FIG. 1. The process for designing a vortex of a desired shape. (a) A 3D path of the desired vortex shape is created, and (b) converted into a 3D model of a hydrofoil whose trailing edge traces this shape. Struts which connect the hydrofoil to the acceleration frame are also added. (c) The 3D model is printed with a rapid prototyping machine. Examples of hydrofoils are shown in a number of geometries and topologies. (d) The hydrofoil is accelerated in a tank of water, generating a vortex which is traced with buoyant micro-bubbles. The hydrofoil which created this vortex knot is faintly visible in the background. [URL: http://dx.doi.org/10.1063/1.4893590.1]

The life of a vortex knot

Dustin Kleckner,[a)] Martin W. Scheeler,[b)] and William T. M. Irvine[c)]
James Franck Institute, Department of Physics, The University of Chicago, Chicago, Illinois 60637, USA

(Received 9 August 2014; published online 19 September 2014)
[http://dx.doi.org/10.1063/1.4893590]

A vortex knot tied in an ideal fluid will remain knotted for all time, as first noted by Lord Kelvin.[1] This observation led to the discovery of a conserved quantity – helicity – which measures the total linking and knotting of vortices.[2] For viscous flows, however, this conservation is jeopardized by vortex reconnections, which may change the topology. Does this break helicity conservation? The divergent nature of reconnection events has complicated attempts to address this and other questions about the role of topology in real fluids. Using newly developed techniques, we have created isolated knotted vortices in the laboratory for the first time.[3] This advance enables the direct study of complex vortex geometry in experimental fluids, providing a new perspective on the role of topology in guiding fluid flow.

To generate a vortex with a prescribed shape, we start with a 3D model of a hydrofoil (a wing designed for use in water) whose shape traces the desired vortex geometry (Fig. 1). This model is fabricated with a 3D printer, and attached to a frame inserted in a tank of water. To create the vortex, the hydrofoil is rapidly accelerated (∼20 g) with a short, open pneumatic cylinder, attaining a velocity of 1–3 m/s in several milliseconds. During this acceleration, a "starting vortex" is shed from the hydrofoil as a consequence of conservation of circulation; the shape of this vortex matches the trailing edge of the hydrofoil, which can be precisely controlled.

We image the vortices with buoyant micro-bubbles, inspired by the bubble rings created by dolphins[4] (though we generate our bubbles with electrolysis, not blowholes). The centripetal acceleration near the rapidly spinning vortex cores attracts the bubbles, producing a visible line

a)Electronic mail: dkleckner@uchicago.edu
b)Electronic mail: scheeler@uchicago.edu
c)Electronic mail: wtmirvine@uchicago.edu

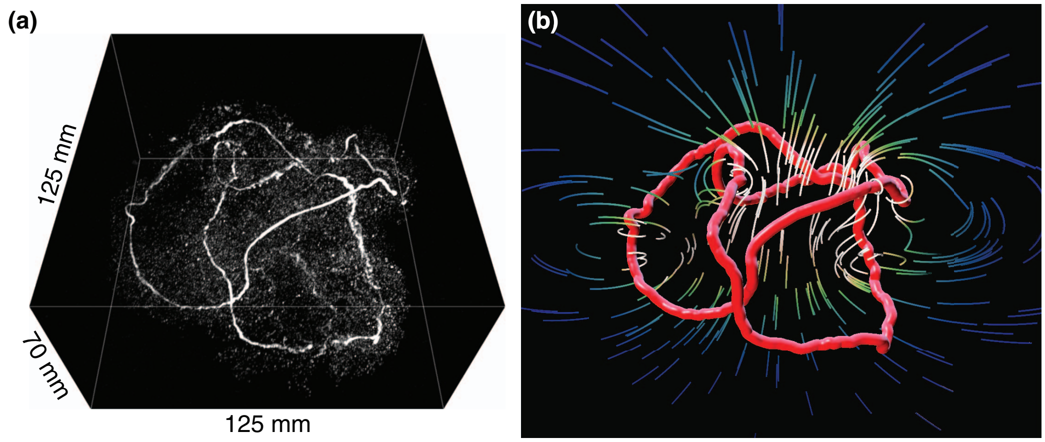

FIG. 2. (a) A volumetric image of a trefoil knot vortex, obtained with the laser scanning apparatus. The volume is taken ∼250 ms after the hydrofoil acceleration; the vortex has already begun stretching as it approaches a series of reconnection events. (b) A rendering of the reconstructed vortex path (red) overlaid with some computed stream lines (white to blue).

corresponding to the shape of the vortex (Fig. 1(d)). These 3D bubble density is reconstructed with a purpose-built laser-scanning tomography imager, consisting of a galvanometer scanned laser sheet synchronized to a high speed camera. The speed and resolution of our scanner is adjustable, limited by the data rate of the camera; using a Phantom V1610 high speed camera we are able to record volumes with a resolution of $384 \times 384 \times 384$ at 170 Hz, producing a useful data rate of ∼10 Gvoxels/s. The resulting volumetric images are viewed using custom software which corrects for perspective effects in the recorded data (Fig. 2(a)).

Once calibrated volumetric data is obtained, the geometry of the vortex lines can be reconstructed using techniques adapted from medical imaging[3] (Fig. 2(b)). Our first observations of linked and knotted vortices indicate that these topologies are intrinsically unstable; in each case they unlink or untie to form a series of topologically trivial rings. This behavior is driven by self-induced stretching of the vortex loops, which appears to be a generic feature of vortices with non-trivial topology. These results offer the first insights into the topological evolution of vortex loops in viscous fluids, but raise more questions than they answer: are all knotted vortices intrinsically unstable? Are the instability mechanisms generic to knotted fields in other physical systems? How does helicity decay in the presence of dissipation? We now have the tools to address these questions in experiment.

This work was supported by the National Science Foundation (NSF) Materials Research and Engineering Centers (MRSEC) shared facilities at the University of Chicago (DMR-0820054) and the NSF Faculty Early Career Development (CAREER) Program (DMR-1351506). W.T.M.I. further acknowledges support from the A.P. Sloan Foundation through a Sloan fellowship and the Packard Foundation through a Packard fellowship.

[1] W. Thomson, “On vortex atoms,” Philos. Mag. **XXXIV**, 94–105 (1867).
[2] H. K. Moffatt, “Degree of knottedness of tangled vortex lines,” J. Fluid Mech. **35**, 117–129 (1969).
[3] D. Kleckner and W. T. M. Irvine, “Creation and dynamics of knotted vortices,” Nature Phys. **9**, 253–258 (2013).
[4] K. Marten, K. Shariff, S. Psarakos, and D. J. White, “Ring bubbles of dolphins,” Sci. Am. **275**, 82–87 (1996).