A wave has angular frequency 30 rad/s and wavelength 2.0 m. What are its (a) wave number and (b) wave speed?

Solution

(a) The wave number is \( k = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = 3.14 \text{ m}^{-1} \).
(b) The wave speed is \( v = \lambda f = 2 \times 20 = 60 \text{ m/s} \).

The displacement of a wave traveling in the negative \( y \)-direction is

\[
D(y, t) = (5.2 \text{ cm}) \sin (5.5y + 72t),
\]

where \( y \) is in m and \( t \) is in s. What are the (a) frequency, (b) wavelength, and (c) speed of this wave?

Solution

As we’ve seen before, the general expression for a wave is

\[
D(x, t) = A \sin (kx - \omega t),
\]

where \( A \) is the amplitude, \( k = 2\pi / \lambda \) is the wave number, and \( \omega = 2\pi f \) is the angular frequency. So,

(a) The frequency is \( f = \frac{\omega}{2\pi} = \frac{72}{2\pi} = 11.5 \text{ Hz.} \)

(b) The wavelength \( \lambda = \frac{2\pi}{k} = \frac{2\pi}{5.5} = 1.14 \text{ m.} \)

(c) The wave speed is given by \( v = \lambda f = 11.5 \times 1.14 = 13 \text{ m/s.} \)
3. **Chapter 20 - Exercise 19.**
A hammer taps on the end of a 4.0 m-long metal bar at room temperature. A microphone at the other end of the bar picks up two pulses of sound, one that travels through the metal and one that travels through the air. The pulses are separated in time by 11.0 ms. What is the speed of sound in this metal?

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**Solution**

Because the sound travels faster in the metal than it does through the air, the pulse in the metal gets to the microphone first, followed by the pulse through the air. Because the speed of sound in air is 343 m/s (at room temperature), the time it takes the pulse to go the 4 meters along the metal bar is \( t = \frac{4}{343} = 11.66 \) ms. Since the pulses are separated by 11 ms, this means that the pulse in the metal was picked up \( 11.66 - 11 = 0.66 \) ms after the tap. So, since \( v = \frac{D}{t} \), this means that \( v = \frac{D}{t} = \frac{4}{0.66 \times 10^{-3}} = 6060 \) m/s. This is the speed of sound in the metal.
4. Chapter 20 - Exercise 27.

Cell phone conversations are transmitted by high-frequency radio waves. Suppose the signal has wavelength 35 cm while traveling through air. What are the (a) frequency and (b) wavelength as the signal travels through 3 mm-thick window glass into your room?

Solution

The wave is traveling at the speed of light, c. The speed of the wave is $\lambda f = c$ while in the air (air is close enough to vacuum that the speed isn’t very different). So, the frequency in the air is $f = c/\lambda = 3 \times 10^8 / .35 = 8.6 \times 10^8$ Hz. Now we need to determine the values as it passes through the window, with an index of refraction $n = 1.5$.

(a) The frequency of a wave is set by the source of the wave, and doesn’t change as it passes from one material to another. So the frequency in the glass is exactly the same as in air, $f = 8.6 \times 10^8$ Hz.

(b) The wavelength does change as the wave travels from one medium to another. Since the wave slows down as it moves through the glass, $v_{\text{glass}} = c/n = \frac{2}{3}c$, and since $\lambda f = v$, we can solve for the wavelength. This gives $\lambda_{\text{glass}} = v/f = \frac{2}{3} \frac{c}{f} = \frac{2}{3} \lambda_{\text{air}} = \frac{2}{3} \times 35 = 23$ cm.
5. **Chapter 20 - Problem 39.**
A mother hawk screeches as she dives at you. You recall from biology that female hawks screech at 800 Hz, but you hear the screech at 900 Hz. How fast is the hawk approaching?

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**Solution**

Because the hawk is approaching you, the frequency is increasing due to the Doppler effect. The frequency that we hear is

\[
 f_R = \frac{v}{v - u_s} f_S,
\]

where \( f_S \) is the frequency emitted by the hawk, \( v \) is the velocity of sound in the air (343 m/s), and \( u_s \) is the velocity of the hawk. Solving for the speed of the hawk gives

\[
 u_s = \left(1 - \frac{f_S}{f_R}\right) v = \left(1 - \frac{800}{900}\right) \times 343 = 38.3 \text{ m/s}.
\]
6. Chapter 20 - Problem 47.

Oil explorers set off explosives to make loud sounds, then listen for the echoes from underground oil deposits. Geologists suspect that there is oil under 500 m deep Lake Physics. It’s known that Lake Physics is carved out of a granite basin. Explorers detect a weak echo 0.94 s after exploding dynamite at the lake surface. If it’s really oil, how deep will they have to drill into the granite to reach it?

Solution

According to the table on page 617, the speed of sound in water is 1480 m/s while that in granite is 6000 m/s. Now the sound travels through the water a distance of 500 m, then through the granite an unknown distance, $d$. So, the total time that the wave travels is the time through the water, plus the time through the granite, then the same time back through the granite and back through the water. The time it spends in the water is just the distance, divided by the speed, and similarly for the granite. So,

$$t_{\text{echo}} = \frac{2(500)}{1480} + \frac{2d}{6000} = 0.94.$$ 

Solving for the distance gives

$$d = \frac{3000(0.94) - 1000(3000)}{1480} = 792 \text{ meters}.$$
7. Chapter 20 - Problem 51.
Earthquakes are essentially sound waves traveling through the earth. They are called seismic waves. Because the earth is solid, it can support longitudinal and transverse seismic waves. These travel at different speeds. The speed of longitudinal waves, called P waves, is 8000 m/s. Transverse waves, called S waves, travel at a slower 4500 m/s. A seismograph records the two waves from a distant earthquake. if the S wave arrives 2.0 min after the P wave, how far away was the earthquake? You can assume that the waves travel in straight lines, although actual seismic waves follow more complex routes.

