Chem 212
Problem 1. (Due Thursday, September 4, 2008)

This problem is meant to give you practice manipulating bras and kets.

A two-state system can be represented by the kets |u> and |d> depending whether the particle is in the "up" or "down" state. Assume that the |u> and |d> kets are normalized and orthogonal.

(a) If the ket for a prepared state of the particle is known to be

$$|\alpha_+> = \frac{1}{2}|u> + \frac{\sqrt{3}}{2}|d>$$

then show that $|\alpha_+>$ is normalized, and find the probability that the particle is (i) up or (ii) down.

(b) Find a ket that is orthogonal to $|\alpha_+>$ and normalize it. Call it $|\alpha_->$.

(c) Express the kets |u> and |d> in terms of the kets $|\alpha_+>$ and $|\alpha_->$.

(d) If a ket is known to be in the |u> state, then what is the probability that it is in the $|\alpha_+>$ state? The $|\alpha_->$ state?

(e) Suppose that the particle is known to be in $|\alpha_+>$ initially, and is passed through a measuring device that measures only |u> or |d>. Particles that are measured as "up" are deflected up, and those that are measured "down" are deflected down. Another measurement device follows which measures whether the state is $|\alpha_+>$ or $|\alpha_->$. The "+" response triggers and audible detector as a "click". If a beam of 1000 particles per second is sent through the first detector, what is the average rate of clicks?