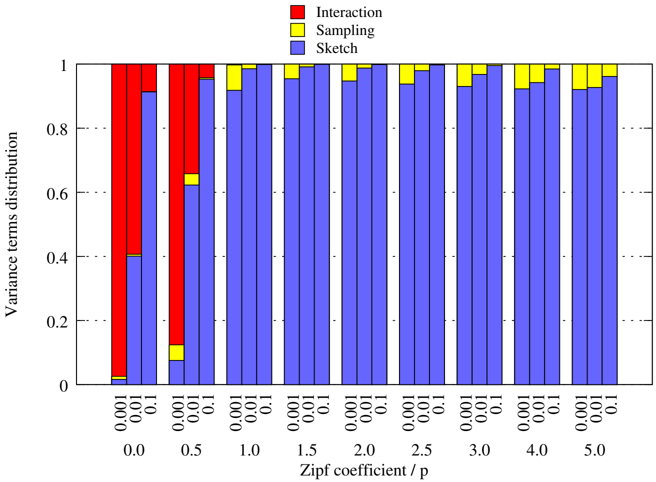
$$\frac{1}{n}\left[\sum_{i\in I} f_i^2 \sum_{j\in I} g_j^2 + \left(\sum_{i\in I} f_i g_i\right)^2 - 2\sum_{i\in I} f_i^2 g_i^2\right]$$

$$+\frac{1-p}{p}\sum_{i\in I} f_i g_i^2 + \frac{1-q}{q}\sum_{i\in I} f_i^2 g_i + \frac{(1-p)(1-q)}{pq}\sum_{i\in I} f_i g_i$$

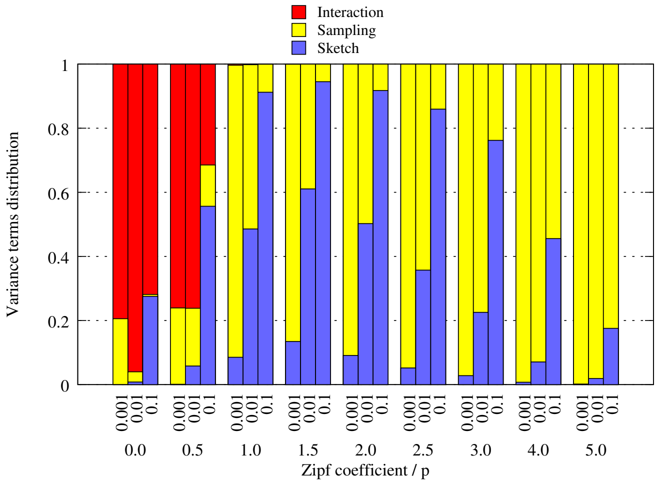
$$+\frac{1}{n}\left[\frac{1-p}{p}\sum_{i\in I}\sum_{j\in I, j\neq i} f_i g_j^2 + \frac{1-q}{q}\sum_{i\in I}\sum_{j\in I, j\neq i} f_i^2 g_j + \frac{(1-p)(1-q)}{pq}\sum_{i\in I}\sum_{j\in I, j\neq i} f_i g_j\right]$$