Solution

The time it takes for the P wave to go the distance is \( t_P = D/v_p \), while the time for the S wave is \( t_S = D/v_s \). The difference in time is \( \Delta t = t_S - t_P = D/v_s - D/v_p = D \left( \frac{1}{v_s} - \frac{1}{v_p} \right) \), which we can rewrite as \( \Delta t = \frac{D}{v_s v_p} (v_p - v_s) \), which gives, upon solving for \( D \),

\[
D = \frac{v_s v_p}{v_p - v_s} \Delta t = \frac{8000 \times 4500}{8000 - 4500} \times 120 = 1.23 \times 10^6 \text{ m} = 1230 \text{ km}
\]

The radius of the Earth is about 6400 km, so this is a decent distance away.
8. Chapter 20 - Problem 56.
Write the displacement equation for a sinusoidal wave that is traveling in the positive \( x \) direction with frequency 200 Hz, speed 400 m/s, amplitude 0.010 mm, and phase constant \( \pi/2 \) rad.

Solution

The general expression for a wave is

\[
y(x,t) = A \sin (kx \pm \omega t + \delta),
\]

where \( A \) is the amplitude, \( k = 2\pi/\lambda \) is the wave number, \( \omega = 2\pi f \) is the angular frequency, and \( \delta \) is the phase shift. The plus sign is used for waves moving to the left, while the minus sign gives a wave moving to the right. Now, clearly \( A = 0.01 \) mm, and \( \delta = \pi/2 \). The angular frequency is \( \omega = 2\pi f = 400\pi \). The speed is \( v = \omega/k \), and so \( k = \omega/v = 400\pi/400 = \pi \). Thus, we have

\[
y(x,t) = 10^{-5} \sin \left( \pi x - 400\pi t + \frac{\pi}{2} \right).
\]
9. Chapter 20 - Problem 77.
You have just been pulled over for running a red light, and the police officer has informed you that the fine will be $250. In desperation, you suddenly recall an idea that your physics professor recently discussed in class. In your calmest voice, you tell the officer that the laws of physics prevented you from knowing that the light was red. In fact, as you drove toward it, the light was Doppler shifted to where it appeared green to you. “OK,” says the officer, “Then I’ll ticket you for speeding. The fine is $1 for every 1 km/hr over the posted speed limit of 50 km/hr.” How big is your fine? Use 650 nm as the wavelength of red light and 540 nm as the wavelength of green light.

Solution

Since we are approaching the light, the wavelengths are shortened due to the Doppler shift. The decrease in the wavelength is given by

\[
\lambda = \sqrt{\frac{c - v}{c + v}} \lambda_0 = \sqrt{\frac{1 - v/c}{1 + v/c}} \lambda_0.
\]

Once again, we can solve for the velocity as

\[
\frac{v}{c} = \frac{\lambda_0^2 - \lambda^2}{\lambda_0^2 + \lambda^2}.
\]

Taking \(\lambda_0 = 650\) nm and \(\lambda = 540\) nm, we find

\[
\frac{v}{c} = \frac{\lambda_0^2 - \lambda^2}{\lambda_0^2 + \lambda^2} = \frac{650^2 - 540^2}{650^2 + 540^2} = 0.183.
\]

Thus, the velocity is \(v = 0.183c \approx 55.5 \times 10^7\) m/s, which is about \(2 \times 10^8\) km/hr. This is \textit{well} above the posted speed limit of 50 km/hr, and so the fine would basically be $200 million!!! You’re far better off just paying the $250 fine for running the red light!
10. **Chapter 20 - Problem 81.** A rope of mass $m$ and length $L$ hangs from a ceiling.

(a) Show that the wave speed on the rope a distance $y$ above the lower end is $v = \sqrt{gy}$.
(b) Show that the time for a pulse to travel the length of the string is $\Delta t = 2\sqrt{L/g}$.

---

**Solution**

(a) At a point $y$ above the bottom end of the rope, the tension in the rope above $y$ exactly balances the weight of the rope below $y$ (the point $y$ isn’t accelerating). Assuming the rope has a uniform mass density, $\mu = m/L$, we see that the mass that hangs below the point $y$ is $M = \mu y$. Equating the forces gives $T = Mg = \mu yg$. The speed of a wave on a string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu yg}{\mu}} = \sqrt{gy},$$

and is independent of the weight of the rope!

(b) The time for the pulse to travel the length of the string can be determined by integrating the velocity, $v = \frac{dy}{dt} = \sqrt{gy}$. Separating and integrating gives

$$\int dt = \Delta t = \frac{1}{\sqrt{g}} \int_0^L \frac{dy}{\sqrt{y}} = \frac{1}{\sqrt{g}} \left[ 2\sqrt{y} \right]_0^L = 2\sqrt{\frac{L}{g}}$$

So, the time it takes is $\Delta t = 2\sqrt{\frac{L}{g}}$, as claimed.