E-3. Gauss’s Law

Questions for discussion

1. Consider a pair of point charges ±Q, fixed in place near one another as shown.

   ![Diagram of two point charges](image)

   a) On the diagram above, sketch the field created by these two point charges.

   b) Now consider an imaginary spherical surface enclosing the +Q charge:

      ![Diagram of a spherical surface](image)

      i) Reproduce here your drawing of the electric field lines from part (a), so you can get a sense of how the field lines pierce the imaginary spherical surface.

      ii) How much electric flux passes outward through the imaginary spherical surface? You should be able to arrive at the answer very quickly using Gauss’s Law.

      iii) By examining the field lines and how they pierce the imaginary spherical surface, try to explain why the flux turns out to be what Gauss’s Law said it was. (For example, try to explain why the net flux through the surface is outward.)
c) Next, consider an imaginary *ellipsoidal* surface enclosing both charges:

\[ \text{Drawing of an ellipsoidal surface} \]

\[ \text{Field lines from part (a) indicated.} \]

i) Once again, reproduce your drawing of the electric field lines from part (a), so you can get a sense of how the field lines pierce the imaginary ellipsoidal surface.

ii) How much electric flux passes outward through the imaginary ellipsoidal surface? Again, you should be able to arrive at the answer very quickly using Gauss’s Law.

iii) By examining the field lines and how they pierce the ellipsoid, try to explain *why* the flux turns out to be what Gauss’s Law said it was.

d) Finally, consider an *irregular* imaginary closed surface that winds around between the charges as shown:

\[ \text{Drawing of an irregular closed surface} \]

i) Once again, reproduce your drawing of the electric field lines from part (a), so you can get a sense of how the field lines pierce the irregular imaginary closed surface.
ii) How much electric flux passes outward through the irregular imaginary spherical surface? Again, you should be able to arrive at the answer very quickly using Gauss’s Law.

iii) By examining the field lines and how they pierce the irregular surface, try to explain why the flux turns out to be what Gauss’s Law said it was.

2. The diagram below shows a single point charge Q. Sketch the field created by Q, and find the amount of electric flux passing through the imaginary infinite plane surface.
3. A thin disk of radius $R$ has uniform surface charge density $\sigma$. Let us imagine a “cubical” surface enclosing the disk.

a) What is the electric flux passing outward through the imaginary cubical surface?

b) Can you use this result to find the electric field created by the disk? Why or why not? You may want to sketch qualitative field lines on the diagram above.

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**Differential volume elements**

d$v = \text{(surface area)} \times \text{(thickness)}$

- thin sheet: $d\nu = A \, dy$
- thin cylindrical shell: $d\nu = 2\pi r L \, dr$
- thin spherical shell: $d\nu = 4\pi r^2 \, dr$
Problems

1. Consider a long line of charge, with uniform positive charge per unit length $\lambda$.

   a) Sketch the electric field created by this charge distribution.

   The figure below shows an imaginary surface that can be used with Gauss’s Law to determine the strength of the electric field at any distance $r$ from the line charge.

   The imaginary surface is kind of like a soup can, with a label part, a lid part, and a bottom part.

   b) Is the magnitude of the electric field the same at all points of the label part of the Gaussian surface? Why or why not?

   c) What angle do the electric field vectors make with the label, at various points of the label?

   d) Is this angle the same at all points of the label?

   e) In terms of the (unknown) electric field strength $E$, how much electric flux passes through the label? Compute this directly using the flux integral

   $$\Phi_{\text{label}} = \iint_{\text{label}} \mathbf{E} \cdot d\mathbf{a}.$$  

   f) Answer parts (c) - (e) for the lid part of the Gaussian surface.

   g) Answer parts (c) - (e) for the bottom part of the Gaussian surface.

   h) What is the total flux passing outward through the closed Gaussian surface?

   i) How much charge is enclosed by the Gaussian surface?

   j) What is the electric field strength $E(r)$ at any distance $r$ from the line charge?

   k) For this derivation to work, why is it necessary that the line of charge be infinitely long - or, in practice, very long compared to $r$?

2. Imagine that a flat sheet of paper has a surface charge density $\sigma$ spread uniformly on it.

   Find the electric field at points close to the paper, away from the edges.
3. A very long tube of radius \( R \) is full of charged stuff with uniform positive charge per unit volume \( \rho \), as shown in the figure below.

a) Sketch the electric field created by this charge distribution.
b) Find the electric field outside the tube, for \( r > R \).
c) Find the electric field inside the tube, for \( r < R \).
d) Is the electric field continuous at the surface of the cylinder? That is, are the values of \( E_{\text{inside}}(r=R) \) and \( E_{\text{outside}}(r=R) \) equal?

4. A spherical shell with inner radius \( R_A \) and outer radius \( R_B \) is filled with a material with uniform charge per unit volume \( \rho_0 \). The inside of the shell is empty.

a) What is the electric field inside the shell, for \( r < R_A \)?
b) Find the electric field outside the sphere, for \( r > R_B \).
c) Find the electric field inside the shell, for \( R_A < r < R_B \).