Variance for Bernoulli Sampling

- Term significance as a function of the frequency distribution



Size of Join

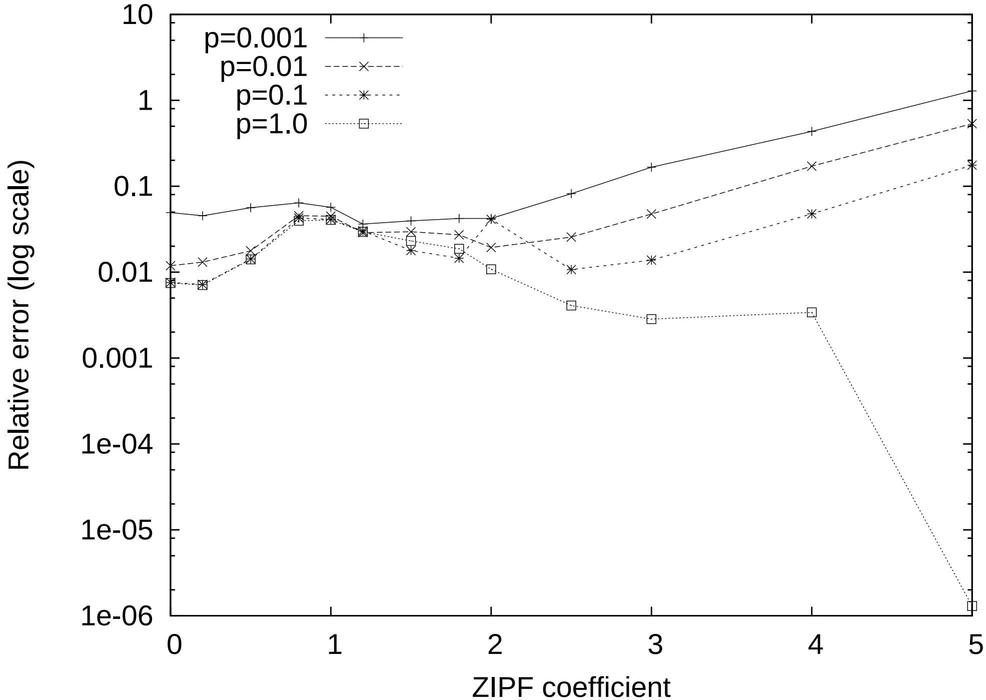


Self-Join Size

Error for Bernoulli Sampling

Settings

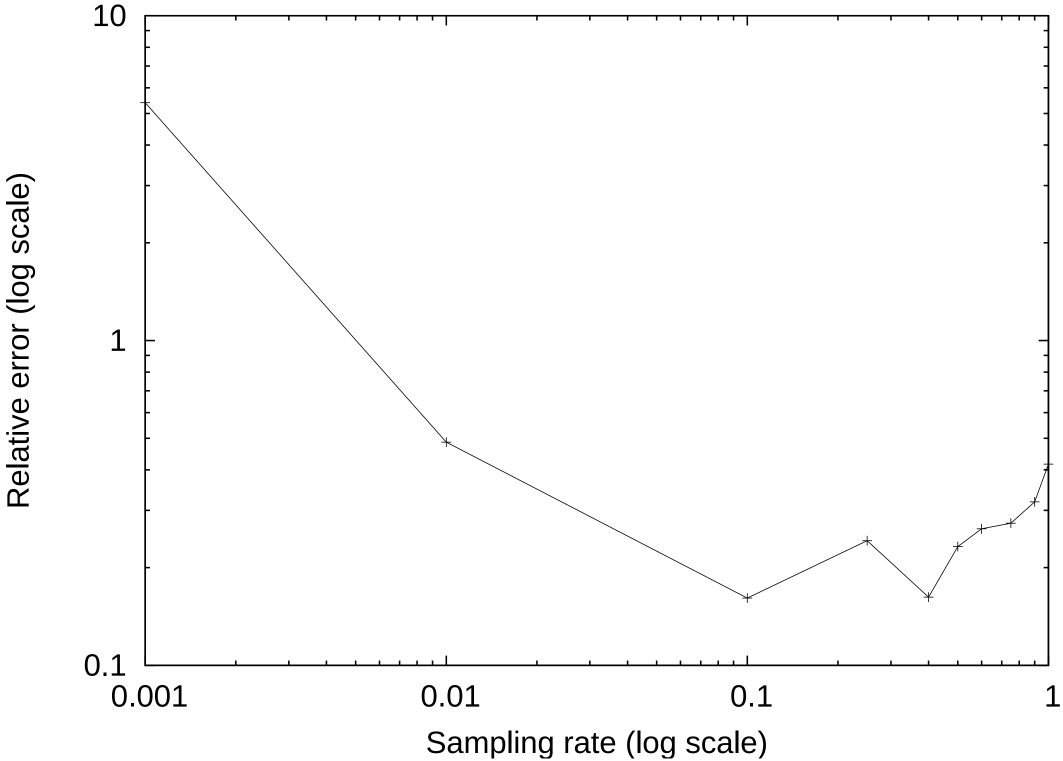
- 100 million tuples
- F-AGMS sketches with $5,000$ buckets



Error for WOR Sampling

Settings

- TPC-H scale 1, $|\text{lineitem} \bowtie_{\text{l\_orderkey} = \text{o\_orderkey}} \text{orders}|$



Conclusions

Sketches over sampled data

- Generic moment analysis
  - Sampling in frequency domain
  - Combined estimator
- Three types of sampling
  - Bernoulli
  - With replacement
  - Without replacement
- Experimental evaluation
  - 2 orders of magnitude speed-up without significant error degradation

$\Longrightarrow$ Fast-AGMS sketches with EH3 random variables over a sample

Questions